Economics 3012
Strategic Behavior

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with special guest
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October 27, 2006

Lecture 12

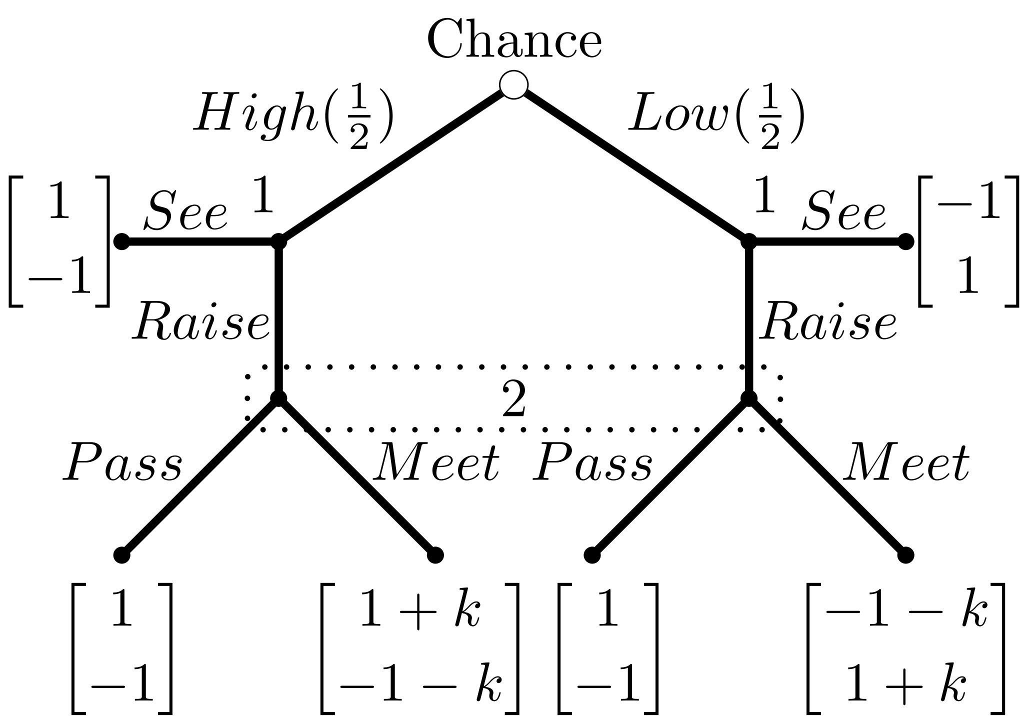
Topics

- Problem Set 10
- Topics in the Economics of Information
  - Signalling
  - Strategic Information Transmission

Problem Set 10

Exercise 319.1 of Osborne

Problem Statement: This is a variant of the poker game in Example 315.1 in which the amount bet when player 1 raises, and when player 2 meets, is k.



The problem is to find the Nash equilibria of this game, and in particular see how the probability of bluffing (that is, raising with a low card) varies with k.

Analysis: We compute the strategic form of the game:

$$\begin{array}{c} \\ S_HS_L \\ R_HS_L \\ S_HR_L \\ R_HR_L \end{array} \begin{array}{c} \begin{array}{cc} Pass & Meet \end{array} \\ \begin{pmatrix} (0,0) & (0,0) \\ (0,0) & (\frac{k}{2}, -\frac{k}{2}) \\ (1,-1) & (-\frac{k}{2}, \frac{k}{2}) \\ (1,-1) & (0,0) \end{pmatrix} \end{array}.$$

- Here S_HS_L is weakly dominated by R_HS_L, and it is not a best response to $Pass$, so it will not be assigned positive probability in any Nash equilibrium.
- Similarly, S_HR_L is weakly dominated by R_HR_L. It could only be played in a Nash equilibrium if the probability of $Pass$ was one. There is no such equilibrium because:
  - The best responses to $Pass$ are S_HR_L and R_HR_L.
  - If player one is assigning all probability to these, then $Meet$ is player 2's best response.

