Efficiency and equality

A simple model of efficient unemployment insurance (Andrew Atkeson and Robert Lucas, JET 1995)

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Contents

Motivation

- Thomas and Worrall (1990) consider an environment with insurance, where consumers can give up *all* claims to future consumption for the sake of current consumption
- The efficient allocation that emerges from this problem implies the distribution of consumption perpetually widens and is degenerate in the limit
- Result: A vanishing fraction of the population consumes all output in the economy

Motivation

- Atkeson and Lucas (this paper) impose a limit on the extent to which workers can trade future for current consumption
- Interpretation: There is a limit to which living members of an infinitely lived household can sell the consumption claims of their heirs

Agents

- Time is discrete and infinite
- Agent consuming c and working h gets flow utility

$$(1-\beta)\big(U(c)-hv\big)$$

with $U:\mathbb{R}_+\to D\subseteq\mathbb{R}$ and $v>0$ fixed

- $C(u)$ is the inverse of flow utility $U(c)$, $C:D\to\mathbb{R}_+$
- C continuously differentiable, strictly increasing, strictly convex, and $\inf_{u\in D} C(u)=0$

Job opportunities

- Every t, agents can find a job opportunity with probability π
- If agent finds a job, works $h \in [0, 1]$ hours
- Output is hy
- Job opportunities are iid across agents and across time, and not observable by others

Job opportunities

- History of *reported* job opportunities: $z^t = (z_0, z_1, \ldots, z_t)$
- Either $z_t = 0$ (no job opportunity) or $z_t = 1$
- History of *actual* job experience: $\theta^t = (\theta_0, \theta_1, \ldots, \theta_t)$
- $\Theta^{t+1} = \{0,1\} \times \cdots \times \{0,1\}$ space of possible job histories
- μ^{t+1} is the distribution of $\theta^t \in \Theta^t$ generated by π

Job reporting

- Agent's strategy for reporting job opportunities is $z = \left\{ z_t(\theta_t) \right\}_{t=0}^{\infty}$
- Agent with $\theta_t = 0$ can only report $z_t(\theta^t) = 0$
- Agent with $\theta_t = 1$ that chooses not to work cannot be distinguished from agent with $\theta_t = 0$
- A *truthful* reporting strategy is $z^* = \left\{ z_t^*(\theta^t) \right\}_{t=0}^{\infty}$ where $z_t^*(\theta^t) = \theta_t$ for all $t \geq 1$ and $\theta^t \in \Theta^t$

Endowments

- $w_0 \in D$ is agent's name and entitlement to discounted expected utility
- Agents are identified by pair (w_0, z^t)
- At each t, a planner assigns to each agent consumption $C(x_t)$; assigns hours h_t if $z_t = 1$

Allocations

An *allocation* is a sequence of functions

$$\sigma = \left\{x_t(w_0, z^t), h_t(w_0, z^t)\right\}_{t=0}^{\infty}$$

where $x_t : (w_0, z^t) \longrightarrow D,\ h_t : (w_0, z^t) \longrightarrow [0, 1]$

The agent's problem

Given an allocation σ, the agent chooses z to maximize discounted expected utility

$$U(w_0, \sigma, z) = (1-\beta)\sum_{t=0}^{\infty}\int_{\Theta^{t+1}}\Big\{x_t(w_0, z^t(\theta^t)) - z_t(\theta_t)h_t(w_0, z^t(\theta^t))v\Big\}\, d\mu^{t+1}$$

The agent's problem

$U_t(w_0, \sigma, z^*, \theta^{t-1})$ is discounted expected utility from t onwards for an agent

- Under allocation σ
- Originally entitled to $w_0 \in D$
- Reporting history $\theta^{t-1} \in \Theta^t$
- Using truthful reporting strategy z^*

Properties of allocations

We want to have four conditions on allocations:

- Promise keeping constraint (PKC)

$$w_0 = U(w_0, \sigma, z^*) \tag{1}$$

for all $w_0 \in D$

- Incentive compatibility constraint (ICC)

$$U(w_0, \sigma, z^*) \geq U(w_0, \sigma, z) \tag{2}$$

for all $w_0 \in D$, z

Properties of allocations

- Minimum entitlement constraint (MEC)

$$U_t(w_0, \sigma, z^*, \theta^{t-1}) \geq \underline{w} \qquad (3)$$

for all $t \geq 1$, $w_0 \in D$, $\theta^{t-1} \in \Theta^t$

- Upper bound on discounted expected utility:

$$\lim_{t \to \infty} \beta^t \sup_{\theta^{t-1} \in \Theta^t} U_t(w_0, \sigma, z^*, \theta^{t-1}) = 0 \qquad (4)$$

Properties of allocations

Allocation σ *attains* a distribution of entitlements ψ_0 with transfers τ if (1) – (4) are satisfied and if σ never requires a net infusion of resources greater than τ:

$$\int_{D\times\Theta^{t+1}} \left\{ C[x_t(w_0,\theta^t)] - \theta_t h_t(w_0,\theta^t) y \right\} d\mu^{t+1} d\psi_0 \leq \tau \qquad (5)$$

Efficient allocations

Allocation σ is *efficient* if it attains a distribution ψ_0 with transfers τ and there is no other allocation that attains ψ_0 with transfers less than τ.

