Class Notes, Econ 8801
Two Results on Ramsey Taxes

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1 The Uniform Commodity Taxation Result

1.1 Basics

Notes based on Chari and Kehoe, 1999, "Optimal Fiscal and Monetary Policy," Chapter 26, Handbook of Public Finance.

Original Reference is Atkison and Stiglitz, 1972.

Representative Consumer with preferences:

$$U(c_1, ..., c_n, l)$$

where c_i is consumption of the $i-th$ good and l is labor supply.

Production function:

$$F(c_1 + g_1, c_2 + g_2, ..., c_n + g_n, l) = 0$$

Assume that F is CRS.

Normalize the wage at $w = 1$.

HH problem is:

$$\max_{c,l} \qquad U(c_1, ..., c_n, l)$$

s.t. $\quad \sum_i p_i(1 + \tau_i)c_i \leq l.$

NOTE: Taxes available on all goods expect leisure/labor supply.

Firm Problem:

$$\max_{x_i} \quad \sum_i p_i x_i - l$$

s.t. $\quad F(x_1, x_2, ..., x_n, l) = 0.$

Gov't Budget Balance:

$$\sum_i \tau_i p_i c_i = \sum p_i g_i$$

Feas:

$$x_i = c_i + g_i \text{ all } i.$$

1.2 Implementability Constraint

Proposition 1 *Assume enough interiority and differentiability so that FOC's work. Then, a TDCE allocation satsifies:*

Proposition 2 *1)(FEAS)* $\quad F(c_1 + g_1, c_2 + g_2, ..., c_n + g_n, l) = 0,$ *and*

Proposition 3 *2)(IMP)* $\quad \sum_i U_i c_i + U_l l = 0.$

Proposition 4 *Also, if an interior allocation satisfies (FEAS) and (IMP), then, the exist prices and taxes so that this is the TDCE allocation.*

Proof: Standard Primal Approach argument.

FOC's from HH problem are:

$$U_i = \alpha p_i (1 + \tau_i)$$

$$U_l = \alpha$$

where α is the multiplier on the budget constraint in the HH problem.

Substituting for $p_i(1 + \tau_i) = \frac{U_i}{U_l}$ in the budget constraint and multiplying through by U_l gives (IMP).

Conversely, suppose that c and l satisfy (FEAS) and (IMP) for the given g_i. We construct the CE by:

1) Define $x_i = c_i + g_i$

2) Use Firms FOC to construct prices: $p_i = -\frac{F_i}{F_l}$

3) Construct taxes by: $1 + \tau_i = \frac{U_i}{U_l} \frac{F_l}{F_i}.$

Check that with these definitions, all the relevant FOC's and constraints are satisfied.

1.3 Ramsey Problem

The Ramsey Problem becomes:

$$\max \qquad U(c,l)$$

s.t. (FEAS) and (IMP).

1.4 Uniform Commodity Taxation

Proposition 5 *Suppose U satisfies $U(c,l) = W(G(c),l)$ G is homothetic. Then, at the solution to the RP, $\tau_i = \tau_j$ for all i, j.*

Proof: Bunch FOC's crap....

First a technical property. Since G is homothetic, we have:

$$\frac{U_i(\alpha c,l)}{U_k(\alpha c,l)} = \frac{U_i(c,l)}{U_k(c,l)}$$

or

$$U_i(\alpha c, l) = \frac{U_i(c,l)}{U_k(c,l)} U_k(\alpha c, l)$$

Differentiating with respect to α, and evaluating at $\alpha = 1$, gives:

$$\sum_j c_j \frac{U_{ij}}{U_i} = \sum_j c_j \frac{U_{jk}}{U_k} \text{ for all } i, k.$$

Rewriting this we have:

*** $\qquad \sum_j c_j U_{ij} = AU_i$

where $A = \sum_j c_j \frac{U_{jk}}{U_k}$ is independent of i.

Now, using the FOC's for both the Firm and HH, we have:

$$1 + \tau_i = \frac{U_i}{U_l} \frac{F_l}{F_i}$$

Thus, $\tau_i = \tau_j$ if and only if:

$$\frac{U_i}{F_i} = \frac{U_j}{F_j}.$$

Now, the RP is:

$$\max \quad U(c,l)$$

s.t. $\quad F(c_1+g_1, c_2+g_2, ..., c_n+g_n, l) = 0 \ (\gamma)$ (FEAS) and

$\sum_i U_i c_i + U_l l = 0 \ (\lambda)$ (IMP).

