Economics 8117-8 **Noncooperative Game Theory**

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Bargaining Between Asymmetrically Informed Agents

I. Introduction.

A. Rubinstein's 1982 paper provoked considerable interest in bargaining.

1. Rubinstein provided a deterministic theory of bargaining, a problem of acknowledged importance, based on a noncooperative equilibrium theory.
2. The theory was incomplete in many respects, but clearly one of the most important was that his model did not allow the agents to have relevant private information.
   a. In particular, Rubinstein's model predicts immediate agreement on an efficient outcome.
   b. Causes of inefficiency aside from incomplete information have not been investigated as much as they might.
      i. Nonconvex feasible sets.
      ii. Consequences of current agreements, and the means by which they are reached, for the future distribution of power.

B. A first cut at what might be an equilibrium outcome is given by the set of incentive compatible outcomes.

1. Technically an *outcome* is a joint distribution over vectors of types and possible agreements or failures to agree.
2. The revelation principle states that in equilibrium no type of an agent prefers the probability distribution over agreements and disagreements that another type of the same agent is receiving in the equilibrium.
3. Consider the simplest case: a risk neutral seller whose valuation is known to

be 0 bargaining with a risk neutral buyer whose valuation may be either $\underline{b}$ or $\bar{b}$, where $0 < \underline{b} < \bar{b}$.

a. An immediate consequence of the revelation principle is that if the equilibrium is efficient – that is, the good changes hands with probability 1 – then both buyers pay the same (expected) price.

b. With two sided asymmetric information it is easy to concoct examples in which there is no fully efficient individually rational incentive compatible outcome.

4. Incentive compatibility alone does not come close to determining a finite set of equilibrium outcomes.

C. Analysis of games like Rubinstein's where agents are asymmetrically informed has foundered on a multitude of equilibria.

1. When an agent has a multitude of possible types, sequential equilibrium allows the other agents to "punish" deviations from the equilibrium path by that agent by adopting unfavorable beliefs about his type.

a. Stability imposes some restrictions on punishment by beliefs – in the case of bargaining the main constraints are derived from the intuitive criterion – but these restrictions still allow a continuum of outcomes.

2. One particularly unfortunate aspect of punishment by beliefs is that it makes the solution set very sensitive to the distinction between a type being impossible and a type having probability zero.

3. This problem with the sequential equilibrium concept is really quite a bit more general, and more important, than the problem of bargaining between asymmetrically informed agents, and there now seems to be little sense in working on the latter problem without further progress on the former.

D. Cramton (1985) is an example of the pitfalls of this subject.

1. The thesis containing this paper was greeted with great enthusiasm.

a. The analysis was lucid and clearly quite talented.

b. The paper is exceedingly well written.

c. The presentation is elegant and professional.

d. At the time the work seemed to represent substantial progress.

2. Cramton considers two sided uncertainty in which the buyer and the seller have continuously and independently distributed valuations.

   a. He considers equilibria in which the time of first concession is a strictly monotone function of type (tough types concede later), so one's first concession is fully revealing.

   b. Once the value of the game after a first concession is determined, the incentive compatibility conditions determine a pair of linked differential equations whose solutions are well behaved.

3. The problem is that the off the path behavior specified by Cramton is difficult to derive from any sort of appeal to abstract criteria, so there is ultimately little apparent reason for preferring Cramton's equilibrium to any of the other equilibria of the model.

II. Fudenberg–Tirole (1983) and Gul–Sonnenschein–Wilson (1986).

A. The one type of model that is tractable has the uninformed agent making all the offers.

   1. Of course this seems highly unnatural, both because the rules treat the agents asymmetrically and because it seems most natural for the agent with information to try to dictate terms.

   2. From a technical point of view it is easy to see why such models might tend to have unique equilibria, or at least unique stable equilibria. Beliefs are uniquely determined by Bayes' rule except after the time at which all informed agents should have accepted an offer, and usually stability will imply that the

equilibrium path must be compatible with beliefs at this point assigning high probability to tough types.

B. In one circumstance this model seems natural. Suppose that instead of facing a single buyer with a probability distribution over valuations, the seller is a monopolist who faces a continuum of consumers with a distribution of valuations.

1. This is the problem of a durable goods monopoly.
   a. The seller wishes to select a path of prices that maximizes profits.
   b. He is constrained by subgame perfection which implies that the monopolist will not continue on a path if revising his plan increases profits.
   c. Coase conjectured that this competition between the monopolist and himself (in the future) would lead to a competitive outcome.
2. Gul, Sonnenschein, and Wilson analyze this model, arriving at the conclusion that Coase was right.
   a. Precisely, as the amount of time between price changes becomes small, the price path converges to the constant cost of production path.
   b. An intuitive explanation of this result is as follows.
      i. At any point the monopolist could accelerate the path by offering tomorrow's price today, the day after tomorrow's price tomorrow, and so on.
      ii. The benefit of doing so is the interest on the continuation value tomorrow which is of order $V\Delta t$.
      iii. The cost is the difference between today's price and tomorrow's price multiplied by the number of agents buying today. This quantity is of order $(\Delta t)^2$.
      iv. The "marginal condition" then yields $V \sim \Delta t$.
   c. This result is in some ways implausible, and it is sensitive to changes in some of its assumptions.