- Therefore any Nash equilibrium must be a Nash equilibrium of the truncated game

$$\begin{array}{cc} & \begin{array}{cc} Pass & Meet \end{array} \\ \begin{array}{c} R_H S_L \\ R_H R_L \end{array} & \left(\begin{array}{cc} (0,0) & (\frac{k}{2}, -\frac{k}{2}) \\ (1,-1) & (0,0) \end{array} \right). \end{array}$$

- The only Nash equilibrium of this game is

$$\left(\tfrac{2}{k+2} R_H S_L + \tfrac{k}{k+2} R_H R_L, \tfrac{k}{k+2} Pass + \tfrac{2}{k+2} Meet\right).$$

- The probability of bluffing is $\frac{k}{k+2}$, which is an increasing function of k.

Exercise 331.2 of Osborne

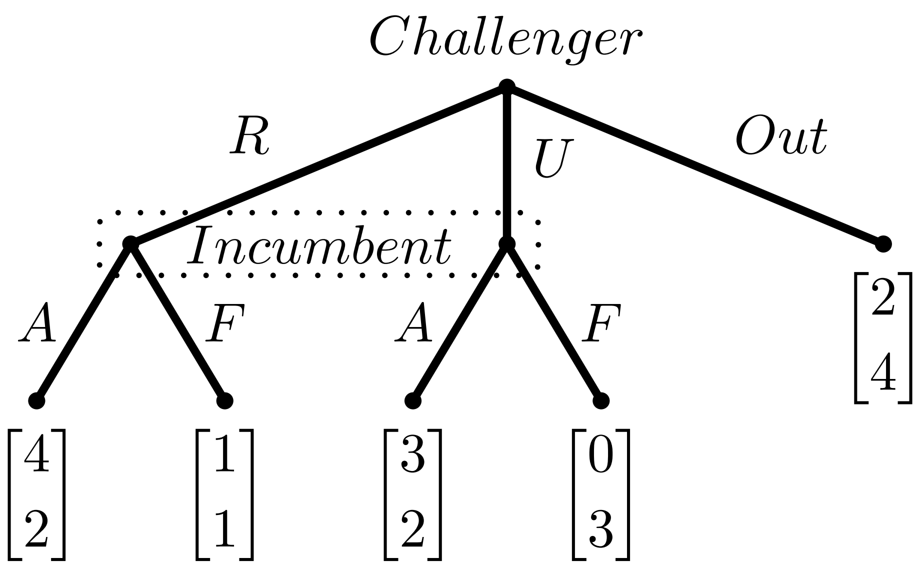
Problem Statement: The game in Figure 323.1 allows the Challenger to:

- enter prepared,
- enter unprepared, or
- stay out.

After the Challenger enters, the Incumbent decides (without seeing whether the Challenger is ready) to either

- acquiesce or
- fight.

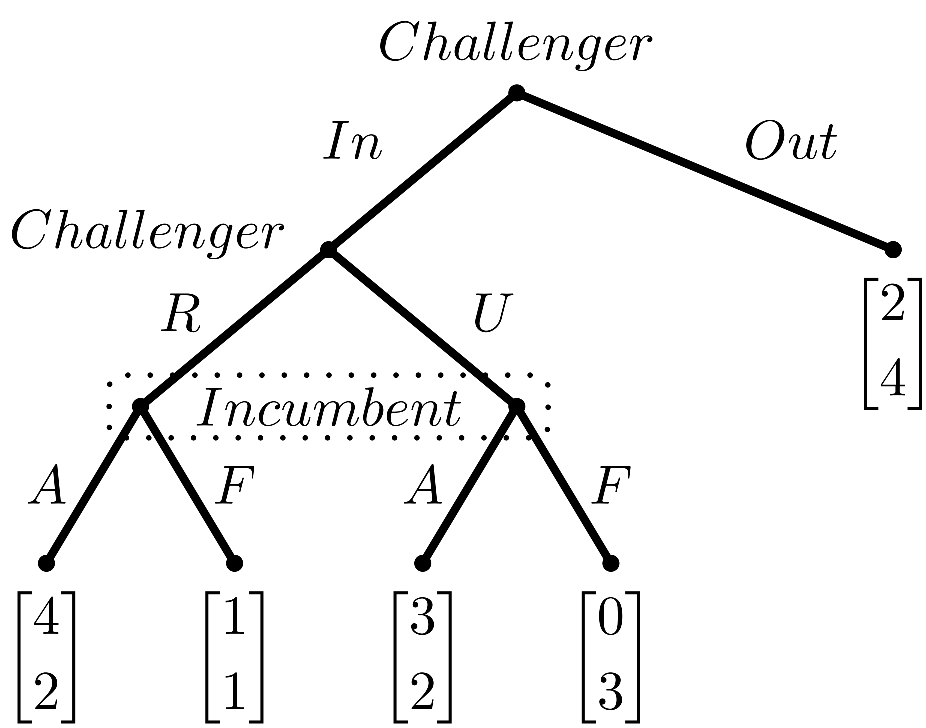
In the variant in Figure 331.1 (shown below) entering prepared always leads to a higher payoff than entering unprepared, regardless of the Incumbent's response.



