Problem Set #6

Econ 8105-8106
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Questions 1-4 deal with an economy with one representative consumer, one representative firm, and a government that must finance a given stream of expenditures, g_t.

1. Show that the Tax-Distorted Competitive Equilibrium allocation that results when the government taxes the consumer's labor and capital income can also be attained if the government instead taxes the firm's purchases of labor and capital.

2. Suppose the government only taxes the consumer's purchases of consumption goods, and balances its budget in each period. Show that the TDCE allocation solves a social planner's problem.

3. Suppose the consumer has utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + v(l_t) \right]$$

   where $\sigma \geq 0$ and v is an increasing function.

   Suppose also that the government can only tax purchases of consumption goods.

   Set up the Ramsey Problem for this economy, and show that the optimal policy is to set the consumption tax at a constant rate after the first period (i.e. $\tau_{ct} = \tau_{ct+1}$ for all $t \geq 1$). How much does this result depend on the form of the utility function? How much can you say about the optimal policy for more general utility functions?

4. Suppose that the government no longer has to raise enough revenue to finance expenditures, but instead makes lump-sum transfers to the consumer, financed by a constant rate tax on capital income. Suppose that the firm has a technology of the Ak form and the consumer has utility given by:

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

   (a) Derive the growth rate of consumption, and show that the capital stock grows at the same rate.

   (b) Show that the growth rate is higher than if there were no government, but utility is lower.

5. Consider a growth model with 2 types of capital: physical capital, k_t, and human capital, h_t. The consumer is endowed with k_0 units of physical capital and h_0 units of human capital, and 1 unit of time in each period. The utility function of the consumer is:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

and technology is described by the production function $Ak_t^{\alpha}(n_t h_t)^{1-\alpha}$.

Physical and human capital depreciate at the rates δ_k and δ_h, respectively.

(a) Show that the physical/human capital ratio is uniquely determined by the parameters of the model, and is constant over time.

(b) Show that this problem is equivalent to a modified Ak model.