Fisher without Euler

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Abstract

Following Irving Fisher, the relation between nominal an real interest rates is traditionally related to inflationary expectations. When consumers exhibit an extreme amount of inattention, inflationary expectations cannot be a source of variation in the spread between nominal and real interest rates. This note shows how, in such an economy, setting low nominal interest rates will cause low inflation.

1. Introduction

Consider an economy in which households consume according to a simple decision rule that is linear in their after-tax earnings and the real value of their claims on the government. Households inelastically supply output, which grows at a constant rate. The government follows a fiscal policy rule that implements government purchases and tax revenues that grow at the same rate. In such an economy, there is a mechanical way in which setting low nominal interest rates leads to low inflation. Roughly, the sequence of temporary equilibrium conditions implies constant real interest rates and a simple quantity theory for the price of government debt in terms of consumption. Low nominal interest rates cause the supply of nominal government debt to grow slowly. This implies the Fisher relation between nominal and real interest rates, even though there is no intertemporal optimization problem that generates an Euler equation.

Section 2 describes government policy and household decision rules and presents government policies that will lead to well defined and uniquely determined equilibria. Poorly designed government policies could lead to a breakdown of markets or to multiple equilibria. Section 3 describes equilibrium paths in more familiar terms.

2. THE ECONOMY

Households are endowed with a flow of $y_t > 0$ units of non-storable consumption goods, and the government takes a share $\tau \in (0, 1)$ in taxes. There is an initial supply $B_{-1} > 0$ of government stock outstanding, held by households who can freely abandon any of it. So the price of government stock cannot be negative. Households can also default on liabilities with impunity, and so there can be no lending to households.

Government policy is defined by a rule $G_t : \mathbb{R}_+ \to \mathbb{R}_+$ that specifies government purchases of consumption goods, and a rule $D_t : \mathbb{R}_+ \to \mathbb{R}_+$ that specifies dividends per share of government stock. The subscript t captures the dependence of these policy rules on history and period-t shocks. When the auctioneer in period t calls out a price $s \geq 0$, the government purchases $G_t(s)$ units of consumption and pays $D_t(s)$ units of consumption as dividends per share. These policy rules have to satisfy

$$G_t(0) - \tau y_t + B_{t-1} D_t(0) = 0 \tag{1}$$

and

$$G_t(s) - \tau y_t + B_{t-1}(s + D_t(s)) \geq 0 \tag{2}$$

for all $s \geq 0$. If $s = 0$, then the government cannot raise revenue by issuing stock, and so the government is forced to balance its budget. In case of a strictly positive primary surplus, the government will have to pay a dividend. The inequality (2) ensures that tax revenues are never more than enough to retire all outstanding government stock. This reflects the fact that the government cannot save by lending to households.

If the period-t price s_t is strictly positive, then the supply of government stock evolves according to

$$B_t s_t = G_t(s_t) - \tau y_t + B_{t-1}(s_t + D_t(s_t)). \tag{3}$$

If $s_t = 0$ then $B_t \geq 0$ is unrestricted. Government policy can be augmented with a rule that pins down B_t in that case. If $s_t > 0$ then condition (2) implies that $B_t \geq 0$. See `http://www.econ.umn.edu/~luttmer/research/me/` for a more detailed description of government policy along these lines, in an economy with forward-looking agents.

Consumers know they live in a complicated world. They have some tentative interpretations of what they see around them, but they know those interpretations are imperfect and incomplete. They do not read the Federal Register or the minutes of the Federal Open Market Committee. They just stick to a simple decision rule $C_t : \mathbb{R}_+ \to \mathbb{R}_+$ that has done reasonably well in the past,

$$C_t(s) = \alpha(1 - \tau)y_t + \beta B_{t-1}(s + D_t(s)), \tag{4}$$

where $\alpha \in (0, 1]$ and $\beta \in (0, 1)$. Households consider their current after-tax earnings and the value of their claims on the government, both measured in units of consumption. The higher the value of those claims, the more they consume. Household resources at the beginning of period t are $(1 - \tau)y_t + B_{t-1}(s + D_t(s))$, and so (4) implies that households always attempt to hold at least $(1 - \alpha)(1 - \tau)y_t \geq 0$ in government securities at the end of period t.

