Cigarette Money

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Agents

- Time continuous and infinite
- $[0, 1]$ continuum of identical, infinitely-lived agents
- Discount rate $r > 0$

Commodities

- Finitely many storable and indivisible goods
  - One general good (cigarettes)
  - Many special goods
- u_g utility from consuming general good
- u_s utility from consuming special good
- $u_s > u_g$

Production

- d: Poisson depreciation rate of general goods
- Consumption of a general good (or depreciation), produces a special good
- Consumption of a special good, produces a special good with prob. σ or a general good with prob. $1 - \sigma$

Matching

- Random matching: arrival rate α
- $x \leq 1$ prob of single coincidence meeting
- $y < 1$ conditional prob. of double coincidence meeting

Matching

- Always trade anything for special good and consume immediately
- Always trade a special good for a general good

Matching

Consume or store the general good?

- θ fraction of pop chooses to always consume the general good
- $1 - \theta$ choose to always store it

Steady State

- G steady state fraction of population holding general
- Steady State equation:

$$(1-S)\left[d+\alpha x S\sigma\right]=\left[\alpha(1-S)x+\alpha Sxy(1-\sigma)\right](S-\theta)$$

Steady State

Solve for θ as a function of S:

$$\theta = \frac{Q(S)}{1 - S + y(1-\sigma)S}$$

where

$$Q(S) = -(1-\sigma)(1-y)S^2 + \left(1 - \sigma + \frac{d}{\alpha x}\right)S - \frac{d}{\alpha x}$$

Bellman Equations

$$
\begin{aligned}
rV_s &= \alpha Gx[\theta(u_g + V_s) + (1-\theta)V_g - V_s] \\
&\quad + \alpha Sxy\,\{\theta[u_s + \sigma V_s + (1-\sigma)(u_g + V_s)] \\
&\quad +(1-\theta)(u_g + \sigma V_s + (1-\sigma)V_g) - V_s\} \\
&= \alpha Gx[\theta u_g + (1-\theta)(V_g - V_s)] \\
&\quad + \alpha Sxy[u_s + (1-\sigma)\theta u_g + (1-\sigma)(1-\theta)(V_g - V_s)] \\
rV_g &= \alpha Sx[u_s + \sigma(V_s - V_g)] + d(V_s - V_g)
\end{aligned}
$$

Individual Optimization

$$\Delta = u_g + V_s - V_g$$

- Optimizing wrt θ:

$$\begin{aligned} \Delta > 0 &\implies \theta = 1 \quad \text{(everybody always consumes)} \\ \Delta = 0 &\implies \theta \in [0,1] \\ \Delta < 0 &\implies \theta = 0 \quad \text{(everybody always stores)} \end{aligned}$$

$$\Delta = A\left[1 - S + \frac{r}{\alpha x} + (1-\sigma)Sy + S\sigma + \frac{d}{\alpha x}\right]u_g - AS(1-y)u_s$$

where $A > 0$

Steady State Equilibrium

Definition: A *Steady State Equilibrium* is $(S, V_s, V_g, \theta, \Delta)$ satisfying

1. Steady state equation
2. Bellman equations
3. Individual optimization

subject to $V_g \geq 0$, $V_s \geq 0$, $0 \leq S \leq 1$ and $0 \leq \theta \leq 1$.

Proposition 1

Proposition (1)

There exists $y_1 < 1$ *and* $y_2 < y_1$ *such that:*

1. *If* $y \geq y_1$*, then* $\theta = 1$ *and* $S = 1$*,*
2. *If* $y \in (y_2, y_1)$*, then* $\theta \in (0, 1)$ *and* $S \in (\hat{S}, 1)$*, and*
3. *If* $y \leq y_2$*, then* $\theta = 0$ *and* $S = \hat{S}$*.*

These are all of the (steady state) equilibria.

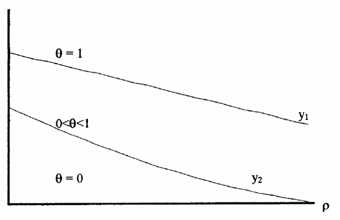
When are general goods used as money?

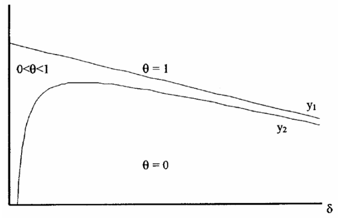
- $\theta < 1$
- iff $y < y_1$
  $\Rightarrow$ by Prop 1, general goods used as money if y small or y_1 big

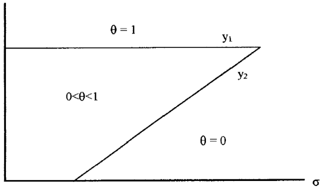
When are general goods used as money?

