Economic Models of Knowledge: Part III, Data Tables

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Abstract

1 Introduction

This part of the notes collects the Tables for each of the models in Parts I and II that have some success so that they can be compared with data on actual cross country (and time) differences.

They are in the same order as what appeared in Parts I and II of the notes. One thing that should be emphasized is that it is not enough to have a model. One also must have a source within the model for sufficient cross country heterogeneity if one hopes to understand cross country differences in the data. For this reason, most of the models have several tables each, one for each of the possible sources of heterogeneity.

You should be aware that these are woefully imperfect, and undoubtedly contain many errors!

2 The Cross Country Data Again

The idea here is to collect a standardized set of facts against which all models can be compared.

What I would like is a Table Something like the following one:

Models: $\to\to\to$	Cass-Koopmans	Ak	$A(k,h)$	Romer	Lucas	Other?
Country Heterogeneity $\downarrow\downarrow$						
Initial Conditions						
Technology						
Preferences						
Policy: Taxes						
Policy: Spending						
Policy: Inflation						

Where in each entry of the Table, there is a further Table which looks like:

	Data	Model
$\gamma =?$		
$\frac{y_t^i}{y_t^j} =?$		
$\frac{wn}{y} =?$		
$TFP_t^i =?$		
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$		
y_{1960}^i vs. $R_t^i =?$		
$\frac{x}{y}$ vs. $y =?$		
$\frac{x}{y}$ vs. $\gamma =?$		
$Educ.$ vs. $y =?$		
γ_n vs. $y =?$		
γ_n vs. $\gamma =?$		

This list is VERY incomplete, and just includes some of the things that I, personally find interesting, and some things that everyone seems to include when they do empirical studies of growth and development on cross sectional, country, data.

2.1 Data Table

I think that for the Data version, we would have:

	Data	Model
$\gamma =?$	1.02 varied	
$\frac{y_t^i}{y_t^j} =?$	1 to 50	
$\frac{wn}{y} =?$	$\approx 0.67?$	
$TFP_t^i =?$	varied	
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	

3 Heterogeneity in Cass-Koopmans

Here we study what the data would like if the economic data for each country in the world was a Cass-Koopmans model. Of course, since different countries have different data, the particular versions of the Cass-Koopmans model will have to be different for each country. And what the data will look like will naturally depend on just exactly how that heterogeneity is introduced.

3.1 Initial Condition Heterogeneity in C-K: k_{0i}

	Data	Model
$\gamma =?$	1.02 varied	$(1+g)$ by assumption
$\frac{y_t^i}{y_t^j} =?$	1 to 50	convergence implies small if t is large
$\frac{wn}{y} =?$	$\approx 0.67?$	$\approx 0.67?$
$TFP_t^i =?$	varied	no differences
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	decreasing relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	no relationship?
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	?
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	?
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

3.2 Cass-Koopmans A^i Differences

	Data	Model
$\gamma =?$	1.02 varied	$(1+g)$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\left[\frac{y_t^i}{y_t^j}\right]^{\frac{1-\alpha}{\alpha}} = \frac{A_0^i}{A_0^j}$
$\frac{wn}{y} =?$	$\approx 0.67?$	$\approx 0.67?$
$TFP_t^i =?$	varied	$TFP_t^i : \left[\frac{y_t^i}{y_t^j}\right]^2$
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	no relationship
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	?
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	?
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

3.3 Differences in δ_k in C-K

	Data	Model
$\gamma =?$	1.02 varied	$(1+g)$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	?
$\frac{wn}{y} =?$	$\approx 0.67?$	$\approx 0.67?$
$TFP_t^i =?$	varied	?
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	no relationship?
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	?
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	?
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

3.4 Cass-Koopmans β differences

	Data	Model
$\gamma =?$	1.02 varied	$(1+g)$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} = \left[\frac{\beta^i}{\beta^j}\right]^{0.5}$
$\frac{wn}{y} =?$	≈ 0.67?	≈ 0.67?
$TFP_t^i =?$	varied	equal in all countries
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	$1+R_t^i = \frac{1}{\beta^i}(1+g)^\sigma$
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0$?	?
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0$?	?
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0$?	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0$?	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0$?	not defined

