A Direct Approach to Measuring the Degree of Partial Insurance to Income Risk*

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Abstract

We propose a simple approach to measuring the percentage of household's income risk that is insured and the magnitude of the welfare cost arising from the lack of full insurance. Using a synthetic panel data set of income and consumption from the Consumer Expenditure Survey (CEX) for the period of 1980-2006, we investigate how efficiently U.S. households insure cohort-specific income risk. There are three main findings in our measurement with a standard CRRA utility function. First, on average, U.S. households insured 37% of their cohort-level income risk, and the welfare cost of uninsured risk was 1.8% of their annual expenditure on nondurables and services. Second, households who faced a higher risk tended to insure a larger portion of their risk. This observation is consistent with a prediction from the models of risk-sharing under limited enforcement. Third, stockholders and business owners hedged a larger portion of their risk, but faced higher income risk, ending up with higher welfare costs than other households.

Keywords: Business-ownership, Consumption, Insurance, Risk Sharing, Stockholding.

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1 Introduction

Many studies rejected the implications of full insurance to income shocks.[1] The focus in the literature has shifted to investigating how large and how important departures from full insurance are.[2] In this paper, we propose a new approach for measuring the percentage of household's income risk that is insured and the magnitude of the welfare cost arising from the lack of full insurance. In particular, we propose measures of the welfare gain from full insurance (WG), the amount of income risk (IR), and the degree of partial insurance (PI). Also, we estimate them by using a synthetic panel data set of consumption and income, constructed from the Consumer Expenditure Survey (CEX) in the U.S. for the period of 1980-2006.

We propose a theory that justifies our way of measuring the welfare cost of uninsured income risk, which distinguishes our approach from the ones in the previous literature.[3] Our welfare cost measure, WG, is based on a cost minimization problem given the estimated Arrow-Debreu prices and the utility level obtained by the consumption path observed in the data. We prove that the consumption path with full insurance solves the same cost minimization problem, under the assumption of time and state separable utility function. Thus, WG must be zero if households completely smooth out their consumption path across time and states. WG can also be interpreted as a uniform percentage decrease in consumption that households could have saved, given the same utility level, in a hypothetical complete market.[4]

We also propose a way to understand the measured WG by breaking it down into two components: the size of household's income risk and the degree

[1] For example, see Cochrane (1991), Mace (1991), Nelson (1994), Attanasio and Davis (1996), Brav et al. (2002), and Guvenen (2007).

[2] Attanasio and Davis (1996), Schulhofer-Wohl (2007), and Blundell et al. (2008) investigated these questions.

[3] Attanasio and Davis (1996) and Schulhofer-Wohl (2007) propose different ways of measuring the welfare cost of uninsured income risk. Attanasio and Davis (1996) assume that the observed consumption path is one of the two possible outcomes whose average is the optimal consumption path satisfying the Euler equation. This assumption arbitrarily determines the magnitude of the risk in consumption, which crucially affects the resulting welfare cost. Schulhofer-Wohl (2007) also specifies the stochastic process of consumption by assumption. Our approach does not require such assumptions because our measures are based on the information about the actual consumption paths only.

[4] This is the same way of evaluating welfare cost as Lucas (1987), who discusses welfare cost of aggregate consumption risk.

of insurance to their income risk. First, we measure the size of household's income risk, IR, by applying the same way of measuring the welfare cost to the observed income path, which is considered as the consumption path in autarky. Next, we compare IR with WG to assess how well households can insure their income risk. This is our measure of the degree of partial insurance, PI, which is the percentage of the welfare cost that households decrease by insuring their income risk with partial insurance available in the real economy.[5]

There are three main findings under the assumption of a standard CRRA utility function. First, on average, households in the U.S. insured 37% of their cohort-specific income risk, but the welfare cost of uninsured risk was 1.8% of their annual expenditure on nondurables and services. In spite of the relatively low degree of partial insurance, the welfare cost was small because of the small amount of income risk.

The second finding is on the relationship between the degree of partial insurance and the amount of income risk at the cohort-level. We find a positive correlation between them; people who face a higher risk tend to insure a larger portion of their risk. This result implies that people with a considerable amount of income risk may tend to make more of an effort to hedge their risk. This is qualitatively consistent with a prediction of the models with limited enforcement, such as Kocherlakota (1996) and Krueger and Perri (2006).

