Fisher without Euler

Erzo G.J. Luttmer*

University of Minnesota

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Abstract

Following Irving Fisher, the relation between nominal an real interest rates is traditionally related to inflationary expectations. When consumers exhibit an extreme amount of inattention, inflationary expectations cannot be a source of variation in the spread between nominal and real interest rates. This note shows how, in such an economy, setting low nominal interest rates will cause low inflation.

1. Introduction

Consider an economy in which households consume according to a simple decision rule that is linear in their after-tax earnings and the real value of their claims on the government. Households inelastically supply output, which grows at a constant rate. The government follows a fiscal policy rule that implements government purchases and tax revenues that grow at the same rate. In such an economy, there is a mechanical way in which setting low nominal interest rates leads to low inflation. Roughly, the sequence of temporary equilibrium conditions implies constant real interest rates and a simple quantity theory for the price of government debt in terms of consumption. Low nominal interest rates cause the supply of nominal government debt to grow slowly. This implies the Fisher relation between nominal and real interest rates, even though there is no intertemporal optimization problem that generates an Euler equation.

Section 2 describes government policy and household decision rules and presents government policies that will lead to well defined and uniquely determined equilibria.

*This work in progress. Comments welcome.

Poorly designed government policies could lead to a breakdown of markets or to multiple equilibria. Section 3 describes equilibrium paths in more familiar terms. Section 4 adds long-term governement debt and shows the effect

2. The Economy

Households are endowed with a flow of $y_t > 0$ units of non-storable consumption goods, and the government takes a share $\tau \in (0, 1)$ in taxes. There is an initial supply $B_{-1} > 0$ of government stock outstanding, held by households who can freely abandon any of it. So the price of government stock cannot be negative. Households can also default on liabilities with impunity, and so there can be no lending to households.

Government policy is defined by a rule $G_t : \mathbb{R}_+ \to \mathbb{R}_+$ that specifies government purchases of consumption goods, and a rule $D_t : \mathbb{R}_+ \to \mathbb{R}_+$ that specifies dividends per share of government stock. The subscript t captures the dependence of these policy rules on history and period-t shocks. When the auctioneer in period t calls out a price $s \geq 0$, the government purchases $G_t(s)$ units of consumption and pays $D_t(s)$ units of consumption as dividends per share. These policy rules have to satisfy

$$G_t(0) - \tau y_t + B_{t-1} D_t(0) = 0 \tag{1}$$

and

$$G_t(s) - \tau y_t + B_{t-1}(s + D_t(s)) \geq 0 \tag{2}$$

for all $s \geq 0$. If $s = 0$, then the government cannot raise revenue by issuing stock, and so the government is forced to balance its budget. In case of a strictly positive primary surplus, the government will have to pay a dividend. The inequality (2) ensures that tax revenues are never more than enough to retire all outstanding government stock. This reflects the assumption that the government cannot save by lending to households. Note that this is an easy constraint to satisfy: the government can simply pay a large dividend on its outstanding stock.[1]

If the period-t price s_t is strictly positive, then the supply of government stock evolves according to

$$B_t s_t = G_t(s_t) - \tau y_t + B_{t-1}(s_t + D_t(s_t)). \tag{3}$$

[1] An interesting extension, in light of the quantitive easing experience in the US, would be to introduce marketable private sector securities and give the government the ability to act as an intermediary by purchasesing these securities.

If $s_t = 0$ then $B_t \geq 0$ is unrestricted. Government policy can be augmented with a rule that pins down B_t in that case. If $s_t > 0$ then condition (2) implies that $B_t \geq 0$. See `http://www.econ.umn.edu/~luttmer/research/me/` for a more detailed description of government policy along these lines, in an economy with forward-looking agents.