3.5 Cass-Koopmans σ Differences

	Data	Model
$\gamma =?$	1.02 varied	$(1+g)$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	?
$\frac{wn}{y} =?$	≈ 0.67?	≈ 0.67?
$TFP_t^i =?$	varied	?
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	no relationship?
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0$?	?
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0$?	?
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0$?	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0$?	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0$?	not defined

3.6 Cass-Koopmans Tax Policy Differences

	Data	Model
$\gamma =?$	1.02 varied	$(1+g)$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	?
$\frac{wn}{y} =?$	$\approx 0.67?$	$\approx 0.67?$
$TFP_t^i =?$	varied	?
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	no relationship?
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	?
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	?
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

3.7 Cass-Koopmans Government Spending Differences

	Data	Model
$\gamma =?$	1.02 varied	$(1+g)$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	?
$\frac{wn}{y} =?$	$\approx 0.67?$	$\approx 0.67?$
$TFP_t^i =?$	varied	?
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	no relationship?
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	?
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	?
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

3.8 Cass-Koopmans Monetary Policy Differences

	Data	Model
$\gamma =?$	1.02 varied	$(1+g)$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	?
$\frac{wn}{y} =?$	$\approx 0.67?$	$\approx 0.67?$
$TFP_t^i =?$	varied	?
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	no relationship?
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	?
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	?
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

3.9 Summary

Like all models, there are some successes and failures here. And some things that the modelling just gives up on from the start (e.g., Education and Income, or Population and Income). This is not a weakness. Models are meant to be abstractions, and hence, necessarily they will not be perfect. If they were, they would have to be WAY too complicated to get anything useful out of them!

I think that the biggest weaknesses is the difficulty that the model(s) has (have) in generating sufficiently large differences in income per capita across countries. Initial conditions seem to require outrageous differences in k and R. But, absent this, it only gives A_t or TFP differences as the way to go. These need to be large, and in addition, no one knows what it is. Thus, income differences are 'explained' as differences in the 'unexplained residual.' Not a very satisfactory set of affairs!

This is why people who use the single sector growth model are naturally led to cry out for 'theories of TFP.' Although it's not exactly clear what this would mean.

In the next sections of these notes, we'll try and do something like that... they can all be thought of as 'endogenizing' the technological parameters of the single sector model, although they do it in VERY different ways!

4 Convex Models of Endogenous Growth: The Ak Model

In this version of the models, we identify A from the previous discussion with k in the math. I.e., $k = knowledge$.

4.1 Differences in k_{0i} in the Ak Model

	Data	Model
$\gamma = ?$	1.02 varied	$\gamma^{\sigma} = \beta\,[1 - \delta_k + A]$
$\frac{y_t^i}{y_t^j} = ?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{k_0^i}{k_0^j}$
$\frac{wn}{y} = ?$	$\approx 0.67?$	not defined
$TFP_t^i = ?$	varied	not defined
y_{1960}^i vs. $\bar{\gamma}_{60-95} = ?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i = ?$	no relationship? (risk)	no relationship
$\frac{x}{y}$ vs. $y = ?$	$corr(\frac{x}{y}, y) > 0?$	no relationship
$\frac{x}{y}$ vs. $\gamma = ?$	$corr(\frac{x}{y}, \gamma) > 0?$	all countries have the same $\frac{x}{y}$ and γ
$Educ.$ vs. $y = ?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y = ?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma = ?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.2 Ak: A^i Differences

	Data	Model
$\gamma = ?$	1.02 varied	$\gamma^{i\sigma} = \beta\,[1 - \delta_k + A^i]$
$\frac{y_t^i}{y_t^j} = ?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \rightarrow \infty$
$\frac{wn}{y} = ?$	$\approx 0.67?$	not defined
$TFP_t^i = ?$	varied	not defined
y_{1960}^i vs. $\bar{\gamma}_{60-95} = ?$	no relationship	high $y_{60}^i \Longrightarrow$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i = ?$	no relationship? (risk)	high $y_{60}^i \Longrightarrow$ high R_t^i
$\frac{x}{y}$ vs. $y = ?$	$corr(\frac{x}{y}, y) > 0?$	$corr(\frac{x}{y}, y) > 0$
$\frac{x}{y}$ vs. $\gamma = ?$	$corr(\frac{x}{y}, \gamma) > 0?$	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y = ?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y = ?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma = ?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.3 Ak: δ_k differences

