Class Notes, Econ 8801
Lump Sum Taxes are Awesome

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1 Exchange Economies with Taxes and Spending

1.1 Basics

1) Assume that there are n goods which can be consumed in any non-negative amounts;

    2) I consumers indexed by i;

    3) $e_i \in R_+^n$ is $i's$ endowment;

    4) x_i is the consumption of i;

    5) $g \in R_+^n$ is government provided goods;

    6) $U_i = U_i(x_i, g) : R_+^{2n} \to R$ is $i's$ utility function.

Allocation: $(x_1, ..., x_I; g) \in R_+^{In+n}$

Feasibility: $g + \sum_i x_i = \sum_i e_i$

Conditional Pareto Optimality (given g, $x_i's$ are optimal): A feasible allocation is CPO if there is not another feasible allocation, $(x_1', ..., x_I'; g) \in R_+^{In+n}$ (note that g has not been changed) with $U_i(x_i', g) \geq U_i(x_i, g)$ for all i and strict for at least one i.

Pareto Optimality (both the choice of g and the distribution of the $x_i's$ are optimal): A feasible allocation is PO if there is not another feasible allocation $(x_1', ..., x_I'; g') \in R_+^{In+n}$ (note that g has been changed to g') with $U_i(x_i', g') \geq U_i(x_i, g)$ for all i and strict for at least one i.

Lemma 1 *If $(x_1, ..., x_I; g)$ is P.O., then $(x_1, ..., x_I)$ is C.P.O. given g.*

proof: Obvious.

1.2 Implementation

How can CPO and PO allocations be 'implemented'? I.e., how can they be reached via 'decentralized' means?

One obvious way is to just confiscate and redistribute resources:

Lemma 2 *If the government can confiscate and redistribute resources in any way, any CPO can be implemented in this way, and there will be no incentives for the private agents to trade after that.*

Implementation with transfers only:

1.2.1 Implementation with Linear Taxes

General Implementation: Given $\tau'_n s$, and a g a TDCE 'implements' its equilibrium allocation –

You can tax all consumption goods?

Consumer's Problem becomes:

$$Max \quad U_i(\cdot)$$
$$\text{s.t.} \quad \sum_n p_n(1 + \tau_n)c_n \leq \sum_n p_n e_{in}$$

So, the TDCE is $p \in R^n_+$, an allocation, $(x_1, ..., x_I; g) \in R^{In+n}_+$

so that:

1) x_i solves CP

2) Feas: $\sum_i x_i + g = \sum_i e_i$

3) Gov't BB: $\sum_n p_n g_n = \sum_n \sum_i p_n \tau_n x_{in}$

1.2.2 Implementation with Transfers Only

An ADT equilibrium (Arrow-Debreu equilibrium with Transfers) is given by prices, $p \in R_+^n$, an allocation, $(x_1, ..., x_I; g) \in R_+^{In+n}$, and a set of transfers, $(T_1, ..., T_I)$, such that:

(A) For all i x_i solves

$$Max \quad U_i(\cdot)$$
$$\text{s.t.} \quad pc \leq pe_i + T_i$$
.... non-negativity of all variables.

(B) <Accounting> (Supply = Demand)

$$g + \sum_i x_i = \sum_i e_i$$

<Government Budget Balance>

$$\sum_i T_i + pg = 0.$$

1.3 Results: Lump Sum Taxes are Awesome!

HH1) U_i is continuous, strictly increasing and strictly concave in x holding g fixed.

HH2) Endowments are positive: $\sum_i e_i >> 0$.

Proposition 3 *Under HH1 and HH2, given any TDCE allocation, there is a Pareto Superior allocation implemented by transfers only.*

Proof: If it's CPO already use Theorem below. If not, find a PS one that is CPO and use Theorem below.

Theorem 4 *If the feasible allocation, $(x_1, ..., x_I; g)$, is CPO, and HH1 and HH2 hold, then, there is a choice of transfers, $(T_1, ..., T_I)$ and prices, p, that implement $(x_1, ..., x_I; g)$ as an ADT equilibrium allocation.*

Proof:

Step 1. Notice that the restricted utility U_i holding g fixed satisfies the assumptions of the 2nd Welfare Theorem where the relevant aggregate endowment is given by $\sum_i e_i - g$.

Step 2. Depends on exactly what you think the 2nd Welfare Theorem says. For some versions, they automatically give the result given Step 1.

If you think that the 2nd Welfare Theorem says: "If $(x_1, ..., x_I)$ is PO, then there is a price system p, such that $(x_1, ..., x_I)$ with p is a CE equilibrium with no trade."
Then you still need to show that you can implement this with transfers and a different budget constraint.

In either case, it's step 1 that is the critical step.

Do you know enough about people to do this? I.e., indiv specific transfers

So, this shows that the optimal way of financing government spending is through the use of lump sum taxes. I.e., it's not just the best way to do redistribution, but also paying for spending.

Problem: Show that this holds with firms/production too.

Problem: Show that this holds in an infinite horizon too.

Intuition – "Lump sum taxes are 'non-distortionary' – they don't distort any decisions 'at the margin'." Anything beyond this???

You should use lump sum taxes to finance g even if g is not chosen 'correctly'. I.e., you can't do any better than this.