The third finding is on the role of stockholding and business ownership. We find that both stockholding and business ownership increased the degree of partial insurance, but they also led to a sizable increase in the amount of income risk.[6] As a result, they ended up with a higher welfare cost due to the lack of full insurance. This result may suggest a reason for the limited participation in the stock market.

[5]Blundell et al. (2008) also propose a measure of partial insurance. They measure how much of unpredicted income shocks are reflected in consumption growth. That is, their measure captures changes in consumption, whereas our measure captures changes in utility due to partial insurance.

[6]This result provides some information to reconcile the results among stockholders in the risk sharing literature and the asset pricing literature. Guvenen (2007) found evidence against perfect risk sharing among stockholders. On the other hand, Mankiw and Zeldes (1991), Attanasio et al. (2002), Vissing-Jorgensen (2002), and Attanasio and Vissing-Jorgensen (2003) found evidence for implications of capital asset pricing models (CAPM) among stockholders. The rejection of full insurance may be related with low WG and the accepts of CAPM implications may be related with high PI.

Our approach has two advantages. First, it provides a new insight to the welfare gain from perfect insurance; it is determined by both the amount of income risk and the degree of partial insurance. Second, this approach enables us to investigate distributions of the welfare gain, the amount of income risk, and the degree of partial insurance. They have non-degenerate distributions and their characteristics are informative.

The rest of the paper consists of four sections. Section 2 derives measures of the welfare gain from achieving full insurance, the amount of income risk, and the degree of partial insurance. Section 3 describes how we construct a synthetic panel data set from the CEX. It also explains the definitions of consumption and income, and the estimation of cohort-level consumption and income. Section 4 demonstrates the results and Section 5 concludes.

2 Theory of Risk-Sharing Measures

We propose a framework of measuring efficiency in risk-sharing among households. We introduce three measures of risk-sharing: the welfare gain from perfect insurance, the amount of income risk, and the degree of partial insurance. Starting with a competitive equilibrium in a complete market, we derive the form of the measures that can be calculated from a panel data set of consumption and income.

2.1 Complete Market Representation

Households perfectly share their risk in a competitive equilibrium in a complete market, where they can trade securities contingent on any state of the economy. Thus, we characterize the consumption allocation with full risk insurance as an equilibrium outcome in a complete market where all households trade Arrow-Debreu securities at time 0.

Consider a pure exchange economy with a set of households, I, in a complete market. Every household $i \in I$ has an instantaneous utility function, $u_i(\cdot)$, has a discount factor, β_i, and lives until time T. $u_i(\cdot)$ only depends on consumption. Each household receives income $y_{it}(s)$ as an endowment at time t, which depends on $s \in S \subseteq R^T$, a history of the states of the economy until time T. Let $\pi(s)$ be the probability that s occurs.

We define a competitive equilibrium as a sequence of prices and an allocation of quantities such that (i) the allocation solves household's utility maximization problem given the prices for every i in I, and that (ii) the consumption goods market clears with the allocation.

In a competitive equilibrium, every household $i \in I$ solves the following problem given the (ex post) Arrow-Debreu prices, $P_t(s)$.

$$\text{(HH)} \qquad \max_{\{\{c_{it}(s)\}_{t=1}^{T}\}_{s\in S}} \sum_{s\in S} \pi(s) \left[\sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}(s)) \right]$$

$$s.t. \quad \sum_{s\in S} \pi(s) \left[\sum_{t=1}^{T} P_t(s) c_{it}(s) \right] \leq \sum_{s\in S} \pi(s) \left[\sum_{t=1}^{T} P_t(s) y_{it}(s) \right].$$

By assuming that $u_i(\cdot)$ is continuous and locally non-satiated, the solution to this problem, $\{c^*_{it}(s)\}_{t=1}^{T}$, is also the solution to the following cost minimization problem.

$$\text{(CM0)} \qquad \min_{\{\{c_{it}(s)\}_{t=1}^{T}\}_{s\in S}} \sum_{s\in S} \pi(s) \left[\sum_{t=1}^{T} P_t(s) c_{it}(s) \right]$$

$$s.t. \quad \sum_{s\in S} \pi(s) \left[\sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}(s)) \right] \geq \sum_{s\in S} \pi(s) \left[\sum_{t=1}^{T} \beta_i^{t-1} u_i(c^*_{it}(s)) \right].$$

That is, each household minimizes their *expected* expenditure, keeping the *expected* utility level equal to or greater than the level they achieve in a complete market with the Arrow-Debreu prices, $P_t(s)$.

