

**Economics 8117-8****Noncooperative Game Theory****May 1, 1997****Lecture 20****Professor Andrew McLennan**

## **Reputation**

### **I. Introduction.**

- A. We consider the possibility, discussed by Kreps, Milgrom, Roberts, and Wilson, that adding a small probability of a “crazy” type of agent may create new equilibria in which rational players act like the crazy player in order to have a reputation for behaving that way.
- B. The idea is that a small probability of the crazy type can have a large effect on the equilibrium.
- C. We examine this in two contexts.
  1. The finitely repeated prisoner’s dilemma.
  2. The chain store paradox.

### **II. Cooperation – Consider a finitely repeated prisoner’s dilemma.**

- A. Numerous studies have shown that most people will cooperate until a very few period before the end so long as their partner cooperates.
  1. Fewer observation are available, but one certainly expects that people will not continue to cooperate if their partner deviates frequently.
  2. A rational agent who knew this would cooperate until a few periods before the end.
- B. Roughly, what unravels this as a true equilibrium is that the rational agent defects, on average, somewhat earlier than the rest of the population.
  1. Suppose, however, that a small fraction of the population was committed in some way to play tit-for-tat, regardless of rationality. (They do it instinctively, they enjoy it, they believe in it morally, etc.)

2. One might guess that even a small fraction of the population committed to tit-for-tat could force an equilibrium in which all agents play tit-for-tat up until the last few rounds.
3. This is indeed the result.

### III. Deterrence – the Chain Store Paradox.

- A. Well established theory shows that, faced with a single entrant, a firm always does better sharing the market rather than using low prices to bankrupt the entrant, a strategy called predatory pricing.
- B. A finite repetition of the entry game therefore unravels like the prisoner's dilemma: since the incumbent will share markets  $N - n + 1, \dots, N$  no matter what, he cannot credibly threaten to use predatory pricing in market  $N - n$ .
- C. Again, reputation effects are induced by introducing a small probability that the monopolist is someone who always reacts to entry with predation.
- D. We will analyze this in detail later.

### IV. How satisfactory is this theory?

- A. It is certainly realistic in a casual sense.
  1. People are not always “rational”, and many aspects of behavior are either automatic or are guided by emotional responses that are not rational choices.
  2. Note that in each case above the “crazy” type does almost as well as the rational agent.
  3. This means that the crazy type will be driven from the market or the gene pool slowly, if at all.
- B. The most troublesome aspect of the theory for me is that the crazy types are chosen quite carefully. What about other involuntary strategies?
  1. In fact a large part of the appeal of this work is that the anger of the crazy types seems so “human”.

2. When one thinks about this aspect of human nature it seems that its *raison d'être* is precisely to create and maintain a reputation.
3. One then wants to ask why this particular form of craziness was selected by evolution.
4. The natural approach is to investigate the evolution of gene pools in which many different strategies are present initially.
  - a. This has been done by means of computer experiments by Robert Axelrod reported in *The Evolution of Cooperation*.
  - b. The results are, in general, very favorable to tit-for-tat.

## V. Detailed Analysis of the Chain Store Paradox.

### A. The Stage Game.

1. The potential entrant for that period decides whether to be In or Out.
  - a. If the entrant chooses Out then the entrant's payoff is 0 and the monopolist's payoff is  $a$  regardless of type.
2. If the entrant chooses In then the monopolist must decide whether to Acquiesce or Fight.
  - a. If the monopolist Acquiesces then the entrant receives  $b$ , the weak monopolist receives 0, and the tough monopolist receives  $-1$ .
  - b. If the monopolist Fights then the entrant receives  $b-1$ , the weak monopolist receives  $-1$ , and the tough monopolist receives 0.
3. We assume that  $a > 1$  and  $0 < b < 1$ .

### B. The Value Correspondence.

1. Let  $p_n$  denote the probability that the monopolist is tough when there are  $n$  remaining stages.
2. Let  $V_n(p_n)$  be the set of pairs of payoffs  $v = (v_W, v_S)$  for the two types of monopolist in sequential equilibria of the game with  $n$  stages and prior  $p_n$ .

in which the tough monopolist never acquiesces.