The FOC for this for c_i is:

$$(1+\lambda)U_i + \lambda \left[\sum_i U_{ij} c_i + U_{il} l\right] = \gamma F_i.$$

Using *** and the form of the utility function, we can rewrite this as:

$$(1+\lambda)W_i G_i + \lambda \left[A W_1 G_i + l W_{12} G_i\right] = \lambda F_i.$$

Thus,

$$\frac{G_i}{F_i} = \frac{\lambda}{(1+\lambda)W_i + \lambda[AW_1 + lW_{12}]}$$

which doesn't depend on i.

Thus,

$$\frac{G_i}{F_i} = \frac{G_j}{F_j} \text{ for all } i, j.$$

Hence,

$$\frac{U_i}{F_i} = \frac{W_1 G_i}{F_i} = \frac{W_1 G_j}{F_j} = \frac{U_j}{F_j}$$

And hence, $\tau_i = \tau_j$ as desired.

Intuition?????

Example: $U = \sum_i \alpha_i \frac{c_i^{1-\sigma}}{(1-\sigma)} + v(l)$.

2 Intermediate Goods

This presentation is taken from Chari and Kehoe, 1999.

Basic Reference is Diamond and Mirrlees, 1974.

They show that Ramsey allocations are 'productively efficient' under some assumptions. That is, MRT's of all firms are the same for all goods. This wouldn't hold if, for example, government taxed transactions between firms, e.g., for the sale of 'intermediate' goods.

This is an example that illustrates this result – see Diamond and Mirrlees for the more general version.

- 3 final goods, consumption c, government consumption g and labor l.

- 1 intermediate good, z.

1) g and z are produced using l according to the CRS technology: $h(z, g, l) \leq 0$

2) c is produced using l and the intermediate good, z according to the CRS technology: $f(c, z, l) \leq 0$

HH Problem is:

$$\max \quad U(c, l)$$

$$\text{s.t.} \quad p(1+\tau)c \leq w(l_1 + l_2)$$

where l_1 is labor supply to the final good producer and l_2 is labor supply to the intermediate good producer.

Final Good Producer Problem is:

$$\max \quad pc - wl - (1+\tau_z)qz$$

$$\text{s.t.} \quad f(c, z, l) \leq 0$$

Intermediate Good Producer Problem is:

$$\max \quad qz + rg - wl$$

$$\text{s.t.} \quad h(z, g, l) \leq 0$$

2.1 Ramsey Problem

The Ramsey Problem for this example is:

$$\max \quad U(c, l_1 + l_2)$$

$$\text{s.t.} \quad cU_c + (l_1 + l_2)U_l = 0 \ (\lambda) \quad \text{(IMP)}$$

$$f(c, z, l) = 0 \ (\eta) \quad \text{(FEAS1)}$$

$$h(z, g, l) = 0 \ (\mu) \quad \text{(FEAS2)}$$

FOC's are:

$z: \qquad \eta f_z = -\mu h_z$

$l_1: \qquad U_l + \lambda\left(cU_{cl} + U_l + lU_{ll}\right) + \eta f_l = 0$

$l_2: \qquad U_l + \lambda(cU_{cl} + U_l + lU_{ll}) + \mu h_l = 0$

Thus, $\eta f_l = \mu h_l$. Using this along with the z FOC, we obtain:

$$\eta f_l = \mu h_l$$

$$f_l\left(-\mu \tfrac{h_z}{f_z}\right) = \mu h_l$$

$$-\tfrac{f_l}{f_z} = \tfrac{h_l}{h_z} \ {****}$$

(This is 'Productive Efficiency' – equality of the MRT's between z and l for both firms.)

The FOC for the firms are:

Final Good Producer Problem is:

$$\max \qquad pc - wl - (1+\tau_z)qz$$

s.t. $\qquad f(c, z, l) = 0 \ (\lambda_f)$

$c: \qquad p = -\lambda_f f_c$

$l: \qquad w = \lambda_f f_l$

$z: \qquad (1+\tau_z)q = \lambda_f f_z$

So,

$$\tfrac{(1+\tau_z)q}{w} = \tfrac{f_z}{f_l} \ {****}_1$$

Intermediate Good Producer Problem is:

$$\max \qquad qz + rg - wl$$

s.t. $\qquad h(z, g, l) = 0 \ (\lambda_h)$

$z: \qquad q = -\lambda_h h_z$

$g: \qquad r = -\lambda_h h_g$

$l: \qquad w = \lambda_h h_l$

So,

$\frac{q}{w} = -\frac{h_z}{h_l}$ $****_2$

Using $****$, $****_1$ and $****_2$, we see that

$\frac{(1+\tau_z)q}{w} = \frac{f_z}{f_l} = -\frac{h_z}{h_l} = \frac{q}{w}$

and so, $\tau_z = 0$.

Summarizing:

Proposition 6 *Under interiority, in the solution to the RP, intermediate goods are not taxed.*

Problem: Give an example where this doesn't hold – e.g., you can't tax all consumption goods?