We need to remove the *expected* expenditure and the *expected* utility level from our formulation because they are not directly observed in data. Data contain the information on a particular realized history, s. To remove the *expected* expressions, we change (CM0) into history-by-history minimization problems, where each household minimizes their expenditure *for each history*, s, in equilibrium. The proof of the equivalence between such history-by-history minimization problems and (CM0) requires household's preferences to be separable for each history of the states, in addition to the conditions of the duality theorem.

The cost minimization problem, (CM0), can be solved by two steps. First, we solve for the expenditure function for each history, $\rho_i(U_i(s), s)$, given prices.

$$\text{(CM1)} \qquad \rho_i(U_i(s), s) := \min_{\{c_{it}(s)\}_{t=1}^{T}} \sum_{t=1}^{T} P_t(s) c_{it}(s)$$

$$s.t. \quad \sum_{t=1}^{T} \beta_i^{t-1} u(c_{it}(s)) \geq U_i(s).$$

Second, we minimize the expected expenditure using the expenditure function for each history. That is, given the minimized expenditure function, we optimally allocate the utility level for each history, keeping the expected utility level the same as or greater than the level achieved in equilibrium.

$$\text{(CM2)} \qquad \min_{\{U_i(s)\}_{s \in S}} \sum_{s \in S} \pi(s) \rho_i(U_i(s), s)$$

$$s.t. \quad \sum_{s \in S} \pi(s) U_i(s) \geq \sum_{s \in S} \pi(s) \left[\sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}^*(s)) \right].$$

The following proposition formally establishes the equivalence between the original cost minimization problem and the two-step cost minimization problem.

Proposition 1 The two-step cost minimization problem formulated as (CM1) and (CM2) is equivalent to the original cost minimization problem, (CM0).

(Proof) First, the objective functions of both (CM0) and the two-step cost minimization problem are the same, after we plug the solution to (CM2) in the required utility of (CM1). Next, the solution to (CM0) is affordable in the two-step cost minimization problem, (CM1) and (CM2). On the other hand, the solution to the two-step cost minimization problem is affordable in (CM0). Hence, the two cost minimization problems are equivalent. ■

Proposition 1 implies that the solution to (CM0) solves (CM1) for each history given $U^*(s)$, which is the solution to (CM2) that satisfies

$$U_i^*(s) = \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}^*(s)).$$

Hence, combining these two facts, $c_{it}^*(s)$ solves the following cost minimization problem for each history, $s \in S$, given Arrow-Debreu prices, $P_t(s)$.

$$\text{(CM3)} \qquad \min_{\{c_{it}(s)\}_{t=1}^{T}} \sum_{t=1}^{T} P_t(s) c_{it}(s)$$

$$s.t. \quad \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}(s)) \geq \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}^*(s)).$$

This means that the equilibrium consumption path for each history minimizes the expenditure for each history, given the utility level it attains. Since there is no expected expression, (CM3) has a direct implication to data, which include the information on only one realized history.

Suppose that all households efficiently share their risk in a complete market. That is, suppose that the observed consumption path in data is an equilibrium consumption path for the observed history. Then, the observed consumption path must solve (CM3), which means that the observed consumption path must minimize the expenditure in the observed history, given the utility level from the observed consumption path. In other words, in the case of perfect risk-sharing, the minimized expenditure in (CM3) must equal to the expenditure for the observed consumption path.

This motivates us to see how different the observed expenditure is from the minimized expenditure in (CM3). The difference will be large if the observations are far from an equilibrium outcome in a complete market. Conversely, the difference will be small if households efficiently share their risk with others. Therefore, this difference can be interpreted as a measure of efficiency in risk-sharing. Specifically, it measures the cost of being away from complete market. In the next subsection, we show a formal definition of such a measure.

2.2 Measure of Welfare Gain From Perfect Insurance

We formally define our measure of efficiency in risk-sharing, which represents a possible welfare gain from perfect insurance to idiosyncratic income shocks.