The government does not face an explicit borrowing constraint in this economy. In particular, the auctioneer in period t simply calls out a price s and does not do some type of present-value calculation to limit how much the government may borrow at that price. Household demand for consumption and market clearing will limit how much the government can purchase.

2.1   A Sequence of Temporary Equilibria

In every period t, the initial supply of government stock outstanding B_{t-1} is given. The ex-dividend equilibrium price $s_t \geq 0$ of government stock is determined by clearing period-t markets. Goods market clearing in period t requires

$$y = C_t(s_t) + G_t(s_t), \tag{5}$$

and household holdings of government securities at the end of every period t have to match the value $B_t s_t$ implied by (3). Using the household decision rule (4) to eliminate $C_t(s_t)$ from (5) gives the equilibrium condition

$$(1 - \alpha)(1 - \tau)y_t = \beta B_{t-1}(s_t + D_t(s_t)) + G_t(s_t) - \tau y_t. \tag{6}$$

What households save out of their after-tax earnings, minus what they spend out of their holdings of government securities, must be equal to the primary surplus of the government. If (6) implies $s_t = 0$, then $B_t \geq 0$ can be anything. If (6) implies $s_t > 0$, then (3) determines the new supply of government securities B_t, and (2) ensures that this will be non-negative.

2.2   Equilibrium Paths

It will be convenient to write

$$c_t = C_t(s_t), \quad g_t = G_t(s_t), \quad d_t = D_t(s_t),$$

for household consumption, government consumption, and government dividends along the equilibrium path. Because $\beta B_{t-1}(s_t + d_t)$ cannot be negative, the equilibrium condition (6) implies an upper bound on how much the government can spend in any

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equilibrium. Put differently, the decision rule (4) says that households consume at least $\alpha(1 - \tau)y_t$, and so the government can never consume more than what is left, $y_t - \alpha(1 - \tau)y_t = (1 - \alpha)(1 - \tau)y_t + \tau y_t$. That is,

$$(1 - \alpha)(1 - \tau)y_t + \tau y_t - g_t \geq 0. \tag{7}$$

If $\alpha = 1$, this bound rules out primary deficits altogether. Primary deficits are not ruled out if $\alpha \in (0, 1)$. But although (2) allows the government to plan for arbitrarily large purchases $G_t(s)$ at positive off-equilibrium prices, (7) says that equilibrium imposes a definite upper bound on government purchases. This upper bound can be relaxed only by raising the tax rate τ.

Use the equilibrium condition (6) to eliminate $B_{t-1}(s_t + d_t)$ from the right-hand side of (3), to conclude that $\beta B_t s_t = (1 - \alpha)(1 - \tau)y_t + (1 - \beta)(\tau y_t - g_t)$. Combining this with the period-$t + 1$ version of (6) gives

$$R_{t+1} = \frac{s_{t+1} + d_{t+1}}{s_t} = \frac{(1 - \alpha)(1 - \tau)y_{t+1} + \tau y_{t+1} - g_{t+1}}{(1 - \alpha)(1 - \tau)y_t + (1 - \beta)(\tau y_t - g_t)}. \tag{8}$$

That is, the real return on government stock depends only on output and the equilibrium level of government consumption. The supply of government stock can only matter if government policy is such that it affects g_t or g_{t+1} on the equilibrium path. If output and government consumption grow at a common rate, then the real return on government stock will move one-for-one with the growth rate of household consumption.

Note that the gross real return (8) is non-negative and equal to zero precisely when (7) holds with equality in period $t + 1$. The government can attain this upper bound only by fully expropriating the holders of its securities. Of course, households may then conclude that the decision rule (4) needs some updating.

2.3 Two Examples

The government's policy rules are constrained by the fact that it cannot raise revenue when its securities are worthless (1), and that it cannot lend to the private sector (2). In equilibrium, government purchases are constrained by the fact that consumers can and do guarantee a minimum level of consumption for themselves (7). A government policy that tries to purchase more at any price would imply non-existence of an equilibrium.

In the following two examples, the government targets a certain amount of government purchases $g_t > 0$ that satisfies (7). In the first it raises more than enough taxes to finance these purchases. In the second it does not. In both examples, policy can be specified to achieve any arbitrary non-negative sequence of dividend yields $d_t/s_t = D_t(s_t)/s_t$.