Setup

- At each t, resources are valued using the sequence of prices $\{q_t\}_{t=0}^{\infty}$, $q_t \in (0,1)$
- Consider a planner responsible for allocating resources only to agents with type w_0
- Call this the *component planning problem*, or CPP

Component planning problem

The planner chooses $\sigma(w_0) = \left\{x_t(w_0, \theta^t), h_t(w_0, \theta^t)\right\}_{t=0}^{\infty}$,
$x_t : \Theta^{t+1} \to D,\ h_t : \Theta^{t+1} \to [0, 1]$ to minimize

$$(1 - q_0) \int_{\Theta} \left\{ C[x_0(w_0, \theta)] - \theta h_0(w_0, \theta) y \right\} d\mu + \sum_{t=1}^{\infty} (1 - q_t) \prod_{s=0}^{t-1} q_s \int_{\Theta^{t+1}} \left\{ C[x_t(w_0, \theta^t)] - \theta_t h_t(w_0, \theta^t) y \right\} d\mu^{t+1} \tag{6}$$

such that (1) – (4) hold

Intuition

- Consumers are grouped by entitlements w_0, each group represented by its own planner
- Planners trade claims among themselves at prices $\{q_t\}_{t=0}^{\infty}$

Main result

Theorem

Suppose there exists an allocation σ, prices $\{q_t\}_{t=0}^{\infty}$, a distribution of entitlements ψ_0, and transfers τ such that

(i) *At prices $\{q_t\}_{t=0}^{\infty}$, for all $w_0 \in D$, $\sigma(w_0)$ minimizes (6) subject to (1) – (4)*

(ii) *For all t, (5) holds with equality*

(iii) $(1 - q_0) + \sum_{t=1}^{\infty}(1 - q_t)\prod_{s=0}^{t-1} q_s < \infty$

Then the allocation σ attains ψ_0 with transfers τ and is efficient

Main result

Proof (Sketch).

- The fact that σ attains ψ_0 with transfers τ is direct
- To prove efficiency, follow a standard First Welfare Theorem proof argument
  - Assume there exists some other $\hat{\sigma} \neq \sigma$ that attains ψ_0 with $\hat{\tau} < \tau$
  - After some work, reach contradiction
- Follows that σ is efficient

□

Goal

Want to define and study a Bellman equation that characterizes the solutions to (6) when the price sequence $\{q_t\}_{t=0}^{\infty}$ is fixed at some $q \in [\beta, 1)$

Setup

- Assume $q_t = q$ for all t and $q \in [\beta, 1)$
- For each q, get $(\sigma_q, \psi_q, \tau(q))$ satisfying assumptions of the Theorem
- Bound the domain of C to be $\overline{D} = [\underline{w}, \overline{w}]$
- Show that $\overline{w}$ is not binding
- $C(\overline{D})$ is space of bounded, continuous functions

Bellman equation

Define an operator T_q on $C(\overline{D})$; the Bellman equation is

$$\begin{aligned}(T_q V)(w) \quad = \quad & \inf_{u,\ell,g} (1-q)\left[\pi C(u(1)) + (1-\pi)C(u(0)) - \pi\ell y\right] \\ & + q\left[\pi V(g(1)) + (1-\pi)V(g(0))\right] \qquad (7)\end{aligned}$$

where ...

Bellman equation

... where $u : \{0,1\} \to D$, $\ell \in [0,1]$, and $g : \{0,1\} \to \overline{D}$ satisfy the PKC

$$\begin{aligned} w &= (1-\beta)\left[\pi u(1) + (1-\pi)u(0) - \pi\ell\right] \\ &\quad + \beta\left[\pi g(1) + (1-\pi)g(0)\right] \end{aligned} \tag{8}$$

and the ICC

$$(1-\beta)\left[u(1) - \ell v\right] + \beta g(1) \geq (1-\beta)u(0) + \beta g(0) \tag{9}$$

Results

Standard dynamic programming shows that:

- The operator T_q has a unique fixed point V_q in $C(\overline{D})$
- The function V_q is strictly increasing and strictly convex
- The ICC binds

