Econ 8106
Prof. L. Jones

Due date: 11/01 during lecture.

Problem 1

Consider a 2 sector-economy with consumption-goods firms indexed by $j_c = 1, 2, ..., J_c$ and investment-goods firms indexed by $j_x = 1, 2, ..., J_x$. You may assume a CRS technology for each firm in each sector. There are I consumers indexed by $i = 1, 2, ..., I$ and preferences are $U^i(\underline{c}, \underline{l}) = \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i)$.

(a) Define an Arrow-Debreu equilibrium.

(b) Suppose the solution to the consumer's problem is interior, that is, all quantities at all times are strictly positive. Derive a condition relating the price of investment goods and the rental rate of capital. State briefly in words what this means.

(c) Consider the following claim: consumers can not make positive profits by investing. In order to agree with the claim show that in equilibrium the constraints of the consumer's problem can be re-written so that only the initial endowment of capital enters the budget constraint.

(d) Write down the necessary first-order conditions that a solution to each firm's problem must satisfy. Derive a condition relating the price of consumption goods and the price of investment goods.

(e) Define a Sequential Market Equilibrium.

(f) Set up the Social Planner's problem for this economy.

(g) Consider any Arrow-Debreu equilibrium. Construct prices, interest rates, and borrowing/lending amounts so that the allocation of the Arrow-Debreu equilibrium is part of the allocation of a Sequential Markets Equilibrium at those prices.

(h) Consider any Sequential Markets Equilibrium. Construct prices for the Arrow-Debreu environment of the economy so that the allocation of the Sequential Markets Equilibrium (without borring-lending holdings) is the allocation of an Arrow-Debreu equilibrium at those prices.

Problem 2

Consider the same economy like in question 1. Show that the Competitive Equilibrium (CE) is "homogeneous of degree zero in population". That is, consider a new economy with 2I consumers, 2 exactly like each of the agents in the original economy. Assuming that all production functions are CRS, show that the original equilibrium prices and allocations are still an equilibrium.

Problem 3

Consider an economy like in question 1. Assume the following: all firms are identical within and across sectors and the technology is CRS; all consumers have the same endowment of capital and time; all consumers have the same preferences, that is, $u^i = u \ \forall i = 1, 2, .., I$ with u strictly concave.

(a) State clearly and prove a theorem which says that the CE of this economy is the same as one with one firm and one consumer.

(b) State the Planner's Problem for the economy with one consumer and one firm. Show that the CE solves this Planner's Problem.

Problem 4

Consider an economy like in question 1. Assume the following: all firms are identical within each sector and the technology is CRS; technologies across sectors are different, but, for a given sequence of positive numbers $\{b_t\}_{t=0}^{\infty}$, $F_{x_t} = b_t F_{c_t} \;\forall\; t$; all consumers have the same endowment of capital and time; all consumers have the same preferences, that is, $u^i = u \;\; \forall i = 1, 2, .., I$ with u strictly concave.

State clearly and prove the theorem which says that the CE of this economy is the same as one with one firm in each sector and one consumer. Show the relation between prices of consumption and investment goods.

Problem 5

Consider an economy with agents of type $i = 1, 2$. Agents of type 1 are indexed by $i \in I_1$ and agents of type 2 are indexed by $i \in I_2$. Both sets are finite. There are L different goods and each agent receives an endowment $w^i \in \Re_{++}^L$. Assume that preferences are represented by $u^i : \Re_+^L \to \Re$ and u^i is str. concave, str. increasing and homothetic (that is, $u^i(x) = u^i(y) \to u^i(\lambda x) = u^i(\lambda y) \;\; \forall x, y \in \Re_+^L, \forall \lambda \geq 0$).

(a) Define an equilibrium for this economy.

(b) Taking $p \in \Re_{++}^L$ as given, show that the demand function is homogenous of degree one in the value of the endowment $pw^i \in \Re_{++}^L$.
Also show that $\sum_{i \in I_i} x^i(p, pw^i) = x^i(p, \sum_{i \in I_i} pw^i)$, where $x^i(p, pw^i)$ is the demand function at price p and wealth pw^i.

(c) State and prove a theorem which says that an equilibrium in part (a) can be re-written as an equilibrium in an economy with two agents.

Problem 6

Consider an economy with one firm producing consumption goods and investment goods using a CRS technology. In this economy there are I consumers with preferences over streams of consumption and leisure given by $U^i(\underline{\boldsymbol{c}}, \underline{\boldsymbol{l}})$ with U^i homothetic, str. concave and str. increasing. Each consumer i receives endowment of capital and time given by $w^i = [k_0^i, \{\overline{\boldsymbol{n}}_t^i\}_{t=0}^{\infty}]$ where w^i may be different from $w^{i'}$.

(a) Show that in equilibrium the budget constraint for each consumer can be re-written such that the expenditure on consumption goods and leisure is equal the value of his endowment.

(b) Prove the following theorem:
Let $\{p_t^*, r_t^*, w_t^*\}_{t=0}^{\infty}$, $\{c_t^i, x_t^i, k_t^i, n_t^i, l_t^i\}_{t=0}^{\infty}$ and $\{c_t^f, x_t^f, k_t^f, n_t^f\}_{t=0}^{\infty}$ be an CE in the above economy with I consumers.
Then $\{p_t^*, r_t^*, w_t^*\}_{t=0}^{\infty}$, $\{\sum_i c_t^i, \sum_i x_t^i, \sum_i k_t^i, \sum_i n_t^i, \sum_i l_t^i\}_{t=0}^{\infty}$, and $\{c_t^f, x_t^f, k_t^f, n_t^f\}_{t=0}^{\infty}$ is a CE for the economy that has one consumer with endowment $w = [\sum_i k_0^i, \{\sum_i \overline{\boldsymbol{n}}_t^i\}_{t=0}^{\infty}]$.