University of Sydney

Department of Economics

Mathematical Methods of Economic Analysis

Derivatives Questions

PART I

From Simon and Blume, do the following:

- Chapter 14: 14.1, 14.5, 14.7, 14.11, 14.19, 14.22 (the "Jacobian derivative matrix" referred to in question 14.22 is simply the total derivative of the vector valued function; the total derivative of each component function will be a gradient vector, so all of these derivatives together will form a matrix).

PART II

Q1. Suppose that a consumer has a utility function $U\colon \mathbf{R}_+^2 \to \mathbf{R}$, where $U = U(x,y)$. Assume that $U_{xx}(x,y) < 0$, i.e., the marginal utility of x declines as x increases. Now let f be any function that has the range of U within its domain and has a positive derivative. Define a new utility function

$$V = V(x,y) = f(U(x,y)).$$

Since f has a positive derivative and is therefore a strictly increasing function, it follows that V and U give the same rankings of commodity bundles, i.e., have the same indifference curves.

(i) Calculate V_x, V_y, V_{xx}, V_{yy}, and $V_{xy} = V_{yx}$.

(ii) Is it necessarily the case that $V_x/V_y = U_x/U_y$, where all partial derivatives are evaluated at the same point? Interpret this result.

(iii) Is it necessarily the case that $V_{xx} < 0$? Interpret this result.

Q2. A function $f(x_1, \ldots, x_n)$ is homogeneous of degree r if it satisfies the following equation for every $\mathbf{x}$ vector in its domain and every $\lambda > 0$:

$$f(\lambda \mathbf{x}) = \lambda^r f(\mathbf{x}).$$

By differentiating both sides of this equation with respect to λ for given $\mathbf{x}$, and then evaluating the derivative at $\lambda = 1$, show that

$$\sum_{i=1}^{n} f_i(\mathbf{x}) \cdot x_i = rf(\mathbf{x}).$$

Hint: You will need to use a version of the chain rule.