Results

- The policy functions $\rho_q = (u_q, \ell_q, g_q)$ are continuous
- $g(w, \cdot)$ is nondecreasing and $\forall w$, $g(w, 1) \geq g(w, 0)$
- For any $w_0 \in \overline{D}$, $w_0 < \overline{w}$, the optimal allocation $\sigma_q(w_0)$ solves the CPP

New Bellman equation

For any convex $V \in C(\overline{D})$ and $w \in \overline{D}$, $T_q V$ satisfies

$$T_q V(w) = \min_{\ell,g}(1-q)\left[\pi C\left(\frac{w-\beta g(1)}{1-\beta}+\ell y\right) +(1-\pi)C\left(\frac{w-\beta g(0)}{1-\beta}\right)\right] -(1-q)\ell y + q\left[\pi V(g(1)) + (1-\pi)V(g(0))\right] \quad (10)$$

with $g : \{0,1\} \rightarrow \overline{D}$ and $\ell \in [0,1]$

Characterizing the policy function

From (10), first-order conditions, envelope conditions and the properties outlined above,

- $w \in \left[\underline{w}, w_q^1\right]$ implies $\ell_q(w) = 1$
- $w \in \left(w_q^1, w_q^2\right)$ implies $\ell_q(w) \in (0, 1)$
- $w \in \left[w_q^2, \infty\right)$ implies $\ell_q(w) = 0$

for w_q^1, w_q^2 determined from the model

Characterizing the policy function

We get:

- For $w \geq w_q^2$, $g_q(w,1) = g_q(w,0)$
- For $w \in (w_q^1, w_q^2)$,
    - $g_q(w,1)$ is constant
    - $l_q(w)$ decreases in w from 1 to 0

To further characterize $g_q(w,1)$, we need ...

$g_q(w, 1)$, first possibility

Lemma (1)

Assume $q > \beta$. Then

(i) $\exists\ \delta > 0,\ k > 0$ *such that* $g_q(w, 0) = \underline{w}$ *for all* $w \in [\underline{w}, \underline{w} + \delta)$ *and* $g_q(w, 0) \leq w - k$ *for all* $w \in (\underline{w} + \delta, w_q^2]$

(ii) $g_q(\underline{w}, 1) > \underline{w}$ *iff*

$$C'(w + v) > \frac{q}{\beta}\Big[\pi C'(w + v) + (1 - \pi)C'(w)\Big] \qquad (11)$$

for $w = \underline{w}$

$g_q(w, 1)$, second possibility

Lemma (2)

Assume $q = \beta$. Then

(i) $\exists\ k > 0$ *such that* $g_q(w, 1) \geq w + k$ *for all* $w \in \left[\underline{w}, w_q^1\right]$

(ii) $g_q(w, 1) = w_q^2$ *for all* $w \in \left[w_q^1, w_q^2\right]$

(iii) $g_q(w, 1) = g_q(w, 0) = w$ *for all* $w \geq w_q^2$

$g_q(w, \theta)$

Graphically:

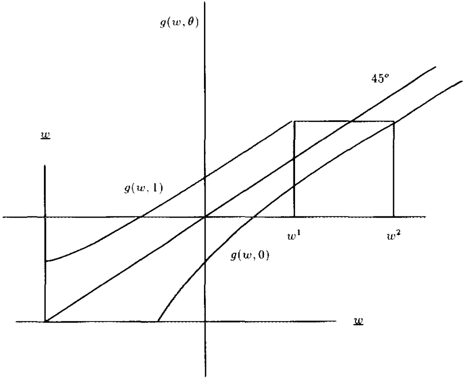

FIGURE 1.

Setup

- Consider $q \in [\beta, 1)$
- Want to study the Markov process defined by π and $g_q(w, \theta)$
- State space is $[\underline{w}, \infty)$

Transfers

- Can show that the function

$$\tau(q) = \int_D V_q(w) d\psi_q$$

  is well defined for $q \in [\beta, 1)$
- Want $q^* \in [\beta, 1)$ that solves $\tau(q^*) = 0$

Transfers

Can also show that

1. Both $\underline{w}$, w_q^2 are absorbing states
   - If q is close to 1 then $g_q(\underline{w}, 1) = \underline{w}$, and $\underline{w}$ is an absorbing state; here $\tau(q) < 0$
   - If $q = \beta$ then w_q^2 is an absorbing state; here $\tau(\beta) > 0$
2. $\tau(q)$ is continuous in $[\beta, 1)$
3. $\tau(q)$ is decreasing in q

Transfers

Then, conclude that

- From 1 – 3 above, there is $q^* \in [\beta, 1)$ such that $\tau(q^*) = 0$
- For this q^*, ψ_{q^*} is non-degenerate