Problem Set #3

Econ 8105-8106
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Question 1

Consider a one-period economy in which firms produce a single good y with a single input k according to the technology constraint $y \leq f(k)$. There is free-entry, so any number of firms may produce.

Write the cost of a firm as a function $c(y)$ of its output at equilibrium prices. Assume that there is a unique $\widehat{y}$ such that $\frac{c(\widehat{y})}{\widehat{y}} \leq \frac{c(y)}{y}$ for all $y > 0$ (What does this mean?). Show that the equilibrium price of output is equal to $\frac{c(\widehat{y})}{\widehat{y}}$. Show that aggregate output displays constant returns to scale; that is, if (k^*, y^*) is an aggregate equilibrium allocation, then $(\lambda k^*, \lambda y^*)$ is also, for any $\lambda > 0$.

Question 2

Consider the problem of finding a Pareto Optimal allocation in a T-period economy with one consumer and one firm. The consumer has preferences over consumption and leisure represented by the utility function:

$$U(\underset{\sim}{c}, \underset{\sim}{l}) = \sum_{t=0}^{T} \beta^t u(c_t),$$

where $0 < \beta < 1$. The firm produces consumption and investment goods according to the technology:

$$c_t + x_t \leq F(k_t, n_t),$$

and capital stock evolves according to:

$$k_{t+1} \leq (1-\delta)k_t + x_t.$$

Write the first-order necessary conditions for this problem as a second-order difference equation in k (i.e. an equation in k_{t-1}, k_t, and k_{t+1}).

Show that if a sequence of capital stocks satisfies these conditions and $k_{T+1} = 0$, then it is part of a Pareto Optimal allocation (make any additional assumptions you need on u and F).

Suppose now that the time horizon is infinite. What condition do you need to add to the equations found above to show that a sequence of capital stocks satisfying these conditions is part of an optimal allocation? How would you define this additional condition in an Arrow-Debreu equilibrium setting?

Question 3

Complete the "loose sketch of the proof" of the Second Welfare Theorem presented in class.

Question 4

This question outlines a procedure (usually referred to as "Negishi's method") for using the Second Welfare theorem to find an Arrow-Debreu equilibrium in a specific environment.

Consider an economy with 2 consumers and one firm. The consumers have endowments of hours $\overline{n}_t^1$, $\overline{n}_t^2$, and endowments of capital $\overline{k}^1, \overline{k}^2$. There is no investment and no depreciation, so the capital stocks stay constant over time. The consumers have preferences over consumption and leisure represented by the utility function

$$U^i(\underline{c}^i, \underline{l}^i) = \sum_{t=0}^{\infty} \beta^t \log(c_t^i) \qquad i = 1, 2,$$

where $\beta < 1$. The firm produces the consumption good using labor and capital according to the technology:

$$c_t \leq Ak^{1-\alpha} (n_t)^{\alpha}.$$

(a) Set up the problem of a "social planner" maximizing a weighted sum of the consumers' utilities subject to attaining a feasible allocation. For given weights, solve for the Pareto Optimal allocation.

(b) In the Arrow-Debreu environment of this economy, what would be the prices that implement this optimal allocation as an Arrow-Debreu equilibrium? Find the transfers needed to implement this allocation as an Arrow-Debreu Transfer Equilibrium.

(c) Find the Arrow-Debreu equilibrium of the economy by setting the transfers from (b) equal to zero.

Question 5

This question asks you to establish the equivalence of Arrow-Debreu Equilibrium and Sequential Markets Equilibrium in an environment with I consumers, and J firms that produce both consumption and investment goods and sell all their output at price p_t.

(a) In a Sequential Markets environment, consider the constraint $L_t^i \leq A$, for all t, where A is "large enough." Show that this constraint implies that, if $r_t^L > 0$ for all t, then:

$$\lim_{t\to\infty} \frac{L_t^i}{(1+r_0^L)(1+r_1^L)\cdots(1+r_{t-1}^L)} = 0,$$

where r_t^L is the interest paid (or received) on borrowing (or lending) done in period t.

(b) Consider any Sequential Markets Equilibrium. Construct prices for the Arrow-Debreu environment of the economy so that the allocation of the Sequential Markets Equilibrium (without the L's) is the allocation of an Arrow-Debreu equilibrium at those prices.

(c) Consider any Arrow-Debreu equilibrium. Construct prices, interest rates, and borrowing/lending amounts so that the allocation of the Arrow-Debreu equilibrium is a part of the allocation of a Sequential Markets Equilibrium at those prices.

(Note: you may find it easier to start with the case in which there are no firms, no capital, and no labor, and each consumer has an endowment of the consumption good in each period.)