In the variant in Figure 332.1 the payoffs are the same, but the Challenger's decision is broken into two parts.

- She first decides whether to go In or stay Out.
- If she goes In, she then decides whether to prepare.

This variant is shown below.



The problem given by Osborne to show that the second variant of this game has no weakly sequential equilibrium in which the Incumbent chooses to Fight, but

I think this is incorrect.

It becomes correct if we replace weak consistency with a somewhat stronger condition.

Mild Consistency

An assessment (β, μ) is **mildly consistent** if, for

- each agent i,
- each information set I_i for i,
- each partial history g such that every history in I_i has g as a proper subhistory, and
- each $h^* \in I_i$,

if the probability

$$\Pr(I_i \text{ according to } \beta|g) = \sum_{h \in I_i} \Pr(h \text{ according to } \beta|g)$$

of reaching I_i, conditional on having reached g, is positive, then player i's belief at I_i must assign probability

$$\frac{\Pr(h^* \text{ according to } \beta|g)}{\Pr(I_i \text{ according to } \beta|g)}$$

to h^*.

Analysis: In the second variant sequential rationality requires that the Challenger, after going In, makes an expected utility maximizing choice between R and U.

- Clearly R is the only rational choice, independent of the Incumbent's strategy.
- Weak consistency imposes no restriction on the Incumbent's beliefs if the Challenger is playing Out with probability one, since then her information set does not occur with positive probability.
- Thus there is a weak sequential equilibrium with the strategy profile (Out, R, F).

However,

- every history in this information set has In as a proper subhistory, and
- the information set for the Incumbent necessarily occurs with positive probability if In is chosen.

Therefore sequential rationality and mild consistency imply that the Incumbent's beliefs assign all probability to the partial history resulting from R.

- The rational response to this belief is to acquiesce.
- If the Incumbent is acquiescing, then sequential rationality requires that the Challenger play In.

The only assessments satisfying sequential rationality and mild consistency have the strategy profile (In, R, A).

An Advanced Aside: Forward Induction

Osborne asks which version of the game should be preferred. There is a principle called *forward induction* according to which the only equilibria that are permitted, in either version of the game, are the ones in which the Challenger acquiesces.

- The idea is that a belief that the Challenger has chosen U is inconsistent with rationality, whereas choosing R is rational for the Challenger *if* the Incumbent believes that R has been chosen and responds rationally.
- The version of the game in Figure 332.1 seems to bring "mildly sequential equilibrium" closer in line with forward induction, and in this sense it might be preferred.

But forward induction has never been given a fully satisfactory general definition, and remains controversial.