Consumers know they live in a complicated world. They have some tentative interpretations of what they see around them, but they know those interpretations are imperfect and incomplete. They do not read the Federal Register or the minutes of the Federal Open Market Committee. They do often receive conflicting advice from reputed experts. Since they have to do something, they just stick to a simple decision rule $C_t : \mathbb{R}_+ \to \mathbb{R}_+$ that has done reasonably well in the past,

$$C_t(s) = \alpha(1-\tau)y_t + \beta B_{t-1}(s + D_t(s)), \tag{4}$$

where $\alpha \in (0, 1]$ and $\beta \in (0, 1)$. Households consider their current after-tax earnings and the value of their claims on the government, both measured in units of consumption. The higher the value of those claims, the more they consume. Household resources at the beginning of period t are $(1-\tau)y_t + B_{t-1}(s + D_t(s))$, and so (4) implies that households always attempt to hold at least $(1-\alpha)(1-\tau)y_t \geq 0$ in government securities at the end of period t.

The government does not face an explicit borrowing constraint in this economy. In particular, the auctioneer in period t simply calls out a price s and does not do some type of present-value calculation to limit how much the government may borrow at that price. Household demand for consumption and market clearing will limit how much the government can purchase.

2.1 A Sequence of Temporary Equilibria

In every period t, the initial supply of government stock outstanding B_{t-1} is given. The ex-dividend equilibrium price $s_t \geq 0$ of government stock is determined by clearing period-t markets. Goods market clearing in period t requires

$$y_t = C_t(s_t) + G_t(s_t), \tag{5}$$

and household holdings of government securities at the end of every period t have to match the value $B_t s_t$ implied by (3). Using the household decision rule (4) to eliminate $C_t(s_t)$ from (5) gives the equilibrium condition

$$(1-\alpha)(1-\tau)y_t = \beta B_{t-1}(s_t + D_t(s_t)) + G_t(s_t) - \tau y_t. \tag{6}$$

What households save out of their after-tax earnings, minus what they spend out of their holdings of government securities, must be equal to the primary surplus of the government. If (6) implies $s_t = 0$, then $B_t \geq 0$ can be anything. If (6) implies $s_t > 0$, then (3) determines the new supply of government securities B_t, and (2) ensures that this will be non-negative.

2.2 Equilibrium Paths

It will be convenient to write

$$c_t = C_t(s_t), \quad g_t = G_t(s_t), \quad d_t = D_t(s_t),$$

for household consumption, government consumption, and government dividends along the equilibrium path. Because $\beta B_{t-1}(s_t + d_t)$ cannot be negative, the equilibrium condition (6) implies an upper bound on how much the government can spend in any equilibrium. Put differently, the decision rule (4) says that households consume at least $\alpha(1-\tau)y_t$, and so the government can never consume more than what is left, $y_t - \alpha(1-\tau)y_t = (1-\alpha)(1-\tau)y_t + \tau y_t$. That is,

$$(1-\alpha)(1-\tau)y_t + \tau y_t - g_t \geq 0. \tag{7}$$

If $\alpha = 1$, this bound rules out primary deficits altogether. Primary deficits are not ruled out if $\alpha \in (0, 1)$. But although (2) allows the government to plan for arbitrarily large purchases $G_t(s)$ at positive off-equilibrium prices, (7) says that equilibrium imposes a definite upper bound on government purchases. This upper bound can be relaxed only by raising the tax rate τ.

Use the equilibrium condition (6) to eliminate $B_{t-1}(s_t + d_t)$ from the right-hand side of (3), to conclude that $\beta B_t s_t = (1-\alpha)(1-\tau)y_t + (1-\beta)(\tau y_t - g_t)$. Combining this with the period-$t+1$ version of (6) gives

$$R_{t+1} = \frac{s_{t+1} + d_{t+1}}{s_t} = \frac{(1-\alpha)(1-\tau)y_{t+1} + \tau y_{t+1} - g_{t+1}}{(1-\alpha)(1-\tau)y_t + (1-\beta)(\tau y_t - g_t)}. \tag{8}$$

That is, the real return on government stock depends only on output and the equilibrium level of government consumption. The supply of government stock can only matter if government policy is such that it affects g_t or g_{t+1} on the equilibrium path. If output and government consumption grow at a common rate, then the real return on government stock will move one-for-one with the growth rate of household consumption.

