

# Midterm

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November 21

## Problem 1 (40 points)

Consider the representative agent model with distortionary taxes discussed in class. Show that if  $\forall t$

$$\tau_{k_t} = \tau_{n_t} = \tau_t$$

$$\tau_{x_t} = \tau_{c_t} = T_t = 0$$

$$p_t g_t = \tau_t (w_t n_t + r_t k_t)$$

then the TDCE allocation solves:

$$\max_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t)$$

$$c_t + x_t \leq (1 - \tau_t) F(k_t, n_t; t)$$

$$k_{t+1} \leq (1 - \delta)k_t + x_t$$

## Problem 2 (30 points)

Consider the one sector growth model with taxes on capital income and lump sum transfers only. Suppose that  $\tau_{k_t} = \tau_k \forall t$ . Assume government consumption  $g_t$  is fixed at  $g_t = g^* \forall t$ . Assume that labor supplied inelastically.

Let  $c^*(\tau_k)$  denote the steady state level of consumption as it depends on  $\tau_k$ . Is  $\partial c^* / \partial \tau_k$  positive or negative? Prove your result.

### Problem 3 (30 points)

Consider the version of the single neoclassical growth model with exogenously growing labor augmenting technological change and linear income taxes.

That is consider the neoclassical model with taxes in which preferences are given by:

$$U(c, l) = \sum_{t=0}^{\infty} \beta^t c^{(1-\sigma)/(1-\sigma)}$$

Note that labor supply is inelastic. Normalize its quantity to 1.

Assume that technology is given by:

$$y_t = A(k_t)^\alpha (z_t n_t)^{1-\alpha}$$

where  $z_t = (1 + g)^t$ , and  $0 < \alpha < 1$ .

Assume that there is full depreciation,  $\delta = 1$ .

Assume that  $\tau_{n_t} = \tau_{k_t} = \tau$  for all  $t$  where  $0 < \tau < 1$ ,  $\tau_{c_t} = \tau_{x_t} = T_t = 0$  for all  $t$ , and  $p_t g_t = \tau(r_t k_t + w_t n_t)$  for all  $t$ .

a) Show that for all  $\tau$ ,  $y_{t+1}/y_t$  converges to a constant,  $\gamma(\tau)$ .

b) What is  $\partial\gamma(\tau)/\partial\tau$ ?