Signalling

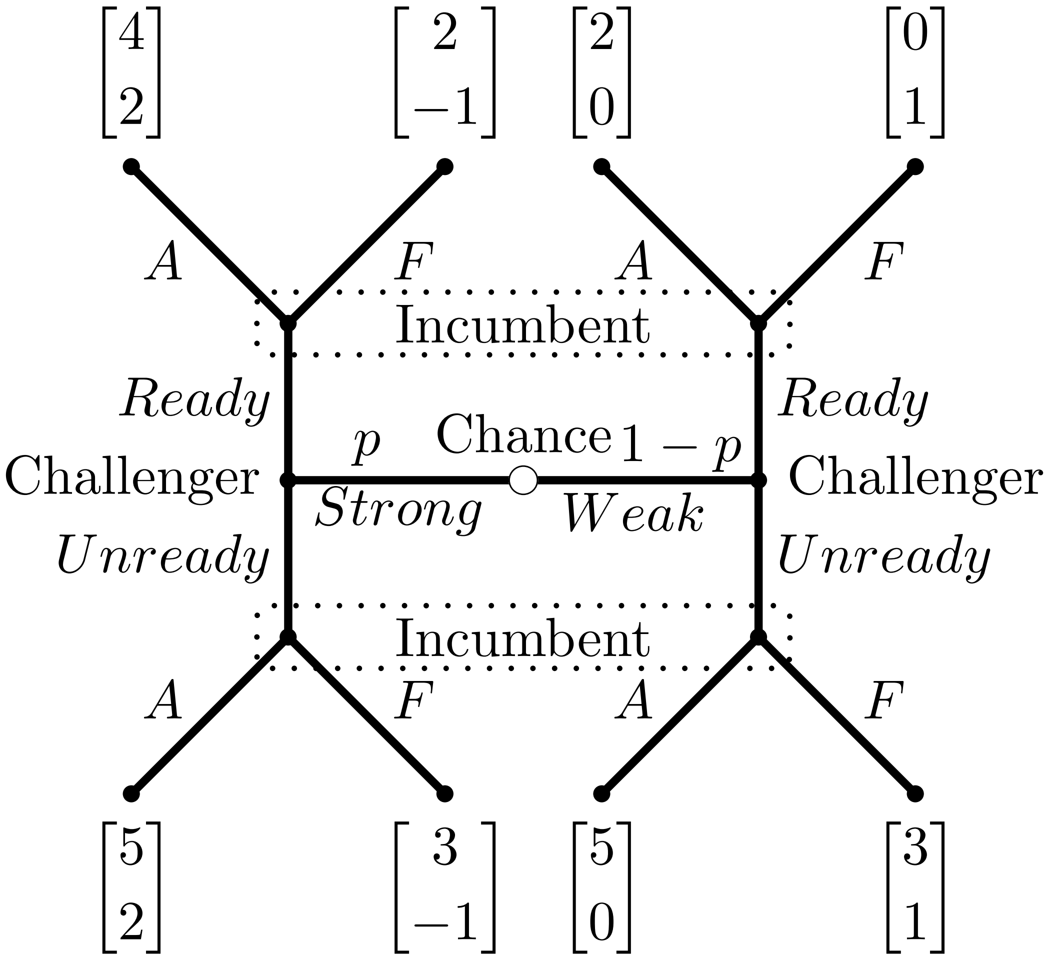
The idea of signalling is to take an observable action that influences others' belief about an unobservable attribute.

- A baby's crying signals hunger or other discomfort.
- Buying a sports car signals wealth.
- Offering a guarantee signals durability.
- Strong diplomatic messages signal a willingness to go to war.

An important feature of these examples, and the general idea, is that the cost of the signal depends on the underlying attribute.

- A signal is *credible* because even though the unhungry (poor, unreliable, unwilling) type might prefer to be thought to be hungry (rich, durable, willing) the cost of sending the signal exceeds the benefit.

Example 332.1



- Chance makes the Challenger strong (probability p) or weak (probability $1-p$).
- After learning her type, the Challenger either gets ready or does not.
- After seeing the Challenger's action the Incumbent decides to fight or acquiesce.

We will study the weak sequential equilibria of this games. In signalling games with two types (here Strong and Weak) the equilibria are of the following three types.

- A *pooling equilibrium* is a weak sequential equilibrium in which there is a single action that is chosen, with probability one, by both types.
- A *separating equilibrium* is a weak sequential equilibrium in which there each type has an action that it chooses with probability one, and the two actions are different.
- A *semi-separating equilibrium* is a weak sequential equilibrium in which one type has an action that it chooses with probability one, and the other type mixes between choosing this action and choosing a different action.

