Problem Set #2

Econ 8105-8106
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These questions deal with the 2-sector model of Arrow-Debreu equilibrium discussed in class, with I consumers indexed by $i = 1, 2, ..., I$; J_c consumption-goods firms indexed by $j_c = 1, 2, ..., J_c$; and J_x investment-goods firms, indexed by $j_x = 1, 2, ..., J_x$. Time is indexed by $t = 0, 1, ...$.

Question 1

(a) Set up the problem of each consumer minimizing their expenditure on goods subject to attaining the utility level of an equilibrium allocation.

(b) Set up the problem of each firm minimizing their total cost of inputs subject to attaining an equilibrium level of output.

(c) Show that an Arrow-Debreu equilibrium allocation solves the problems in (a) and (b).

(d) Show that an Arrow-Debreu equilibrium allocation solves the problem of minimizing the total cost of production given the ouput levels of the goods - that is, it is not possible to redistribute inputs and outputs among firms to reduce total costs.

Question 2

(a) Write down the necessary first-order conditions that a solution to each consumer's problem must satisfy.

(b) Suppose the solution to the consumer's problem is interior, that is, all quantities at all times are strictly positive. Derive a condition relating the price of investment goods and the rental rate of capital. State briefly in words what this means.

(c) Show that the constraints of the consumer's problem can be re-written so that only the initial endowment of capital enters the budget constraint (and the problems are equivalent).

(d) Write down the necessary first-order conditions that a solution to each firm's problem must satisfy. Derive a condition relating the price of consumption goods and the price of investment goods.

Question 3

Suppose that instead of infinitely-lived firms, there are J_c consumption goods firms and J_x investment goods firms in each period, and each of these firms makes input and output decisions only for one period. The consumers supply labor and capital to each firm in each period. Show that an equilibrium in this environment is the same as an equilibrium in the original environment.

Question 4

Suppose that instead of consumers, firms do the investing. Specifically, in period 0, each consumer i sells all of his initial endowment of capital $\bar{k}_0^i$ to the firms at price q. The firms (in both sectors) then purchase investment goods from the investment firms, and accumulate capital according to the same type of law-of-motion that we originally had for consumers. Show that the equilibrium consumption and leisure allocations, and the quantities of capital the firms use as inputs, are the same as in the original set-up.

Question 5

Suppose that consumers are allowed to trade capital amongst themselves. Specifically, in addition to his own accumulated capital k_t^i, consumer i can buy capital $\hat{k}_t^i$ in period t from other consumers at price p_{kt} per unit. Find the equilibrium price of capital in terms of the original environment's equilibrium prices and quantities. Assuming that equilibrium quantities are interior, show that the equilibrium consumption and leisure allocations are the same as in the original set-up.

Question 6

Suppose that consumers are allowed to trade their shares of firms' profits amongst themselves. Specifically, in period 0, consumer i can buy shares $\hat{\theta}_{j_x}^i$ of firm j_x's profits at price p_{j_x} per unit, and shares $\hat{\theta}_{j_c}^i$ of firm j_c's profits at price p_{j_c} per unit, for $j_x = 1, 2, \ldots, J_x$ and $j_c = 1, 2, \ldots, J_c$. Find the equilibrium prices of shares in terms of the original environment's equilibrium prices and quantities. Show that the equilibrium allocation is the same as in the original set-up.