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2.3.1 A Sustained Primary Surplus

Suppose $g_t \in (0, \tau y_t)$ and $B_{t-1} > 0$. Consider the policy

$$G_t(s) = g_t, \quad D_t(s) = \max\left\{ \frac{\tau y_t - g_t}{B_{t-1}} - s, \delta_t s \right\}, \tag{9}$$

where $\delta_t \geq 0$ does not depend on s. When the government runs a primary surplus $\tau y_t - g_t$, it can buy back all its securities when the auctioneer calls out a particularly low price s, unless the government pays out a large dividend $D_t(s)$. The policy rule (9) targets a dividend yield δ_t and deviates from it only to avoid lending to the private sector, to avoid violating (2). Inserting (9) into the equilibrium condition (6) gives

$$(1 - \alpha)(1 - \tau)y_t + \tau y_t - g_t = \beta \max\left\{ \tau y_t - g_t, B_{t-1}(1 + \delta_t)s_t \right\}.$$

The left-hand side of this condition exceeds $\beta(\tau y_t - g_t)$, and so the only solution is

$$s_t = \frac{(1 - \alpha)(1 - \tau)y_t + \tau y_t - g_t}{\beta B_{t-1}(1 + \delta_t)}. \tag{10}$$

This is strictly positive because $\alpha \in (0, 1]$, $\beta \in (0, 1)$, and $g \in (0, \tau y)$. The resulting dividend is $D_t(s_t) = \delta_t s_t$, and (3) implies that the new supply of government stock is given by

$$B_t = \frac{(1 - \alpha)(1 - \tau)y_t + (1 - \beta)(\tau y_t - g_t)}{(1 - \alpha)(1 - \tau)y_t + \tau y_t - g_t} \times B_{t-1}(1 + \delta_t). \tag{11}$$

This is strictly positive, and so one can continue this iteration indefinitely, starting from any initial $B_{-1} > 0$. Observe that (11) and $g_t \in (0, \tau y)$ implies $B_t < B_{t-1}(1 + \delta_t)$.

2.3.2 A Sustained Primary Deficit

Take $\alpha < 1$ and suppose alternatively that the intended level g_t of government purchases satisfies

$$0 < g_t - \tau y_t < (1 - \alpha)(1 - \tau)y_t. \tag{12}$$

That is, the government tries to run a deficit, but one that is consistent with equilibrium—that is, not so large that it would violate (7). Running a primary deficit means that the government has to sell securities to households. But this generates no revenue if the auctioneer calls out a zero price for government securities. Government policy will have to deal with this contingency. A continuous policy that does so is

$$G_t(s) = \tau y_t + \frac{g_t - \tau y_t}{(1 - \alpha)(1 - \tau)y_t + \tau y_t - g_t} \times \beta B_{t-1}(1 + \delta_t)s, \quad D_t(s) = \delta_t s. \tag{13}$$

This forces $G_t(0) = \tau y_t$ if the auctioneer calls out $s = 0$. On the other hand, inserting the price (10) into (13) yields $G_t(s_t) = g_t$. The government policy (13) is a linear interpolation between these two outcomes. Note that planned government purchases become very sensitive to s when g_t approaches the upper bound implied by (7). The fact that $G_t(s)$ and $s + D_t(s) = (1 + \delta_t)s$ are strictly increasing in s implies that the price (10) is the only possible solution to the equilibrium condition (6). Again, the dynamics of B_t follows from (3) and this implies (11). Assumption (12) ensures that B_t is strictly positive. More precisely, (11) and (12) imply that $B_t > (1 + \delta_t)B_{t-1}$.

3. REAL AND NOMINAL INTEREST RATES

To put this into more familiar terms and interpret, let

$$p_t = \frac{1}{s_t + d_t}, \quad q_t = \frac{s_t}{s_t + d_t},$$

and consider government policies that implement $g_t > 0$, where g_t satisfies (7) with a strict inequality. The equilibrium condition (6) then becomes $(1 - \alpha)(1 - \tau)p_t y_t = \beta B_{t-1} + p_t(g_t - \tau y_t)$, and this yields

$$p_t = \frac{\beta B_{t-1}}{(1 - \alpha)(1 - \tau)y_t + \tau y_t - g_t}. \tag{14}$$

This is first and foremost a quantity theory of the price level: prices scale with the nominal supply of government securities, other things equal. But a government that tries to purchase a large part of output (supplied inelastically in this economy) will also drive up prices. Because consumers simply follow the decision rule (4), beliefs play no role.