1.4   Are They Ever Equivalent?

How about if you can only tax consumption goods? Having all of the taxes equal also implements some (?) of the lump sum allocations:

Consumer's Problem becomes:

$$Max \quad U_i(\cdot)$$
$$\text{s.t.} \quad \sum_n p_n(1 + \tau_n)c_n \leq \sum_n p_n e_{in}$$

OR, if $\tau_n = \tau$ for all n,

$$Max \quad U_i(\cdot)$$
$$\text{s.t.} \quad \sum_n p_n c_n \leq \tfrac{1}{1+\tau} \sum_n p_n e_{ni} = \sum_n p_n e_{ni} - T_i$$

where $T_i = \sum_n p_n e_{ni} - \tfrac{1}{1+\tau} \sum_n p_n e_{ni}$

Notice that you can't get all combinations of T_i out of this. As an example, if all agents are identical, it follows that all transfers will be equal.

Also note that p_n depends on τ.

So, in some sense, Ramsey problems always have the solution – tax everything the same and it will be equivalent to lump-sum. At least unless there is some other motive than making things PO – e.g., redistribution – trying to pick out one particular PO allocation.

Problem: Show this result with production in it to.

Problem: What does it say about taxing labor income? Show that you should tax leisure at the same rate as all other consumption goods in order to replicate lump sum taxes.

Beyond that, everything comes from restricting the set of instruments.

Puzzle: What about consumption taxes that are different?

I.e., suppose we have τ_n different.

And suppose we have an equilibrium with these taxes for some g^* – i.e.,
1. all the max problems are satisfied at x_i^* and
2. $\sum_i x_i^* + g^* = \sum_i e_i$

(This can't hold for all p^*'s, for some private demand is bigger than endowment, so g_n would have to be negative!)

Then all the MRS's are equalized (at p^*), so the allocation must be CPO. So, it must be true that you can also implement it with lump sum taxes — what are the prices? Probably $p_n^{**} = p_n^*(1 + \tau_n)$. And T_i is defined accordingly.

1.5 Particular PO allocations and interpretations – ex post vs. ex ante

Looking at specific PO allocations – the one that maxes $\sum_i \frac{1}{I} U_i$ – notice that this adds some extra stuff – 'redistribution' or 'social insurance.' Even if $g = 0$.

Why is this important?

1. One interpretation of this: one particular choice of redistribution

2. Second interpretation of this: ex ante insurance

ex ante – each person is equally likely to end up being one of the I types – (U_i, e_i).

This is a representative consumer economy (ex ante) and hence there is an obvious choice of the PO allocation, the ex ante symmetric one. Alternatives would be to put more weight on 'names' NOT 'types'.

Specific example, $U_i = u$ for all i, $n = 1$, only one good. $I = 2$, H and L, $e_H > e_L$

If you can't trade insurance contracts, $c_L = e_L$ and $c_H = e_H$.

$$\max \quad \pi_L u(c_L) + \pi_H u(c_H)$$

$$\text{s.to} \quad \pi_H c_H + \pi_L c_L = \pi_H e_H + \pi_L e_L = \bar{e}$$

Solution is $c_L = c_H = \pi_H e_H + \pi_L e_L = \bar{e}$

Is this redistribution? (i.e., Ex Post)

Or is it Social Insurance? (i.e., Ex Ante)

Ex Ante, all agents would prefer to solution to this problem over the ex post allocation as long as u is strictly concave.

What if people aren't born yet, so that they can't trade these kinds of insurance contracts? I.e., born, find out your 'type' and then trade.

This gives the ex post outcome as above.

So, to implement the insurance outcome requires some sort of government intervention?

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1.5.1 More on the Example

So, to implement the insurance outcome requires some sort of government intervention ex post if these contracts can't be signed privately ex ante?

What would the Lump Sum Transfers be:

$$T_H = \bar{e} - e_H < 0$$

$$T_L = \bar{e} - e_L > 0$$

I.e., this requires TYPE SPECIFIC transfers.

What if the 'Planner' or government can't observe the types/endowments?

Set up a game or 'mechanism'.

REVELATION PRINCIPLE: Any equilibrium outcome can be implemented as a Bayesian Nash Equilibrium in a Direct Revelation Game in which 'telling the truth' is an equilibrium strategy.

Each person 'reports' their type, and transfers are made based on reported type. $T(\theta)$ where $\theta = H, L$.

Truth telling is an equilibrium implies that the following two Incentive Constraints must hold:

$(IC_H) \qquad U_H(\theta = H) = u(e_H + T(H)) \geq U_H(\theta = L) = u(e_H + T(L))$

$(IC_L) \qquad U_L(\theta = L) = u(e_L + T(L)) \geq U_L(\theta = H) = u(e_L + T(H))$

These say, respectively:

$$T(H) \geq T(L)$$

$$T(L) \geq T(H)$$

Thus, $T(H) = T(L)$.

Hence, feasibility $(\pi_H T(H) + \pi_L(T(L)) = 0)$ implies that $T(H) = T(L) = 0$.

Thus, there is no social insurance possible in this example.

1.5.2 Other Thoughts/Issues

What about maxing

$$\sum_i \pi(i) g(U_i)?$$

Maybe more redistribution?

So, ex ante PO is sort of a combination of classic PS (ex post) and redistribution due to an ASSUMPTION of ex ante identical chances of being each type combined with no ability to trade contracts before that 'type' is realized.

What if we are not all 'ex ante identical'?