First, along with (CM3), we define the minimum expenditure for each household i given the realized utility levels and prices by

$$\rho_i^{min} := \min_{\{c_{it}\}_{t=1}^{T}} \sum_{t=1}^{T} P_t c_{it}$$

$$s.t. \quad \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}) \geq \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}^{obs}), \tag{1}$$

where c_{it}^{obs} denotes the observed consumption for household i at time t in data, and P_t denotes the Arrow-Debreu prices at time t on the observed history. The minimized expenditure, ρ_i^{min}, is a possible minimum cost needed to attain the realized utility level if households have access to a hypothetical complete market. Next, we define the observed expenditure by

$$\rho_i^{obs} := \sum_{t=1}^{T} P_t c_{it}^{obs}. \tag{2}$$

This is the cost of the observed consumption path, measured in the Arrow-Debreu prices, P_t.

Based on ρ_i^{min} and ρ_i^{obs}, we define the *welfare gain* measure as follows. The difference between ρ_i^{min} and ρ_i^{obs} measures the cost that households could have saved if they have access to a complete market. In other words, it is the compensating variation from partial insurance in the real economy to perfect insurance in a hypothetical complete market. Therefore, we define the *welfare gain* measure, WG_i, by

$$WG_i := \frac{\rho_i^{obs} - \rho_i^{min}}{\rho_i^{obs}}.$$

That is, WG_i is the loss due to the lack of access to a complete market, in terms of a ratio of the observed expenditure. We can show, after a little algebra, that WG_i can also be interpreted as the percentage gain in consumption due to perfect insurance to idiosyncratic income shocks.

WG_i will be zero if households perfectly share their risk, and will be large if households poorly share their risk. If $c_{it}^{obs} = c_{it}^{*}(s)$ holds, that is, the observed consumption allocation is the allocation under perfect risk-sharing, then we will have $\rho_i^{obs} = \rho_i^{min}$, and hence $WG_i = 0$. Since c_{it}^{obs} is always affordable in the constraint of the cost minimization problem, ρ_i^{min} is always smaller than ρ_i^{obs}. This assures $WG_i \geq 0$.

2.3 Measure of Income Risk

The welfare gain measure is a composite of the degree of insurance and the magnitude of their income risk. Even though households are able to share most of their income risk, the welfare gain from perfect insurance might be large when the magnitude of their risk is very large. Therefore, the welfare gain measure does not distinguish whether households do not have a way to share their risk or they face too large risk to share with others.

To resolve this issue, we propose a way to measure the amount of income risk by applying the welfare gain measure to the consumption path associated with an autarky situation. We consider the autarky situation where household's consumption must be equal to their income flow at the current period, because households do not have access to any kind of insurance. This specification excludes self-insurance, that is, the situation where households can save at a constant riskless rate, but do not have access to any other insurance. Given this notion of autarky, we calculate welfare gain by moving from autarky to complete market and define it as a measure of income risk.

Along with this idea, we define the *income risk* measure, IR_i, by

$$IR_i := \frac{\rho_i^{obs,y} - \rho_i^{min,y}}{\rho_i^{obs,y}},$$

where $\rho_i^{min,y}$ and $\rho_i^{obs,y}$ are calculated by replacing observed consumption $\{c_{it}^{obs}\}$ by income $\{y_{it}\}$, which is considered as autarky consumption, in equations (1) and (2).

2.4 Measure of Partial Insurance

We construct a measure of the degree of partial insurance, by combining measures of possible welfare gain from perfect insurance and the size of idiosyncratic income risk. That is, we can directly quantify the degree of partial insurance in the real economy, instead of accepting or rejecting the hypothesis of perfect insurance.

We define the *partial insurance* measure, PI_i, by

$$PI_i := 1 - \frac{WG_i}{IR_i},$$

where the ratio of WG_i over IR_i captures the size of the remaining income risk that cannot be hedged with partial insurance. In other words, households can reduce the cost of their income risk by a fraction of PI_i, using partial insurance available in the real economy. We have $PI_i = 1$ in the case of perfect insurance where $\rho_i^{obs} = \rho_i^{min}$ and thus $WG_i = 0$, while we have $PI_i = 0$ if in the case of autarky where $WG_i = IR_i$, that is, the observed consumption path delivers just the same utility as in the case of consuming all income at the time of receipt.

The following equation, which holds by definition, summarizes how households could gain from perfect insurance.

$$WG_i = IR_i \times (1 - PI_i). \tag{3}$$

That is, possible welfare gain from perfect insurance is equal to the cost of remaining idiosyncratic income risk that cannot be hedged with partial insurance in the real economy.