	Data	Model
$\gamma = ?$	1.02 varied	$\gamma^{i\sigma} = \beta\left[1 - \delta_k^i + A\right]$
$\frac{y_t^i}{y_t^j} = ?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} = ?$	$\approx 0.67?$	not defined
$TFP_t^i = ?$	varied	not defined
y_{1960}^i vs. $\bar{\gamma}_{60-95} = ?$	no relationship	high $y_{60}^i \Longrightarrow$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i = ?$	no relationship? (risk)	high $y_{60}^i \Longrightarrow$ high R_t^i
$\frac{x}{y}$ vs. $y = ?$	$corr(\frac{x}{y}, y) > 0?$	$corr(\frac{x}{y}, y) > 0$
$\frac{x}{y}$ vs. $\gamma = ?$	$corr(\frac{x}{y}, \gamma) > 0?$	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y = ?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y = ?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma = ?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.4 Differences in β in the Ak Model

	Data	Model
$\gamma = ?$	1.02 varied	$\gamma^{i\sigma} = \beta^i\left[1 - \delta_k + A\right]$
$\frac{y_t^i}{y_t^j} = ?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} = ?$	$\approx 0.67?$	not defined
$TFP_t^i = ?$	varied	not defined
y_{1960}^i vs. $\bar{\gamma}_{60-95} = ?$	no relationship	high $y_{60}^i \Longrightarrow$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i = ?$	no relationship? (risk)	high $y_{60}^i \Longrightarrow$ high R_t^i
$\frac{x}{y}$ vs. $y = ?$	$corr(\frac{x}{y}, y) > 0?$	$corr(\frac{x}{y}, y) > 0$
$\frac{x}{y}$ vs. $\gamma = ?$	$corr(\frac{x}{y}, \gamma) > 0?$	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y = ?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y = ?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma = ?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.5 Ak: σ Differences

	Data	Model
$\gamma =?$	1.02 varied	$\gamma^{i\sigma_i} = \beta\left[1-\delta_k + A\right]$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} : \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} =?$	$\approx 0.67?$	not defined
$TFP_t^i =?$	varied	not defined
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	high $y_{60}^i \Longrightarrow$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	high $y_{60}^i \Longrightarrow$ high R_t^i
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	$corr(\frac{x}{y}, y) > 0$
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.6 Differences in τ in the Ak Model

	Data	Model
$\gamma =?$	1.02 varied	$\gamma_i^\sigma = \beta\left[1-\delta_k + (1-\tau_i)A\right]$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} : \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} =?$	$\approx 0.67?$	not defined
$TFP_t^i =?$	varied	not defined
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	high $y_{60}^i \Longrightarrow$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	high $y_{60}^i \Longrightarrow$ high R_t^i
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	$corr(\frac{x}{y}, y) > 0$
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.6.1 Ak: Government Spending Differences

	Data	Model
$\gamma =?$	1.02 varied	
$\frac{y_t^i}{y_t^j} =?$	1 to 50	
$\frac{wn}{y} =?$	$\approx 0.67?$	
$TFP_t^i =?$	varied	
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	

4.7 Ak: Monetary Policy Differences

Cash/Credit model

	Data	Model
$\gamma =?$	1.02 varied	$\gamma_i^\sigma = \beta [1 - \delta_k + (1-\tau)A]$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	?
$\frac{wn}{y} =?$	$\approx 0.67?$	not defined
$TFP_t^i =?$	varied	not defined
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	?
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	?
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	?
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	?
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.8 $A(k,h)$: Initial Conditions

	Data	Model
$\gamma =?$	1.02 varied	$\gamma = [\beta(1-\delta+A\alpha^{\alpha}(1-\alpha)^{1-\alpha})]^{1/\sigma}$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{k_0^i}{k_0^j}$
$\frac{wn}{y} =?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i =?$	varied	$TFP_t^i = \left[\frac{z_t^i}{n_t^i}\right]^{1-\alpha} = [h_t^i]^{1-\alpha}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	$1+R_t^i = (1-\delta+A\alpha^{\alpha}(1-\alpha)^{1-\alpha})$
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y},y) > 0?$	$corr(\frac{x}{y},y) = 0$
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y},\gamma) > 0?$	all countries have the same $\frac{x}{y}$ and γ
$Educ.$ vs. $y =?$	$corr(Ed,y) > 0?$	$corr(h,y) > 0$
γ_n vs. $y =?$	$corr(\gamma_n,y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n,\gamma) < 0?$	not defined