- First of all, note that the Weak type prefers being Unready, regardless of the Incumbent's strategy.
- Is there a pooling equilibrium in which the Strong type also chooses Unready?
  - Since neither type chooses Ready, weak consistency places no restriction on the Incumbent's belief after Ready is chosen.
  - If, after Ready, the Incumbent believes that the Challenger is weak, she will fight.
  - If the Challenger expects a fight after Ready, she is better off choosing Unready.
  - *This equilibrium strikes many researchers as weird and implausible: the Strong type is afraid to do something that makes no sense for the weak type because, in this equilibrium, that would lead the Incumbent to believe that she is weak.*

- Is there a separating equilibrium in which the Strong Challenger always chooses Ready? Yes:
  - In this case the Challenger's action reveals her type.
  - The Incumbent will acquiesce after Ready and fight after Unready.
  - Given this strategy of the Incumbent, both types of the Challenger are maximizing expected utility.
- Is there a semi-separating equilibrium in which the Challenger mixes?
  - If p is large enough, the Strong Challenger can mix is a way that makes the Incumbent indifferent after Unready, and the Incumbent can then mix 50-50, which makes the Strong Incumbent indifferent between Ready (which certainly leads to acquiescence) and Unready.

Exercise 335.1 of Osborne

Problem Statement:

An offspring

- is either hungry or not, and
- it can choose to squawk or keep quiet.

The parent

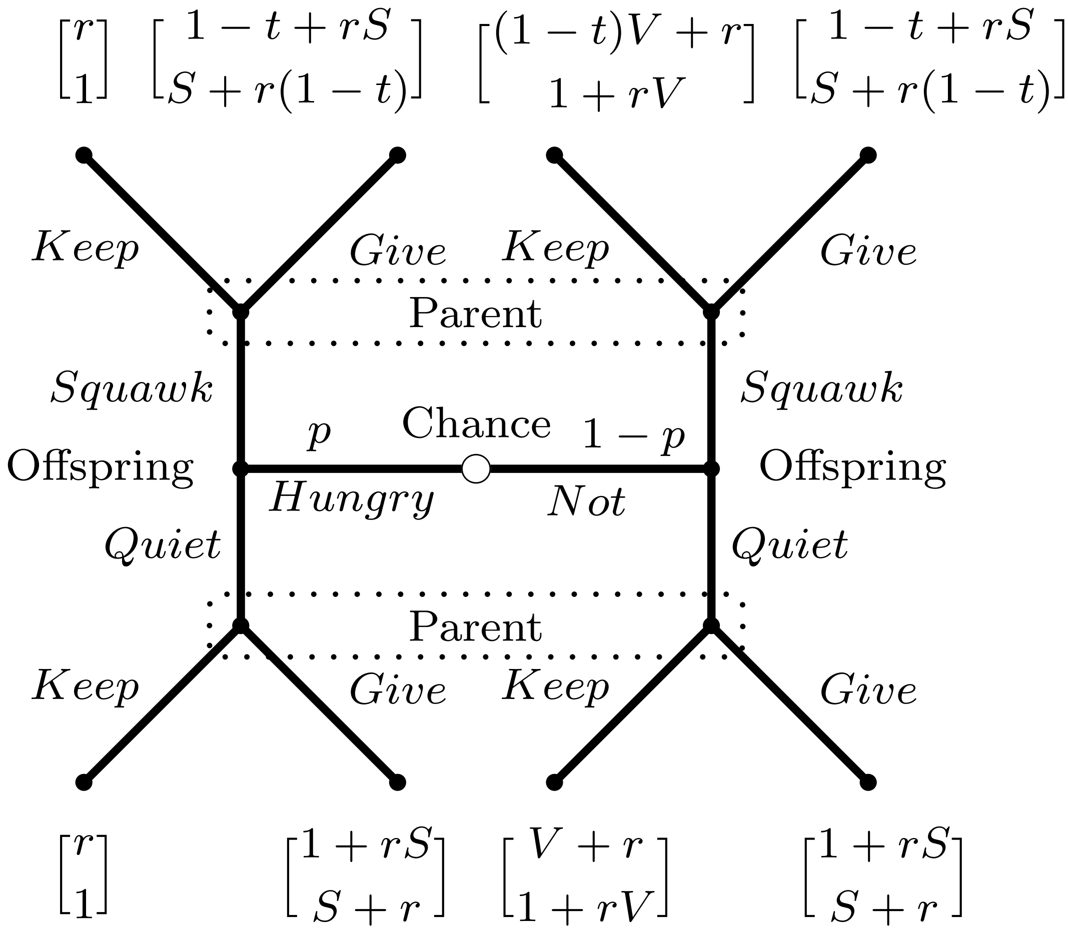
- does not observe whether the offspring is hungry, but
- can hear the squawk, and then
- decides whether to give a piece of food to the offspring.