In order to calculate these three measures, we need to impute the Arrow-Debreu prices, P_t. In the following subsection, we propose a way to impute the Arrow-Debreu prices on the observed history, P_t, from data.

2.5 Arrow-Debreu Prices

How to impute the Arrow-Debreu prices is the main issue in calculating our measures of risk-sharing. For our measures of risk-sharing, the price, P_t, is the only factor that captures interdependency among households in I. Therefore, the results of the analysis depend on the way we impute the Arrow-Debreu prices.

In general, the Arrow-Debreu prices are not directly observed in data. Moreover, P_t's are the prices in a *hypothetical* complete market. Thus, we need some additional assumptions to derive a tractable way to impute the prices from data.

The first order conditions for the household maximization problem, (HH), are

$$\beta_i^{t-1} u_i'(c_{it}^*(s)) = \lambda_i P_t(s), \tag{4}$$

for all i, t, and s, where λ_i is a Lagrange multiplier for household i's budget constraint. Then, we have

$$\beta_i^{t-1} \frac{u_i'(c_{it}^*(s))}{u_i'(c_{i1}^*(s))} = \frac{P_t(s)}{P_1(s)}, \tag{5}$$

for all i, t, and s. We cannot use this equation directly to impute the Arrow-Debreu prices because the observed consumption path, c_{it}^{obs}, might be different from the allocation under perfect risk-sharing, c_{it}^*.

Now, we assume two additional assumptions: (i) identical homothetic preferences for every household, and (ii) the sum of the observed consumption over households is equal to aggregate resource available for consumption, and hence, the sum of the equilibrium consumption in a complete market. The first assumption enables us to directly use the Euler equation (5) to impute the prices from aggregate resource in the economy. And, the second assumption enables us to impute the aggregate resource as the sum of the observed consumption.

Specifically, we assume that

$$\beta_i = \beta \qquad \text{and} \qquad u_i(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \qquad {}^\forall i \in I. \tag{6}$$

Note that if $\gamma = 1$, $u_i(c) = \log(c)$ will be assumed. Given this identical homothetic preferences, each individual equilibrium consumption is a fixed fraction of aggregate resource. Combining this fact with (5) gives

$$\beta^{t-1} \left(\frac{\sum_{i \in I} y_{it}(s)}{\sum_{i \in I} y_{i1}(s)} \right)^{-\gamma} = \frac{P_t(s)}{P_1(s)}. \tag{7}$$

This implies that prices do not depend on the distribution of wealth, but only the total amount of wealth in the whole economy. One unit of consumption is more expensive when aggregate resource is scarce, and vice versa.

By assumption (ii), the aggregate resource must be equal to aggregate consumption. Hence we obtain

$$\beta^{t-1} \left(\frac{\sum_{i \in I} c_{it}^{obs}}{\sum_{i \in I} c_{1t}^{obs}} \right)^{-\gamma} = \frac{P_t}{P_1}, \tag{8}$$

which is a formula for the Arrow-Debreu prices on the observed history. We normalize P_1 to be one.

In this way, given a panel data set of consumption and income, we can impute P_t and calculate three measures to investigate the degree of partial insurance to income shocks in the economy. The next section summarizes the data we use to study how efficiently households share their risk in the United States.

3 Data

This section explains the data that we construct from the Consumer Expenditure Survey (CEX). The section also explains how we construct a synthetic panel data set and estimate cohort-level consumption and income, as well as our definition of consumption measure.

3.1 Consumer Expenditure Survey (CEX)

The CEX is issued by Bureau of Labor Statistics and provides a continuous flow of information on the buying habits of American consumers. It collects data on major items of expense, household characteristics, and income. In each quarter, 20 percent of the sample are new consumer units (CUs) introduced for the first time. Each CU in the sample is interviewed every three months over a 15-month period.

They use uniform questionnaires to collect expenditure information from the previous three months, while income information are collected in the second and fifth interviews only and they contain annual values for the 12 months prior to the interview month. Financial information is interviewed in the fifth quarter only. It is estimated that the CEX expenditure data covers about 90 to 95 percent of expenditures. For this broad coverage of expenditure, we decided to use the CEX.[7]

3.2 Non-Durable Consumption and Income

Our definition of consumption measure is household expenditures on non-durable goods and services, following Kocherlakota and Pistaferri (2008). It includes food (at home and away from home), alcoholic beverages and tobacco, heating fuels and utilities, transports (including gasoline), personal care, clothing and footwear, entertainments, other services (including domestic services). It excludes expenditure on various durables, housing (furniture, appliances, etc.), education and health.