Note that the gross real return (8) is non-negative and equal to zero precisely when (7) holds with equality in period $t+1$. The government can attain this upper bound only by fully expropriating the holders of its securities. Of course, households may then conclude that the decision rule (4) needs some updating.

2.3 Two Examples

The government's policy rules are constrained by the fact that it cannot raise revenue when its securities are worthless (1), and that it cannot lend to the private sector (2). In equilibrium, government purchases are constrained by the fact that consumers can and do guarantee a minimum level of consumption for themselves (7). A government policy that tries to purchase more at any price would imply non-existence of an equilibrium.

In the following two examples, the government targets a certain amount of government purchases $g_t > 0$ that satisfies (7). In the first it raises more than enough taxes to finance these purchases. In the second it does not. In both examples, policy can be specified to achieve any arbitrary non-negative sequence of dividend yields $d_t/s_t = D_t(s_t)/s_t$.

2.3.1 A Sustained Primary Surplus

Suppose $g_t \in (0, \tau y_t)$ and $B_{t-1} > 0$. Consider the policy

$$G_t(s) = g_t, \qquad D_t(s) = \max\left\{\frac{\tau y_t - g_t}{B_{t-1}} - s, \delta_t s\right\}, \tag{9}$$

where $\delta_t \geq 0$ does not depend on s. When the government runs a primary surplus $\tau y_t - g_t$, it can buy back all its securities when the auctioneer calls out a particularly low price s, unless the government pays out a large dividend $D_t(s)$. The policy rule (9) targets a dividend yield δ_t and deviates from it only to avoid lending to the private sector, to avoid violating (2). Inserting (9) into the equilibrium condition (6) gives

$$(1-\alpha)(1-\tau)y_t + \tau y_t - g_t = \beta \max\left\{\tau y_t - g_t, B_{t-1}(1+\delta_t)s_t\right\}.$$

The left-hand side of this condition exceeds $\beta(\tau y_t - g_t)$, and so the only solution is

$$s_t = \frac{(1-\alpha)(1-\tau)y_t + \tau y_t - g_t}{\beta B_{t-1}(1+\delta_t)}. \tag{10}$$

This is strictly positive because $\alpha \in (0,1]$, $\beta \in (0,1)$, and $g \in (0, \tau y)$. The resulting dividend is $D_t(s_t) = \delta_t s_t$, and (3) implies that the new supply of government stock is given by

$$B_t = (1+\delta_t)\Gamma_t B_{t-1}, \tag{11}$$

where

$$\Gamma_t = \frac{(1-\alpha)(1-\tau)y_t + (1-\beta)(\tau y_t - g_t)}{(1-\alpha)(1-\tau)y_t + \tau y_t - g_t}.$$

The growth factor Γ_t is strictly positive, and so one can continue the iteration (11) indefinitely, starting from any initial $B_{-1} > 0$. Observe that (11) and $g_t \in (0, \tau y_t)$ implies $B_t < B_{t-1}(1 + \delta_t)$.

2.3.2 A Sustained Primary Deficit

Take $\alpha < 1$ and suppose alternatively that the intended level g_t of government purchases satisfies

$$0 < g_t - \tau y_t < (1 - \alpha)(1 - \tau) y_t. \tag{12}$$

That is, the government tries to run a deficit, but one that is consistent with equilibrium—that is, not so large that it would violate (7). Running a primary deficit means that the government has to sell securities to households. But this generates no revenue if the auctioneer calls out a zero price for government securities. Government policy will have to deal with this contingency. A continuous policy that does so is

$$G_t(s) = \tau y_t + \frac{g_t - \tau y_t}{(1 - \alpha)(1 - \tau) y_t + \tau y_t - g_t} \times \beta B_{t-1}(1 + \delta_t) s, \qquad D_t(s) = \delta_t s. \tag{13}$$

This forces $G_t(0) = \tau y_t$ if the auctioneer calls out $s = 0$. On the other hand, inserting the price (10) into (13) yields $G_t(s_t) = g_t$. The government policy (13) is a linear interpolation between these two outcomes. Note that planned government purchases become very sensitive to s when g_t approaches the upper bound implied by (7). The fact that $G_t(s)$ and $s + D_t(s) = (1+\delta_t)s$ are strictly increasing in s implies that the price (10) is the only possible solution to the equilibrium condition (6). Again, the dynamics of B_t follows from (3) and this implies (11). Assumption (12) ensures that B_t is strictly positive. More precisely, (11) and (12) imply that $B_t > (1 + \delta_t) B_{t-1}$.