4.9 $A(k,h)$: A^i Differences

	Data	Model
$\gamma =?$	1.02 varied	$\gamma_i = [\beta(1-\delta+A_i\alpha^{\alpha}(1-\alpha)^{1-\alpha})]^{1/\sigma}$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} =?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i =?$	varied	$TFP_t^i = \left[\frac{z_t^i}{n_t^i}\right]^{1-\alpha} = [h_t^i]^{1-\alpha}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	high $y_{60}^i \Longrightarrow$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	$1+R_t^i = (1-\delta+A_i\alpha^{\alpha}(1-\alpha)^{1-\alpha})$
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y},y) > 0?$	$corr(\frac{x}{y},y) > 0$
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y},\gamma) > 0?$	$corr(\frac{x}{y},\gamma) > 0$
$Educ.$ vs. $y =?$	$corr(Ed,y) > 0?$	$corr(h,y) > 0$
γ_n vs. $y =?$	$corr(\gamma_n,y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n,\gamma) < 0?$	not defined

4.9.1 $A(k,h)$: δ_k Differences

	Data	Model
$\gamma =?$	1.02 varied	$\gamma_i = [\beta(1-\delta_i + A\alpha^{\alpha}(1-\alpha)^{1-\alpha})]^{1/\sigma}$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{\gamma_i^t}{\gamma_i^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} =?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i =?$	varied	$TFP_t^i = \left[\frac{z_t^i}{n_t^i}\right]^{1-\alpha} = [h_t^i]^{1-\alpha}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	high $y_{60}^i \implies$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	$1 + R_t^i = (1-\delta_i + A\alpha^{\alpha}(1-\alpha)^{1-\alpha})$
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	$corr(\frac{x}{y}, y) > 0$
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	$corr(h, y) > 0$
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.10 $A(k,h)$: β Differences

	Data	Model
$\gamma =?$	1.02 varied	$\gamma_i = [\beta_i(1-\delta + A\alpha^{\alpha}(1-\alpha)^{1-\alpha})]^{1/\sigma}$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} =?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i =?$	varied	$TFP_t^i = \left[\frac{z_t^i}{n_t^i}\right]^{1-\alpha} = [h_t^i]^{1-\alpha}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	high $y_{60}^i \implies$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	$1 + R_t^i = (1-\delta + A\alpha^{\alpha}(1-\alpha)^{1-\alpha})$
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	$corr(\frac{x}{y}, y) > 0$
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	$corr(h, y) > 0$
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.11 $A(k,h)$: σ Differences

	Data	Model
$\gamma = ?$	1.02 varied	$\gamma = [\beta(1-\delta + A\alpha^{\alpha}(1-\alpha)^{1-\alpha})]^{1/\sigma_i}$
$\frac{y_t^i}{y_t^j} = ?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} = ?$	≈ 0.67?	≈ 0.67
$TFP_t^i = ?$	varied	$TFP_t^i = \left[\frac{z_t^i}{n_t^i}\right]^{1-\alpha} = [h_t^i]^{1-\alpha}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} = ?$	no relationship	high $y_{60}^i \Longrightarrow$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i = ?$	no relationship? (risk)	$1 + R_t^i = (1-\delta + A\alpha^{\alpha}(1-\alpha)^{1-\alpha})$
$\frac{x}{y}$ vs. $y = ?$	$corr(\frac{x}{y}, y) > 0$?	$corr(\frac{x}{y}, y) > 0$
$\frac{x}{y}$ vs. $\gamma = ?$	$corr(\frac{x}{y}, \gamma) > 0$?	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y = ?$	$corr(Ed, y) > 0$?	$corr(h, y) > 0$
γ_n vs. $y = ?$	$corr(\gamma_n, y) < 0$?	not defined
γ_n vs. $\gamma = ?$	$corr(\gamma_n, \gamma) < 0$?	not defined

