Problem Set #4

Econ 8105-8106
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1. Write the following models in cannonical form:

   (a) Human Capital

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$
$$s.t. c_t + x_t \leq F(k_t, h_t n_t)$$
$$k_{t+1} = (1-\delta_k)k_t + x_t$$
$$h_{t+1} = (1-\delta_h)h_t v(n_t^h)$$
$$n_t^h + n_t + l_t = \overline{n_t}$$
$$non\ neggativity$$

   (b) One Sector Growth Model with multiple capital goods:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$
$$s.t. c_t + x_{1t} + x_{2t} + .. + +x_{Jt} \leq F(k_{1t}., .., k_{Jt}, n_t)$$
$$k_{jt+1} = (1-\delta)k_{jt} + x_{jt}$$
$$n_t + l_t = \overline{n_t}$$
$$non\ neggativity$$

2. Do problems 3.5, 3.6 (b), 3.8 and 4.4 from the Stokey, Lucas and Prescott (SLP) book.

3. Consider the infinite-horizon sequence problem (SP) and the functional equation (FE) (you know what they are by now). Suppose that, in (SP),

$$\text{given any } x_0\text{, there is a } \underset{\sim}{x}^* \in \Pi(x_0) \text{ such that } \sum_{t=0}^{\infty} \beta^t F(x_t^*, x_{t+1}^*) = V^*(x_0)$$

   and in (FE),

$$\text{given any } x\text{, there is a } y^* \in \Gamma(x) \text{ such that } v(x) = F(x, y^*) + \beta v(y^*).$$

   Prove Theorems 4.2 and 4.3 from SLP with these additional assumptions.

4. Consider the following $finite$-horizon sequence problem:

$$\max_{\underset{\sim}{x} \in \Pi(x_0)} \sum_{t=0}^{T} \beta^t F\left(x_t, x_{t+1}\right)$$

where $\underset{\sim}{x} = (x_0, x_1, \ldots, x_{T+1})$, and $\Pi(x_0) = \left\{\underset{\sim}{x} : x_{t+1} \in \Gamma(x_t)\ \forall\, t = 1, 2, \ldots, T\right\}$, for a fixed correspondence Γ that describes feasibility in each period.

Let $\Pi_n(x_n) = \left\{\underset{\sim}{x}^n = (x_n, x_{n+1}, \ldots, x_{T+1}) : x_{t+1} \in \Gamma(x_t)\ \forall\, t = n, n+1, \ldots T\right\}$ be the set of choices that are feasible from x_n, for all $n = 0, 1, \ldots, T$,

and for all $\underset{\sim}{x}^n \in \Pi_n(x_n)$,

let $\quad u_n(\underset{\sim}{x}^n) = \sum_{t=n}^{T} \beta^{t-n} F\left(x_t, x_{t+1}\right),$

and let $\quad V_n^*(x_n) = \sup_{\underset{\sim}{x}^n \in \Pi_n(x_n)} u_n(\underset{\sim}{x}^n)$, for all $n = 0, 1, \ldots, T$.

Prove analogues of Theorems 4.2 and 4.3 from SLP in this case - that is, define the appropriate functional equation problem corresponding to this sequence problem, and relate the functions V_n^* and the solutions of these equations.

(Note 1: since there are $T+1$ functions V_n^* (one for each $n = 0, 1, \ldots, T$), you will need $T+1$ value functions v_n.)

(Note 2 - regarding assumptions: do Assumption 4.2 and condition (8) in Theorem 4.3 have any use in the finite-horizon case? Should they be replaced with something else?)

(Note 3: you may assume what was assumed in Problem 3 above, i.e. that the sup's in each problem are attained by some element of the feasible set.)