- y small when goods and tastes are highly specialized
- y_1 is big whenever
  - General goods not very desirable (u_g/u_s low)
  - More patience (r is low)
  - General goods do not depreciate quickly (d low)
  - Search frictions not too severe (αx big)
  - Special goods produced infrequently (σ low)

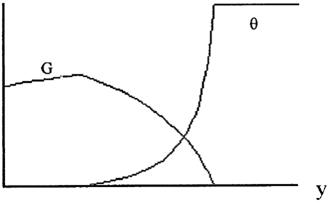
Regions of Parameter Space and Equilibria

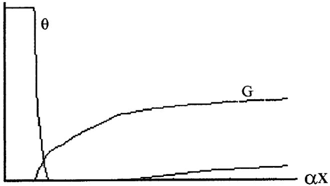


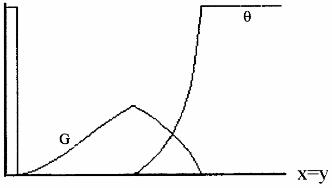




Equilibrium values of θ and G







Setup

- Special goods perfectly divisible
- General good indivisible
- Agents bargain over amount of special good traded for unit of the general good

Setup

- Agents hold 0 or 1 units of general good
- q units of the special good yields $u_s = U_s(q)$
- Producing q units yields disutility $c(q) = q$
- $U_s(0) = 0,\ U_s'(q) > 0,\ U_s''(q) < 0$, some $\hat{q}$ with $U_s(\hat{q}) = \hat{q}$

Bellman Equations

- Ω expected net utility flow from bartering
- q the amount of special good traded for a general good

Bellman Equations

$$\begin{aligned}\rho V_s &= (1-S)[-q+\theta u_g+(1-\theta)(V_g-V_s)] \\ &\quad +Sy[\Omega+(1-\sigma)\theta u_g+(1-\sigma)(1-\theta)(V_g-V_s)]\end{aligned}$$

$$\rho V_g = S[U_s(q)+\sigma(V_s-V_g)]+\delta(V_s-V_g)$$

Exchange of Goods

- For barter, symmetric Nash bargaining
  - $\Rightarrow$ both agents produce q^*

$$\begin{aligned} U_s'(q^*) &= c'(q^*) = 1 \\ \Omega &= U_s(q^*) - q^* \end{aligned}$$

- For general versus special goods, general good holder makes TIOLI offer

$$\Rightarrow q = \theta u_g + (1-\theta)(V_g - V_s)$$

Steady State Equililbrium

- A *steady state equilibrium* is a list $(S, V_s, V_g, \theta, \Delta)$
- q now endogenous
- Look for a triple (S, θ, q) satisfying best reply condition, individual optimization, and bargaining solution

Proposition 2

Proposition (2)

There exists $\bar{y}_1$ and $\bar{y}_2 < \bar{y}_1$ such that:

1. *If $y \geq \bar{y}_1$, then there is an equilibrium with $\theta = 1$ and $q = u_g$.*
2. *If $y \in (\bar{y}_2, \bar{y}_1)$, there is an equilibrium with $\theta \in (0, 1)$ and $q = u_g$, and*
3. *If $y < \bar{y}_2$, then there is an equilibrium with $\theta = 0$ and $q > u_g$.*

Proposition 2

Proposition (2)

Moreover, there is a $z > 0$ such that if $u_g > z$ then these are the only equilibria, but if $u_g < z$ then $\bar{y}_2 > 0$ and there exists $\bar{y}_3 > \bar{y}_2$ such that when $y \in (\bar{y}_2, \bar{y}_3)$, there are two other equilibria, both having $\theta = 0$ but different values of $q > u_g$. These are all of the (steady state) equilibria.

What are the prices of cigarette money?