4.12 $A(k,h)$: Tax Policy Differences

$\tau_k = \tau_h = \tau$

	Data	Model
$\gamma = ?$	1.02 varied	$\gamma_i = [\beta(1-\delta + (1-\tau_i)A\alpha^{\alpha}(1-\alpha)^{1-\alpha})]^{1/\sigma_i}$
$\frac{y_t^i}{y_t^j} = ?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} = ?$	≈ 0.67?	≈ 0.67
$TFP_t^i = ?$	varied	$TFP_t^i = \left[\frac{z_t^i}{n_t^i}\right]^{1-\alpha} = [h_t^i]^{1-\alpha}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} = ?$	no relationship	high $y_{60}^i \Longrightarrow$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i = ?$	no relationship? (risk)	$1 + R_t^i = (1-\delta + (1-\tau_i)A\alpha^{\alpha}(1-\alpha)^{1-\alpha})$
$\frac{x}{y}$ vs. $y = ?$	$corr(\frac{x}{y}, y) > 0$?	$corr(\frac{x}{y}, y) > 0$
$\frac{x}{y}$ vs. $\gamma = ?$	$corr(\frac{x}{y}, \gamma) > 0$?	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y = ?$	$corr(Ed, y) > 0$?	$corr(h, y) > 0$
γ_n vs. $y = ?$	$corr(\gamma_n, y) < 0$?	not defined
γ_n vs. $\gamma = ?$	$corr(\gamma_n, \gamma) < 0$?	not defined

4.13 $A(k,h)$: Government Spending Differences

	Data	Model
$\gamma =?$	1.02 varied	$\gamma_i = \left[\beta\left[1-\delta_k+\left[\frac{1}{(1+\tau_{xhi})}\right]^{1-\alpha} A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right]\right]^{1/}$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{\gamma_i^t}{\gamma_j^t}\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} =?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i =?$	varied	$TFP_t^i = \left[\frac{z_t^i}{n_t^i}\right]^{1-\alpha} = [h_t^i]^{1-\alpha}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	high $y_{60}^i \Longrightarrow$ high $\bar{\gamma}_{60-05}$
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	$1+R_t^i = \left[1-\delta_k+\left[\frac{1}{(1+\tau_{xhi})}\right]^{1-\alpha} A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right]$
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	$corr(\frac{x}{y}, y) > 0?$
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	$corr(\frac{x}{y}, \gamma) > 0$
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	$corr(h, y) > 0$
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

4.14 The Effects of Monetary Policy in the $A(k,h)$ model

4.15 Two Sector Model: Initial Condition Heterogeneity

4.16 Two Sector Model: Preference Heterogeneity

4.17 Two Sector Model: Production Function Heterogeneity

4.18 Two Sector Model: Policy Heterogeneity

5 Heterogeneity Across Countries in the Romer Model with $\alpha + \eta = 1$

5.1 Romer Model, $\alpha + \eta = 1$, Equilibrium: Initial Conditions

Thus, this looks a lot like that $A(k,h)$ model.

	Data	Model
$\gamma =?$	1.02 varied	$\gamma^{\sigma} = \beta\left[1 - \delta_k + \alpha A\right]$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{k_0^i}{k_0^j}$
$\frac{wn}{y} =?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i =?$	varied	$A\left(k_t^i\right)^{.67}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	no relationship
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	no relationship
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	all countries have the same $\frac{x}{y}$ and γ
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

5.2 Romer Model, $\alpha + \eta = 1$, Planner: Initial Conditions

It's not clear exactly how you'd measure TFP here. I guess in exactly the same way as it was done above?

Beyond this, it looks just like an Ak model.

	Data	Model
$\gamma =?$	1.02 varied	$\gamma^{\sigma} = \beta\left[1 - \delta_k + A\right]$
$\frac{y_t^i}{y_t^j} =?$	1 to 50	$\frac{y_t^i}{y_t^j} = \frac{k_0^i}{k_0^j}$
$\frac{wn}{y} =?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i =?$	varied	$A\left(k_t^i\right)^{.67}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} =?$	no relationship	no relationship
y_{1960}^i vs. $R_t^i =?$	no relationship? (risk)	no relationship
$\frac{x}{y}$ vs. $y =?$	$corr(\frac{x}{y}, y) > 0?$	no relationship
$\frac{x}{y}$ vs. $\gamma =?$	$corr(\frac{x}{y}, \gamma) > 0?$	all countries have the same $\frac{x}{y}$ and γ
$Educ.$ vs. $y =?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y =?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma =?$	$corr(\gamma_n, \gamma) < 0?$	not defined

5.3 Romer Model, $\alpha + \eta = 1$: 'Centralization Differences'

Suppose that some of the countries in the world are following the equilibrium version of the model, but that others are following the Planners version? This is a kind of 'policy' difference (I guess).