3. Real and Nominal Interest Rates

To put this into more familiar terms and interpret, let

$$p_t = \frac{1}{s_t + d_t}, \qquad q_t = \frac{s_t}{s_t + d_t},$$

and consider government policies that implement $g_t > 0$, where g_t satisfies (7) with a strict inequality. The equilibrium condition (6) then becomes $(1 - \alpha)(1 - \tau) p_t y_t = \beta B_{t-1} + p_t (g_t - \tau y_t)$, and this yields

$$p_t = \frac{\beta B_{t-1}}{(1 - \alpha)(1 - \tau) y_t + \tau y_t - g_t}. \tag{14}$$

This is first and foremost a quantity theory of the price level: prices scale with the nominal supply of government securities, other things equal. But a government that tries to purchase a large part of output (supplied inelastically in this economy) will also drive up prices. Because consumers simply follow the decision rule (4), beliefs play no role.

The equilibrium dynamics (3) of B_t can be written as

$$p_t g_t + B_{t-1} = \tau p_t y_t + q_t B_t. \tag{15}$$

So government stock is really a one-period nominal discount bond when the cum-dividend price of government stock is used as the numeraire. Note that the dividend yield $\delta_t = d_t/s_t$ implies $q_t = 1/(1+\delta_t)$, and so the dividend yield δ_t is the nominal interest rate in this economy. The restriction $D_t(s) \geq 0$ implies a zero lower bound. Using (14) to eliminate p_t from (15) gives

$$\frac{q_t B_t}{B_{t-1}} = \frac{(1-\alpha)(1-\tau)y_t + (1-\beta)(\tau y_t - g_t)}{(1-\alpha)(1-\tau)y_t + \tau y_t - g_t}, \tag{16}$$

as in (11). This is strictly positive because of (7).

3.1 Stable Output and Government Consumption

Suppose now that output and government consumption grow at some steady rate $\gamma \geq 0$,

$$\begin{bmatrix} y_t & g_t \end{bmatrix} = (1+\gamma)^t \begin{bmatrix} y & g \end{bmatrix},$$

and that g satisfies (7). Then the quantity theory (14) implies

$$\frac{p_{t+1}}{p_t} = \frac{1}{1+\gamma}\frac{B_t}{B_{t-1}}. \tag{17}$$

Absent fluctuations in output and government consumption, inflation simply trails the growth rate of the supply of nominal government securities. In turn, this growth rate follows from (16). The right-hand side of (16) is constant, and so $B_t/B_{t-1} \propto 1/q_t = 1+\delta_t$. Nominal interest rates determine the growth rate of B_t, and then (17) determines inflation.

It is immediate that the real return on government debt is constant. Combining (16) with (17) produces a real rate of return from t to $t+1$ equal to

$$\frac{p_t}{q_t p_{t+1}} = \frac{(1+\gamma)B_{t-1}}{q_t B_t} = \frac{(1+\gamma)((1-\alpha)(1-\tau)y + \tau y - g)}{(1-\alpha)(1-\tau)y + (1-\beta)(\tau y - g)}, \tag{18}$$

as in (8). The real interest rate in this economy is determined the growth rate of the economy, and fiscal parameters. Note that the right-hand side of (18) is strictly increasing in $\tau y - g > -(1-\alpha)(1-\tau)y$ when $\alpha < 1$. The real return on government debt will be below the growth rate of the economy if the government runs a primary deficit, and exceed that growth rate if the government runs a primary surplus.