In formulating payoffs we distinguish between each organism's "strength," or expected reproductive success, and its *inclusive fitness*, which includes the reproductive success of its relatives weighted by the degree of relatedness.

- The parent's strength is 1 if it keeps the food and $S < 1$ if it gives the food to the offspring.
- If the offspring does not squawk, then its strength is:
  - 1 if it gets the food, regardless of whether it is hungry,
  - 0 if it is hungry and does not get the food, and
  - V (where $0 < V < 1$) if it is not hungry and does not get the food.
- If it squawks, then these strengths are multiplied by $1 - t$ where $0 \leq t \leq 1$.

Let r be the degree of relatedness of the parent and the offspring, where $0 < r < 1$.

The game is shown below.



The problem has the following parts:

(a) Find conditions on r, S, V, and t under which the only equilibrium is for the offspring to squawk if and only if it is hungry and the parent to give the food if and only if the offspring squawks.

(b) Show that if the offspring is always better off if given the food, then such an equilibrium cannot exist unless $t > 0$.

(c) Show that if $r < (1 - S)/(1 - V)$, then the game has an equilibrium in which the offspring always keeps quiet and the parent always keeps the food.

Analysis: (a)

- When the offspring is quiet, the parent knows it is not hungry, and prefers keeping the food if

$$1 + rV \geq S + r.$$

- When the offspring squawks, the parent knows it is hungry, and prefers giving the food if

$$S + r(1 - t) \geq 1.$$

- When the offspring if hungry, it prefers squawking and getting the food to being quiet and not getting it if

$$1 - t + rS \geq r.$$

- When the offspring is not hungry, it prefers being quiet and not getting the food to squawking and getting it if

$$V + r \geq 1 - t + rS.$$

Expressing each of these as a condition on r, we find that there is an equilibrium of this sort if and only if

$$\max\{\tfrac{1-S}{1-t}, \tfrac{1-t-V}{1-S}\} \leq r \leq \min\{\tfrac{1-S}{1-V}, \tfrac{1-t}{1-S}\}.$$

(b) If $r < (1-V)/(1-S)$ then $\frac{1-t-V}{1-S} \leq r$ is impossible unless $t > 0$.

(c)

- If the parent is never giving the food, the offspring is always at least as well off if it keeps quiet.
- If the offspring always keeps quiet, then, when the offspring keeps quiet, the parent is better off keeping the food if

  $$p + (1-p)(1+rV) \geq S + r.$$

  - This is equivalent to $r \leq \frac{1-S}{1-(1-p)V}$, which is certainly true if $r < \frac{1-S}{1-V}$.

- The final condition to check is that the parent prefers to keep the food if the offspring squawks.
  - This depends on the parent's beliefs after squawking, which are indeterminate if the offspring always keeps quiet.
  - In order to show that an equilibrium of the indicated type exists, it suffices to find *some* belief that motivates keeping the food.
  - So, let's suppose that the parent believes the offspring is not hungry.
- Then keeping the food is better for the parent if

$$1 + rV \geq S + r(1 - t),$$

which is equivalent to $\frac{1-t-V}{1-V} r \leq \frac{1-S}{1-V}$.
  - Again, this is certainly true if $r < \frac{1-S}{1-V}$.

Education as a Signal

Your academic transcript says something about how much you learned in school, but it also conveys information about an underlying ability that existed before you started university.

Here is a simple model:

- Chance determines whether a worker has high (H) or low (L) ability.
  - Assume that H and L are numbers with $H > L > 0$.
  - The worker observes her ability.
  - Others cannot observe it directly.
- The worker chooses an education level $e \geq 0$.

- After seeing e, the labor market forms a belief about the worker, and she is paid her expected productivity w.
  - Let w_H and w_L be the wages received by worker who are known to have ability H and L respectively.
- If she chooses education level e and receives wage w, the utility of the worker is $w - e/H$ if her ability is high and $w - e/L$ if her ability is low.