- $\theta \in (0, 1] \Rightarrow q = u_g$
- $\theta = 0 \Rightarrow q > u_g$

Equilibria with Endogenous Prices

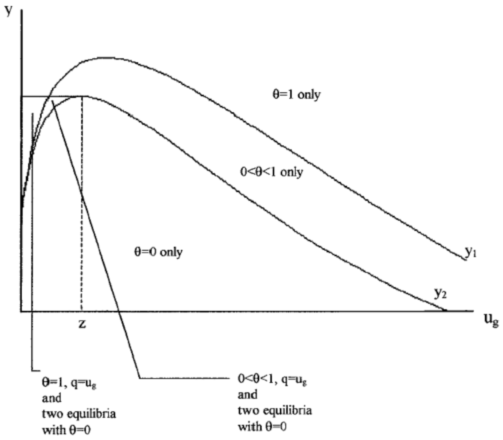


FIG. 4. Equilibria with endogenous prices.

Setup

- Introduce fiat currency to compete with cigarette money
- Fiat currency neither produced nor consumed
- Exogenous fraction M of population endowed with it

Setup

- Fiat money and general good are indivisible
- Special goods are divisible
- Assume $d = 0$ ($\Rightarrow$ unique equilibrium)
- Focus on cases where $V_m > 0$

Bellman Equations

- G, S, M steady state fractions of pop. holding general, special good, and money
- q_m, q_g amounts of special good traded per unit of money or general good

Bellman Equations

$$
\begin{aligned}
\rho V_s &= Sy[\Omega + (1-\sigma)\theta u_g + (1-\sigma)(1-\theta)(V_g - V_s)] \\
&\quad + G[-q_g + \theta u_g + (1-\theta)(V_g - V_s)] \\
&\quad + M[-q_m + (V_m - V_s)]
\end{aligned}
$$

$$
\rho V_g = S[U_s(q_g) + \sigma(V_s - V_g)]
$$

$$
\begin{aligned}
\rho V_m &= S[U_s(q_m) + \sigma(V_s - V_m) + (1-\sigma)\theta(u_g + V_s) \\
&\quad + (1-\sigma)(1-\theta)(V_g - V_m)]
\end{aligned}
$$

Exchange of Goods

TIOLI bargaining:

$$
\begin{aligned}
q_g &= \theta u_g + (1-\theta)(V_g - V_s) \\
q_m &= V_m - V_s
\end{aligned}
$$

Steady State Equilibrium

- An equilibrium is a generalization of the list in the previous sections
- Look for a 4-tuple (S, θ, q_m, q_g) satisfying relevant conditions

Proposition 3

Proposition (3)

There exists $\check{y}_1 < 1$ *such that*

- *If* $y \geq \check{y}_1$ *then* $\theta = 1$, *and*
- *If* $y < \check{y}_1$ *then* $\theta \in (0, 1)$.

In any equilibrium $q_g = u_g$, *while* q_m *depends on parameter values.*

Proposition 3

Proposition (3)

On the one hand, if $y < \breve{y}_1$*, there is a value* $\breve{y}_A < 1$ *such that, when* $y < \breve{y}_A$*, the unique equilibrium has* $q_m = u_g$*, and when* $y > \breve{y}_A$ *there are two equilibrium values of* q_m*, one equal to, and one lower than* u_g*.*

Proposition 3

Proposition (3)

On the other hand, if $y > \breve{y}_1$, there is a value $\breve{y}_B \in (\breve{y}_A, 1)$ such that

- *When $y < \breve{y}_A$ there is a unique equilibrium $q_m < u_g$,*
- *When $y \in (\breve{y}_A, \breve{y}_B)$, there are two equilibrium values of q_m, both lower than u_g, and*
- *When $y > \breve{y}_B$, there are no equilibrium with $q_m > 0$.*

These are all of the (steady state) equilibria with valued fiat money.

What are the effects of fiat currency?

- M small $\Rightarrow$ money crowds out cigarettes
- M large $\Rightarrow$ cigarettes driven out of circulation
- M small, y small $\Rightarrow q_m = q_g = u_g$

Equilibria with Fiat Money

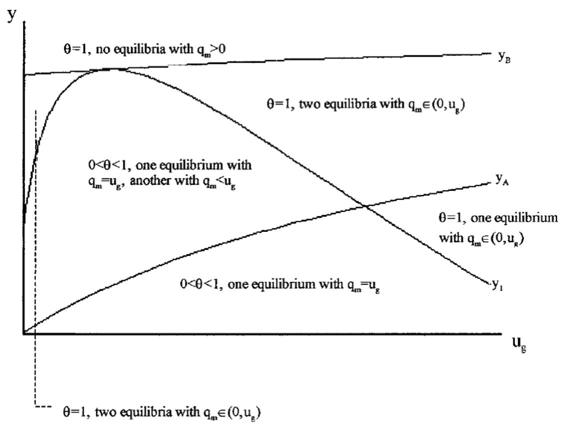


FIG. 5. Equilibria with fiat money.