Note that $\alpha \in (0, 1)$ and $\tau y < g$ means that the government runs primary deficits forever, and these deficits grow at the rate γ. The real return (18) is below this rate, and so the present value of any flow that grows at that rate is infinite. As in the classic Bewley economy, individual consumers without proper insurance may not want to increase their consumption beyond $((1-\alpha)(1-\tau)y-g)(1+\gamma)^t$ when they face periods without income.

3.2 Discussion

Suppose consumers have additively separable preferences with logarithmic period utility functions and a common subjective discount factor $\theta \in (0, 1)$. With common priors and Bayesian updating, the consumption function is of the form (4) with $\alpha = 1$ and $\beta = 1 - \theta$. As expected, the right-hand side of (18) equals $(1 + \gamma)/\theta$ in that case, and (7) does not allow the government to run a permanent primary deficit: the government cannot run sustained primary deficits when infinitely-lived forward-looking households exhaust their present-value budget constraints.[2]

So what is the point? When consumers are busy with their own lives and follow a simple decision rule such as (4), a speech about fiscal irresponsibility at some time in the future will not cause an instantaneous jump in the price level, as it would when logically omniscient consumers are paying attention and the speech is credible. A speech about the path of future nominal interest policy will fall on deaf ears. But the effect of sustained low interest rates is inexorable, simply because of how these interest rates affect the supply of nominal government securities. The simple decision rule (4) provides an illustration of the robustness of this mechanism.

Of course, as Lucas [1976] emphasized long ago, it is all-important to know what decision rules households are using, and understand how they are influenced by changes in policy.

[2] In contrast to the equilibrium associated with permanent primary surpluses, the equilibrium associated with permanent primary deficits is fragile, as are bubble equilibria in other models. Changes in household decision rules could force the government to balance its budget.

4. Discount Bonds and Perpetuities

When the government issues more than one type of nominal security, how does the price level depend on the composition of its supply of nominal securities? What does Operation Twist do in such an economy?

As before, suppose the government issues one-period nominal discount bonds, and take maturing discount bonds to be the numeraire: consumption at time t costs p_t units of maturing discount bonds, and the price of a new discount bond is q_t, in the same units. In addition, the government issues a perpetuity: a claim to a sequence of coupons at every future date that are equal to one unit of the maturing discount bond at the date of the coupon. Write Q_t for the price of this perpetuity, in units of maturing discount bonds at date t. The supply of maturing nominal discount bonds at time t is B_{t-1}, and the number of perpetuities outstanding at the beginning of period t is K_{t-1}.

Households consume a share α of their after-tax labor income, and a fraction β of the value of their holdings of government securities. They re-invest the remainder, as well as a share $1-\alpha$ of their after-tax labor income. A share $\omega \in (0,1)$ of the value of these investments is held in discount bonds, and the remainder $1-\omega$ in perpetuities. In every period, the government sets both the price of its nominal discount bond and of the perpetuity, subject to the following restrictions[3]

$$q_t \in (0,1], \quad Q_t \in (q_t, \infty).$$

The upper bound on q_t is the same as a zero lower bound on the nominal interest rate. It ensures that nominal discount bonds can circulate as bearer securities that are redeemable at the original face value, at any time. The lower bound on Q_t ensures that the government does not offer households an obvious arbitrage opportunity. Both the discount bond and the perpetuity deliver a maturing discount bond in the next period, but the perpetuity offers more, and so it should be more expensive.

In equilibrium, households consume $c_t = y_t - g_t$ and they choose to hold discount bonds with a face value B_t and perpetuities with a coupon K_t. Their decision rules imply

$$\begin{bmatrix} p_t(y_t - g_t) \\ q_t B_t \\ Q_t K_t \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \omega(1-\alpha) & \omega(1-\beta) \\ (1-\omega)(1-\alpha) & (1-\omega)(1-\beta) \end{bmatrix} \begin{bmatrix} (1-\tau)p_t y_t \\ B_{t-1} + K_{t-1} + Q_t K_{t-1} \end{bmatrix}. \tag{19}$$

[3] Government policy should really be specified, as in Section 1, for all prices the auctioneer could call out. Important details will be provided in a future draft.