The equilibrium dynamics (3) of B_t can be written as

$$p_t g_t + B_{t-1} = \tau p_t y_t + q_t B_t. \tag{15}$$

So government stock is really a one-period nominal discount bond when the cum-dividend price of government stock is used as the numeraire. Note that the dividend yield $\delta_t = d_t/s_t$ implies $q_t = 1/(1 + \delta_t)$, and so the dividend yield δ_t is the nominal interest rate in this economy. The restriction $D_t(s) \geq 0$ implies a zero lower bound. Using (14) to eliminate p_t from (15) gives

$$\frac{q_t B_t}{B_{t-1}} = \frac{(1 - \alpha)(1 - \tau)y_t + (1 - \beta)(\tau y_t - g_t)}{(1 - \alpha)(1 - \tau)y_t + \tau y_t - g_t}, \tag{16}$$

as in (11). This is strictly positive because of (7).

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3.1 Stable Output and Government Consumption

Suppose now that output and government consumption grow at some steady rate $\gamma \geq 0$,

$$\begin{bmatrix} y_t & g_t \end{bmatrix} = (1 + \gamma)^t \begin{bmatrix} y & g \end{bmatrix},$$

and that g satisfies (7). Then the quantity theory (14) implies

$$\frac{p_{t+1}}{p_t} = \frac{1}{1 + \gamma} \frac{B_t}{B_{t-1}}. \tag{17}$$

Absent fluctuations in output and government consumption, inflation simply trails the growth rate of the supply of nominal government securities. In turn, this growth rate follows from (16). The right-hand side of (16) is constant, and so $B_t/B_{t-1} \propto 1/q_t = 1 + \delta_t$. Nominal interest rates determine the growth rate of B_t, and then (17) determines inflation.

It is immediate that the real return on government debt is constant. Combining (16) with (17) produces a real rate of return from t to $t + 1$ equal to

$$\frac{p_t}{q_t p_{t+1}} = \frac{(1 + \gamma)B_{t-1}}{q_t B_t} = \frac{(1 + \gamma)((1 - \alpha)(1 - \tau)y + \tau y - g)}{(1 - \alpha)(1 - \tau)y + (1 - \beta)(\tau y - g)}, \tag{18}$$

as in (8). The real interest rate in this economy is determined the growth rate of the economy, and fiscal parameters. Note that the right-hand side of (18) is strictly increasing in $\tau y - g > -(1 - \alpha)(1 - \tau)y$ when $\alpha < 1$. The real return on government debt will be below the growth rate of the economy if the government runs a primary deficit, and exceed that growth rate if the government runs a primary surplus.

Note that $\alpha \in (0, 1)$ and $\tau y < g$ means that the government runs primary deficits forever, and these deficits grow at the rate γ. The real return (18) is below this rate, and so the present value of any flow that grows at that rate is infinite. As in the classic Bewley economy, individual consumers without proper insurance may not want to increase their consumption beyond $((1 - \alpha)(1 - \tau)y - g)(1 + \gamma)^t$ when they face periods without income.

4. DISCUSSION

Suppose consumers have additively separable preferences with logarithmic period utility functions and a common subjective discount factor $\theta \in (0, 1)$. With common priors and Bayesian updating, the consumption function is of the form (4) with $\alpha = 1$ and $\beta = 1 - \theta$. As expected, the right-hand side of (18) equals $(1 + \gamma)/\theta$ in that case, and

(7) does not allow the government to run a permanent primary deficit: the government cannot run sustained primary deficits when infinitely-lived forward-looking households exhaust their present-value budget constraints.

So what is the point? When consumers are busy with their own lives and follow a simple decision rule such as (4), a speech about fiscal irresponsibility at some time in the future will not cause an instantaneous jump in the price level, as it would when logically omniscient consumers are paying attention and the speech is credible. A speech about the path of future nominal interest policy will fall on deaf ears. But the effect of sustained low interest rates is inexorable, simply because of how these interest rates affect the supply of nominal government debt. The simple decision rule (4) provides an illustration of the robustness of this mechanism.

Of course, as Lucas emphasized long ago, it is all-important to know what decision rules households are using, and understand how they are influenced by changes in policy.