### C. Analyzing the Value Correspondence.

1. Suppose  $v \in V_n(p_n)$ . In a sequential equilibrium supporting this we have the following situation.
2. Variables.
  - a.  $\pi_E(I), \pi_W(A)$ .
  - b.  $\mu_F$  – posterior probability of a tough monopolist after  $F$ .
    - i. Sequential equilibrium requires that  $p_n$  be the posterior after 0, and we assume (with later justification) that  $p_{n-1} = 0$  after  $A$ .
  - c.  $v_A = (v_{AW}, v_{AS}) \in V_{n-1}(0), v_F = (v_{FW}, v_{FS}) \in V_{n-1}(\mu_F),$   
 $v_0 = (v_{0W}, v_{0S}) \in V_{n-1}(p_n)$

### 3. Relations.

- a. Beliefs –

$$\mu_F = \frac{p_n}{[p_n + (1 - p_n)(1 - \pi_W(A))]}.$$

- b. Payoffs –

$$v_W = (1 - \pi_E(I))(a + v_{0W}) + \pi_E(I)[\pi_W(A)v_{AW} + (1 - \pi_W(A))(-1 + v_{FW})].$$

$$v_S = (1 - \pi_E(I))(a + v_{0S}) + \pi_E(I)v_{FS}.$$

- c. Rationality.

- i.  $p_E(I) = 0(1)$  if

$$(1 - p_n)\pi_W(A)b + [1 - (1 - p_n)\pi_W(A)](b - 1) < (>) 0$$

- ii.  $\pi_W(A) = 0(1)$  if  $v_{AW} < (>) -1 + v_{FW}$ .

### D. The One Period Problem.

1. Here  $v_A = v_F = v_0 = (0, 0)$ .
2. Consequently  $\pi_W(A) = 1$ .

3. Thus  $\pi_E(I) = 0(1)$  if  $p_n > (<)b$ .

4. We have

$$V_1(p_1) = \begin{cases} (0, 0), & p_1 < b, \\ (a, a), & p_1 > b. \end{cases}$$

5. We do not specify  $V_1(b)$  in order to simplify the visual presentation. Here and below it is to be understood that the correspondence at such a point is the convex hull of the limits from above and below. In this case we have  $V_1(b) = \{(c, c) | 0 \leq c \leq a\}$ .

#### E. The Two Period Problem

1. **Case 1:**  $\pi_W(A) = 0$ .

a. Bayes' rule gives  $\mu_F = p_2$ .

b. From  $V_1$  we have  $v_{AW} = 0$ .

c. Since the entrant is always fought,  $\pi_E(I) = 0$ .

d.  $v_{AW} = 0 \leq -1 + v_{FW} \Rightarrow v_{FW} \geq 1 \Rightarrow p_2 = \mu_F \geq b$ .

e. We either have  $p_2 > b$  and  $v = (a, a) + v_0 = (2a, 2a)$ , or  $p_2 = b$  and  $v = (a, a) + v_0 = (c, c)$ ,  $1 + a \leq c \leq 2a$ .

2. **Case 2:**  $0 < \pi_W(A) < 1$ .

a.  $0 = v_{AW} = -1 + v_{FW} \Rightarrow v_F = (1, 1) \Rightarrow \mu_F = b \Rightarrow \pi_W(A) = \frac{(b-p_2)}{b(1-p_2)} \Rightarrow p_2 < b$ .

b. The entrant prefers to enter when

$$(1 - p_2)\pi_W(A)b + (1 - (1 - p_2)\pi_W(A))(b - 1) > 0 \quad \text{or}$$

$$(1 - p_2) \left[ \frac{(b - p_2)}{b(1 - p_2)} \right] b + \left( 1 - (1 - p_2) \frac{(b - p_2)}{b(1 - p_2)} \right) (b - 1) > 0 \quad \text{or}$$

$$p_2 < b^2.$$

c. When the entrant enters the payoffs are  $(0, 1)$ .

d. When the entrant does not enter the payoffs are  $(a, a)$ .

3. **Case 3:**  $\pi_W(A) = 1$  - This is impossible, except in the trivial case  $p_2 = 0$ , since the weak monopolist could get  $-1 + a$  by fighting but only gets 0 by acquiescing.

4. We therefore have

$$V_2(p_2) = \begin{cases} (0, 1), & p_2 < b^2, \\ (a, a), & b^2 < p_2 < b, \\ (2a, 2a), & b < p_2. \end{cases}$$

#### F. The General Case

1. A pattern is emerging. Suppose the value function has the following form:

$$V_{n-1}(p_{n-1}) = \begin{cases} (0, \hat{v}_{n-1, S}) & 0 \leq p_{n-1} < b^{n-1}, \\ (ka, \hat{v}_{n-k-1, S} + ka), & b^{n-k} < p_{n-1} < b^{n-k-1}, k = 1, \dots, n-1. \end{cases}$$

a. Given  $\hat{v}_{1S} = 0$  and  $\hat{v}_{2S} = 1$ ,  $\hat{v}_{nS}$  is determined recursively by the formula

$$v_{nS} = \left(1 - \frac{1}{a}\right)v_{n-1, S} + \left(\frac{1}{a}\right)(v_{n-2, S} + a)$$