Adding up the rows of (19) yields

$$q_t B_t + Q_t K_t = p_t(g_t - \tau y_t) + B_{t-1} + K_{t-1} + Q_t K_{t-1}.$$

As expected, Walras' law gives back the period-t budget constraint of the government. As before, government purchases are assumed to satisfy (7), so that the government does not try to purchase more than the most consumers will save under any circumstance.

The first equation in (19) determines the price level

$$p_t = \frac{\beta\left(B_{t-1} + K_{t-1} + Q_t K_{t-1}\right)}{(1-\alpha)(1-\tau)y_t + \tau y_t - g_t}, \tag{20}$$

and (7) implies that the denominator is positive. This is an easy generalization of (14). But there is a significant difference: given positive initial conditions B_{t-1} and K_{t-1}, the government at date t can affect the current price level by manipulating the price Q_t of this long-lived security. The mechanism is very simple: a higher Q_t raises the value of household portfolios of government securities, and this induces them to spend more on consumption. Since there is assumed to be no change in $y_t - g_t$, all of this increase in spending results in an increase in the price level. Naturally, there could be a delay in how quickly households become aware of a change in the value of their claims on the government, and then the effect of an increase in Q_t on the price level would not be instantaneous.

To describe how the price level evolves over time, we need to determine the evolution of $B_{t-1}+K_{t-1}+Q_t K_{t-1}$. Using (20) to eliminate p_t from the second and third equilibrium conditions specified in (19) gives

$$\begin{aligned}\begin{bmatrix} q_t B_t \\ Q_t K_t \end{bmatrix} &= \begin{bmatrix} \omega \\ 1-\omega \end{bmatrix} \begin{bmatrix} 1-\alpha & 1-\beta \end{bmatrix} \begin{bmatrix} \frac{(1-\tau)\beta y_t}{(1-\alpha)(1-\tau)y_t + \tau y_t - g_t} \\ 1 \end{bmatrix} (B_{t-1} + K_{t-1} + Q_t K_{t-1}) \\ &= \begin{bmatrix} \omega \\ 1-\omega \end{bmatrix} \Gamma_t \left(B_{t-1} + K_{t-1} + Q_t K_{t-1}\right). \end{aligned} \tag{21}$$

This determines B_t and K_t for a given government choice of $q_t \in (0, 1]$ and $Q_t \in (q_t, \infty)$. Note that (7) and $\beta \in (0, 1]$ ensures that Γ_t is positive, and so positive B_{t-1} and K_{t-1} map into positive B_t and K_t. This is a simple consequence of the fact that $\omega \in (0, 1)$, so that households always want to hold some of each of the two government securities. It is now easy to see from (21) that

$$\frac{B_t + K_t + Q_{t+1} K_t}{B_{t-1} + K_{t-1} + Q_t K_{t-1}} = \left(\frac{\omega}{q_t} + (1-\omega)\left(\frac{1 + Q_{t+1}}{Q_t}\right)\right)\Gamma_t. \tag{22}$$

Recall that $(\tau y_t - g_t)(1-\Gamma_t) \geq 0$. Not surprisingly, a primary surplus tends to lower the supply of both government securities. The nominal returns from t to $t+1$ on discount bounds and perpetuities are $1/q_t$ and $(1+Q_{t+1})/Q_t$, respectively. So the growth rate of the supply of nominal government securities is now proportional to a weighted average of the nominal holding period returns on the two government securities, with weights equal to the respective portfolio weights used by households.

The implications for inflation of fiscal variables and alternative policies for q_t and Q_t are obvious from (20) and (22). Primary surpluses temper inflation, simply because $\Gamma_t < 1$. Lowering the one-period nominal interest rate (raising q_t) has no effect on the current price level, but reduces inflation from the current period to the next. The effect is stronger the higher the share of discount bonds in household portfolios. As already noted, an increase in Q_t raises the contemporaneous price level because it induces households to attempt to consume more. Raising both Q_t and Q_{t+1} in proportion (say, because a policy of raising Q_t is in place for an extended period of time) lowers the nominal rate of return on the perpetuity. Just as a reduction in the one-period nominal interest rate, this lowers inflation from t to $t+1$.