Weak Sequential Equilibria

- Let $\hat{e} = L(w_H - w_L)$.
  - Then $w_H - \hat{e}/L = w_L$, so the low ability worker would be indifferent between getting w_H after choosing $\hat{e}$ and getting w_L after choosing 0.
- If $e^* \geq \hat{e}$ and $w_H - e^*/H \geq w_L$ there is a separating equilibrium in which the high type chooses e^* and the low type chooses 0.

- Let p be the fraction of workers with high ability, and let

$$\overline{w} = pw_H + (1-p)w_L$$

be the expected productivity of a worker.

- Let $\overline{e} = L(\overline{w} - w_L)$.
  - Then $w_H - \overline{e}/L = w_L$, so the low ability worker would be indifferent between getting $\overline{w}$ after choosing $\overline{e}$ and getting w_L after choosing 0.

- If $\overline{e} < e^* < \hat{e}$ there is a semi-separating equilibrium in which the high type always chooses e^* and the low types mix between e^* and 0, with just enough of them choosing e^* to make the wage such they are indifferent.

- If $e^* \leq \overline{e}$, then there is a pooling equilibrium in which all workers choose e^* and receive $\overline{w}$.

These equilibria all depend on the beliefs of the labor market being very unfavorable if a worker chooses any level of education other than e^*.

- In particular, it seems strange that choosing $e > \hat{e}$ should lead to a belief that the worker has low ability.
- This sort of (apparently unreasonable and unrealistic) "deterrence of deviations by threats of unfavorable beliefs" is a general problem with the (weak) sequential equilibrium concept.

Strategic Information Transmission

We often need to communicate with people whose objectives are not completely aligned with ours. How does the possibility of manipulation affect what is said and what is believed?

Here is a basic model:

- The state of the world is uniformly distributed in $[0, 1]$.
- The Sender sees the state of the world and sends a report to the Receiver.
- The Receiver sees the report, forms a revised belief about the state of the world, and then chooses an action $y \in [0, 1]$.
  - The Sender's utility is $-(y - (t + b))^2$.
  - The Receiver's utility is $-(y - t)^2$.
  - That is, the Receiver wants t to be close to t, and the Sender wants y to be close to $t + b$.

What sorts of weak sequential equilibria can we have?

- If $|b| > 0$ there cannot be perfect information transmission in equilibrium.
  - If the Sender was reporting honestly, then in effect the Receiver would be setting $y = t$, but when the state is t the Sender could do better by sending the report that induces the Receiver to set $y = t + b$ rather than the one that induces the Receiver to set $y = t$.
- There is always a “babbling equilibrium” in which no information transmission takes place.
  - Specifically, in the equilibrium the Sender’s report is unrelated (in a statistical sense) to the true state, and the Receiver’s choice of action is statistically unrelated to the Sender’s report.

- *Can there be an equilibrium in which, for some* $\bar{t}$*, the Sender sends one report* r_1 *when* $t < \bar{t}$ *and sends another report* r_2 *when* $t \geq \bar{t}$*?*
  - When the Receiver receives r_1, she thinks that any point in the interval $[0, \bar{t})$ is possible, and the way for her to maximize expected utility is to set $y = \bar{t}/2$.
  - When the Receiver receives r_2, she thinks that any point in the interval $[\bar{t}, 1]$ is possible, and the way for her to maximize expected utility is to set y to the midpoint of this interval, i.e., $y = (1 + \bar{t})/2$.

- In order for this to be an equilibrium the Sender must be indifferent between sending r_1 and r_2 when the state is $\bar{t}$. This means that

  $$-(\bar{t}/2 - (\bar{t} + b))^2 = -((1 + \bar{t})/2 - (\bar{t} + b))^2.$$

  Either solving this, or reasoning that $\bar{t} + b$ must be the midpoint of the interval $[\bar{t}/2, (1 + \bar{t})/2]$, leads to the conclusion that $\bar{t} = \frac{1}{2} - 2b$.
  - This is possible if $|b| < \frac{1}{2}$.

Similar analysis shows that equilibria that partition the interval into more than two subintervals are possible if $|b|$ is sufficiently small. *The upper bound on the accuracy of information transmission in equilibrium is determined by* $|b|$. If $|b|$ is small, then the very detailed information transmission is possible. If $|b|$ is large, then the conflict between the agents' objective creates possibilities for manipulation that undermine detailed communication.