2. Take this as an induction hypotheses, and suppose that there are  $n$  periods to play with prior  $p_n$ .

3. **Case 1:**  $\pi_W(A) = 0$ .

a.  $\mu_F = p_n$ .

b. The “relevant” value of  $v_{AW}$  is 0.

c.  $v_{AW} = 0 \leq -1 + v_{FW} \Rightarrow v_{FW} \geq 1 \Rightarrow p_n \geq b^{n-1}$ .

d. Clearly  $\pi_W(A) = 0 \Rightarrow \pi_E(I) = 0 \Rightarrow v = (a, a) + v_0$ .(\*)

4. **Case 2:**  $0 < \pi_W(A) < 1$ .

a.

$$\begin{aligned} 0 = v_{AW} &= -1 + v_{FW} \Rightarrow v_{FW} = 1 \\ \Rightarrow \mu_F = b^{n-1} &= \frac{p_n}{[p_n + (1 - p_n)(1 - \pi_W(A))]} \\ \Rightarrow \pi_W(A) &= \frac{(b^{n-1} - p_n)}{b^{n-1}(1 - p_n)} \\ \Rightarrow p_n &< b^{n-1}. \end{aligned}$$

b. The entrant prefers to enter when

$$(1 - p_n)\pi_W(A)b + [1 - (1 - p_n)\pi_W(A)](b - 1) > 0 \quad \text{or}$$

$$\left[ \frac{(b^{n-1} - p_n)}{b^{n-1}} \right] b + \left[ 1 - \frac{(b^{n-1} - p_n)}{b^{n-1}} \right] (b - 1) > 0 \quad \text{or}$$

$$p_n < b^n.$$

i. The payoffs in this case are  $(0, v_{FS})$  where  $v_{FS}$  is determined by the condition  $v_{FW} = 1$ .

c. The entrant does not enter in the remaining case, namely  $b^n < p_n < b^{n-1}$ , and we have  $v = (a, a) + v_0$ .

5. **Case 3:**  $\pi_W(A) = 1$  – As before this is impossible except in the trivial case  $p_n = 0$ .

6. Summarizing:

- a. When  $p_n < b^n$  we have  $v = (0, v_{FS})$  where  $v_{FS}$  is determined by the condition  $v_{FW} = 1$ . This value for the weak monopolist is brought about by having the entrant not enter with probability  $\frac{1}{a}$ .
- b. When  $b^n < p_n$  we have  $v = (a, a) + v_0$ .
- c. This obviously completes the inductive argument.

## VI. Stability.

A. We have been restricting attention to equilibria in which  $\pi_T(A) = 0$ .

1. There are equilibria in which both monopolists acquiesce because they want to avoid being thought weak.
2. We will argue informally that these can be eliminated using stability.

B. Consider an equilibrium in which  $0 < \pi_T(A)$  after some history.

1. We will assume that we considering a latest point in the tree where this happens.
2. Thus the values above can be used to evaluate continuations for the weak monopolist.

C. Consider the case  $0 < \pi_T(A) < 1$ .

1. Letting  $v_{TA}, V_{TF}, v_{WA}, v_{WF}$  be the values of the continuations, and letting  $\mu_A, \mu_F$  be the posteriors, we have the following sequence of implications:
  - a.  $v_{TA} > v_{TF} \Rightarrow \mu_A \geq \mu_F \Rightarrow v_{WA} \geq v_{WF} \Rightarrow \pi_W(A) = 1 \Rightarrow \mu_F = 1$ .
  - b. Here we are using certain facts that follow from the inductive calculation of  $V_n$  above – namely that the monopolist's values are monotonic and, at a belief where several values are possible, the weak and strong monopolists' payoffs are monotonically related.
  - c. Thus we assume that  $\pi_T(A) = 1$  at a latest point where  $\pi_T(A) \neq 0$ .

D. This has important consequences.

1. The set of sequential equilibria with  $\pi_T(A) = 0$  everywhere and the set of other sequential equilibria can be separated by open sets.
2. Consider trembles in which the tough monopolist is much more likely than the weak monopolist to involuntarily choose  $F$ . Using the facts about the value correspondence mentioned above one can show that  $\varepsilon$ -perfect equilibria for such trembles necessarily have  $\pi_T(A)$  close to 0 everywhere.
3. Thus in a certain sense all stable sets of sequential equilibria are of the desired type.