The constant portfolio share ω for discount bonds implies that $B_t/K_t = (Q_t/q_t)\omega/(1-\omega)$. So the relative supplies of discount bonds and perpetuities are a function only of their relative price q_t/Q_t. An extreme government policy for this relative price will have to be met with extreme relative supplies of discount bonds and perpetuities.

4.1 Comparison with Perfect Foresight

The simple savings and portfolio decisions used by households in this economy allow the government to set both q_t and Q_t. In an economy in which households have perfect foresight, there would be an arbitrage relation between Q_t and the sequence $\{q_{t+n}\}_{n=0}^{\infty}$ of current and future nominal discount bond prices. For example, under pefect foresight, a policy $q_t = q < 1$ implies $Q_t = Q = q/(1-q)$, so that both nominal securities earn the same return $1/q$.[4] In that case the factor multiplying Γ_t in (22) is simply $1/q$.

Here instead, the simple decision rules followed by households imply that a permanent increase in q does not automatically result in a decline of the nominal return on perpetuities. But the government can set $Q = q/(1-q)$ or even raise Q beyond this level to reduce the nominal return on perpetuities. Similarly, if $q_t = 1$ for an extended period of time, the government can raise the initial Q_t by enough and then let $Q_{t+1} \leq Q_t - 1$

[4] One could consider perpetuities that deliver a geometrically declining sequence of coupons. Such perpetuities would have finite perfect-foresight values even if $q = 1$.

to achieve nominal returns on perpetuities that are also zero, or even lower. The initial increase in Q_t will raise the price level, but the low nominal returns that follow will lead to a reduction in subsequent inflation.

4.2 Elastic Portfolio Weights

In US data, the yields on nominal government securities of different maturities tend to move together. In the above economy, this would have to be interpreted as a deliberate choice of the government. It is more likely that households view the alternative government securities to be close substitutes and trade away from their "normal" portfolio share ω when the perceived returns on these securities diverge too much. If the government does not want to choose extreme relative supplies of discount bonds and perpetuities, this will force it to limit how much the nominal returns on these securities can differ.

For example, suppose households use the portfolio weight $\Omega(q_t, Q_t) \in (0, 1)$ for discount bonds, instead of ω. All this does is replace ω on the right-hand side of (21) with $\Omega(q_t, Q_t)$. The current price level continues to be determined by (20), but (22) becomes

$$\frac{B_t + K_t + Q_{t+1}K_t}{B_{t-1} + K_{t-1} + Q_t K_{t-1}} = \left(\frac{\Omega(q_t, Q_t)}{q_t} + (1 - \Omega(q_t, Q_t)) \left(\frac{1 + Q_{t+1}}{Q_t} \right) \right) \Gamma_t.$$

For example, $\Omega(q_t, Q_t)$ could be an increasing function of the nominal rate of return differential $(1/q_t) - (1 + 1/Q_t)$ that is implied if (q_t, Q_t) will not change between t and $t + 1$. If policy is indeed constant, this will put more weight on the higher of the two ex post nominal returns, and so inflation will tend to be higher than predicted by constant portfolio weights.

5. Concluding Remarks

In a frictionless economy, the fiscal theory of the price level makes the price level look like the stock market. It does not look like the stock market in the data, and this may be why it has, to date, found relatively few adopters. But when households follow simple and stable decision rules of the type considered in this paper, the fiscal theory predicts a price level that is quite smooth. Consumers in the wild live in a much more complicated world than researchers can analyze with pen and paper, or even put on a fast computer. Their decision rules will evolve as a result of their own experiences and of what they can learn from each other. Significant heterogeneity is likely to persist. The key issue is to understand how decision rules in the population are likely to change when policy changes significantly from the regime under which those decision rules have evolved.

References

[1] Lucas, R.E., Jr, "Econometric Policy Evaluation: A Critique," *Carnegie-Rochester Conference Series on Public Policy*, vol. 1 (1976), 19–46.