	Data	Model
$\gamma = ?$	1.02 varied	$\gamma^\sigma = \beta\left[1 - \delta_k + A\right]$ or $\gamma^\sigma = \beta\left[1 - \delta_k + \alpha A\right]$
$\frac{y_t^i}{y_t^j} = ?$	1 to 50	$\frac{y_t^{centi}}{y_t^{decent}} \to \infty$
$\frac{wn}{y} = ?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i = ?$	varied	$A\left(k_t^i\right)^{.67}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} = ?$	no relationship	positive relationship
y_{1960}^i vs. $R_t^i = ?$	no relationship? (risk)	positive relationship
$\frac{x}{y}$ vs. $y = ?$	$corr(\frac{x}{y}, y) > 0?$	positive relationship
$\frac{x}{y}$ vs. $\gamma = ?$	$corr(\frac{x}{y}, \gamma) > 0?$	positive relationship
$Educ.$ vs. $y = ?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y = ?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma = ?$	$corr(\gamma_n, \gamma) < 0?$	not defined

5.4 Romer Model, Planner's Problem: $\alpha + \eta > 1$, log utility, $\delta_k = 1$, Initial Condition Heterogeneity

Here the world splits according to what the initial condition is. I.e.,

$$k_0 = k_{ss}, \text{ vs. } k_0 < k_{ss}, \text{ vs. } k_0 > k_{ss}.$$

	Data	Model
$\gamma = ?$	1.02 varied	$\gamma =$constant, $\to 0, \to \infty$
$\frac{y_t^i}{y_t^j} = ?$	1 to 50	$\frac{y_t^{hi}}{y_t^{low}} \to \infty$
$\frac{wn}{y} = ?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i = ?$	varied	$TFP_t^i = A\left(k_t^i\right)^{\alpha+\eta-.33}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} = ?$	no relationship	positive relationship
y_{1960}^i vs. $R_t^i = ?$	no relationship? (risk)	positive relationship
$\frac{x}{y}$ vs. $y = ?$	$corr(\frac{x}{y}, y) > 0?$	positive relationship
$\frac{x}{y}$ vs. $\gamma = ?$	$corr(\frac{x}{y}, \gamma) > 0?$	positive relationship
$Educ.$ vs. $y = ?$	$corr(Ed, y) > 0?$	not defined
γ_n vs. $y = ?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma = ?$	$corr(\gamma_n, \gamma) < 0?$	not defined

6 Heterogeneity in the Barro Model

6.1 Barro Model, $\alpha + \eta = 1$: Different Tax Rates

High taxes are here associated with HIGH growth rates, at least up to a point (given by $1 - \alpha$) and LOW growth rates beyond that. Assuming we are always on the left hand side of this relationship, we have:

	Data	Model
$\gamma = ?$	1.02 varied	$\gamma^\sigma = \beta\left[1 - \delta_k + \alpha A(1-\tau)\tau^{(1-\alpha)/\alpha}\gamma^{(\alpha-1)/\alpha}\right]$
$\frac{y_t^i}{y_t^j} = ?$	1 to 50	$\frac{y_t^i}{y_t^j} \to \infty$
$\frac{wn}{y} = ?$	$\approx 0.67?$	≈ 0.67
$TFP_t^i = ?$	varied	$A\left[G_t^i\right]^{.67}$
y_{1960}^i vs. $\bar{\gamma}_{60-95} = ?$	no relationship	increasing relationship
y_{1960}^i vs. $R_t^i = ?$	no relationship? (risk)	increasing relationship
$\frac{x}{y}$ vs. $y = ?$	$corr(\frac{x}{y}, y) > 0?$	increasing relationship
$\frac{x}{y}$ vs. $\gamma = ?$	$corr(\frac{x}{y}, \gamma) > 0?$	increasing relationship
$Educ.$ vs. $y = ?$	$corr(Ed, y) > 0?$	is this G? if yes, increasing relationship
γ_n vs. $y = ?$	$corr(\gamma_n, y) < 0?$	not defined
γ_n vs. $\gamma = ?$	$corr(\gamma_n, \gamma) < 0?$	not defined

Again, note that these are all flipped if $\tau \in [1-\alpha, 1]$ which would require a pretty high tax rate.

6.2 Barro Model, $\alpha + \eta = 1$: Different Levels of Centralization