Setup

- General goods come in two qualities: good and bad
- Prob. of producing a special good, a good cigarette, and a bad cigarette are σ, γ, and β
- Steady state fractions of pop. holding these objects: S, G, and B
- $d = 0$, no fiat money

Setup

- Prob. of always consuming a good (bad) cigarette: θ_g (θ_b)
- u_g, u_b utility of consuming good and bad cigarettes
- Cigarette holders make TIOLI offers
- Amount of special good in trade is q_g and q_b

Bellman Equations

- TIOLI offers $\Rightarrow$ special good holder gets no surplus from trading with cigarette holders
- Good cigarette holders must not be good cigarette consumers
  - Can be bad cigarette consumers

Bellman Equations

$$\begin{aligned}
\rho V_s &= Sy[\Omega + \gamma\theta_g u_g + \gamma(1-\theta_g)(V_g - V_s) \\
&\quad + \beta\theta_b u_b + \beta(1-\theta_b)(V_b - V_s)]
\end{aligned}$$

$$\begin{aligned}
\rho V_g &= S[U_s(u_g) + \sigma(V_s - V_g) \\
&\quad + \beta\theta_b(u_b + V_s - V_g) + \beta(1-\theta_b)(V_b - V_g)]
\end{aligned}$$

$$\begin{aligned}
\rho V_b &= S[U_s(u_b) + \sigma(V_s - V_b) \\
&\quad + \gamma\theta_g(u_g + V_s - V_b) + \gamma(1-\theta_g)(V_g - V_b)]
\end{aligned}$$

where we use $q_j = u_j$ from the bargaining solution.

Proposition 4

Proposition (4)

Assume no depreciation, no fiat money, and two types of general commodities – good and bad. With full information, $q_b = u_b$ *and* $q_g = u_g$*, and there exist* y_b*,* y_g*, and* $\hat{y}$ *such that the following is true:*

1. *If* $y > \max\{y_b, y_g\}$ *then* $\theta_b = \theta_g = 1$*,*
2. *If* $y < \min\{y_b, \hat{y}\}$ *then* $0 < \theta_b < 1$ *and* $\theta_g = 1$*, and*
3. *If* $y \in (\hat{y}, y_g)$ *– which is nonempty if and only if* u_g *is not too big relative to* u_b *– then* $\theta_b = 1$ *and* $0 < \theta_g < 1$*.*

Proposition 4

Proposition (4)

With private information, $q_g = q_b = u_b$ and $\theta_g = 1$ for all parameters, while $\theta_b = 1$ if and only if $y \geq y_b$. These are all of the (steady state) equilibria.

Which cigarettes are used as money?

- $y_g < y_b$
  - $y > y_b \Rightarrow \theta_g = \theta_b = 1$
  - $y < y_b \Rightarrow \theta_g = 1, \theta_b < 1$
- $y_g > y_b$
  - $y > y_b \Rightarrow \theta_g = \theta_b = 1$
  - $y \in (\hat{y}, y_g) \Rightarrow \theta_g < 1, \theta_b = 1$
  - $y < \hat{y} \Rightarrow \theta_g = 1, \theta_b < 1$
- Under private information, Gresham's Law holds.

Equilibria with Heterogeneous Quality

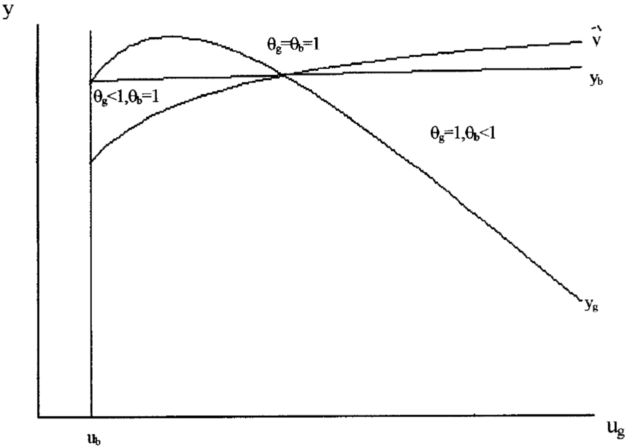


FIG. 6. Equilibria with heterogeneous quality.