Since our analysis focuses on nondurable consumption, we define household's income by consistently excluding the effect of the other durable expenditure. First, we calculate durable consumption by subtracting nondurable consumption from the total expenditure, which is aggregated according to the CEX definition. Next, we calculate the total income for a household by adding after-tax income to lump sum income, that is, (FINCATAX +

[7]Although the Panel Study of Income Dynamics (PSID) provides panel data, its coverage of expenditure is far small. It used to collect only food expenditure, although it starts asking other expenditure categories as well in the recent years.

NONINCMX). Finally, we subtract durable consumption from the total income, and then obtain household's income for nondurables. Our definition of household's income will be used to purchase nondurable goods or for financial purposes including insurance purchases, savings, or loan payments.

Since CEX households join the survey in a different month, we collect the expenditure data in the MTAB file based on calendar periods, that is, the month when expenditures were made. We collect the data of income and sample weights in FMLY file and merge them with the expenditure data. Then, we aggregate them up to the annual data after forming synthetic cohort panel data. We deflate consumption expenditures and income using consumer price index for urban consumers (CPI-U) with the year 2000 as the basis year.[8]

We adjust consumption and income by dividing them by equivalence scales to control for household size. Equivalence scales take into account the fact that the needs of a household is not proportional to the number of persons in the household due to economies of scale in consumption.[9] Among many definitions of equivalence scales, we use "square root scale", which is equal to the square root of the number of persons in a household. Hence, our definition of consumption measure is constant price expenditures on nondurables and services over the square root of household size. The use of equivalence scales has a mechanical consequence on our risk-sharing measures; it decreases welfare gain and income risk measures, and increases partial insurance measure. This is because variations in consumption and income will decrease by being divided by equivalence scales.

3.3 Synthetic Cohorts

In order to measure the degree of risk-sharing, we need panel data, although the CEX is designed as repeated cross-section data. Therefore, we construct a synthetic panel data set, as in Attanasio and Davis (1996). We form

[8]We do not use price index for each type of goods. According to Attanasio and Davis (1996), variations in price indexes are small in their sample from 1980 to 1990. Since we use the data from 1980 to 2006, it might be interesting to use appropriate price index for each type of expenditure because the increase in imports from China in the 2000s might decrease the price of cheaper goods, keeping the price of luxury goods unchanged. This effect may be important in consumption smoothing for low income households.

[9]In an extreme case, a couple living in one bedroom apartment only pay a half of the rent per person compared to a single person living in the same apartment.

synthetic cohorts based on the birth year and educational attainment of the household head, business ownership, homeownership, and stock-holding. That is, we construct a panel data set of each representative consumer unit with one of the combination of these characteristics. Therefore, in this empirical study, risk-sharing means insurance to cohort-specific risk, and not household-specific risk.

The birth cohorts are defined by 5-year band. The oldest cohort consists of people who were born between January 1910 to December 1914. We focus on household heads of age 25 to 75. The number of birth cohorts results in 14 in our sample. The educational attainment is categorized into three levels; less than high-school degree, high-school degree, and collage degree or more. Business ownership is based on the amount of income or loss from non-farm business, partnership or professional practice. Homeownership is based on the housing tenure. Stockholders are defined as the person whose market value of stocks and bonds is positive.[10] In summary, we have 14 birth cohorts, 3 educational attainments, 2 business ownership status, 2 homeownership status, and 2 stock holding status. This results in 336 synthetic cohorts.

We include business ownership, homeownership, and stock-holding in cohort characteristics because they deliver a lot of information on risk that CUs face. There might be a problem, however, to construct the synthetic cohort panel with these characteristics since they are likely to change over time. We cannot separate CUs who change these characteristics and CUs who do not change them, because the CEX tracks CUs only for five quarters. Therefore, this problem may possibly lead to biased estimates by including CUs who have these characteristics at the current time period, but neither had them before nor will keep having them in the future.

3.4 Estimation of Cohort-level Variables

Given the definition of synthetic cohorts, we estimate consumption and income paths for each cohort. Since we use the same way of estimation for both consumption and income, we mainly explain how to construct the estimates of consumption paths for each cohort.

We suppose a reduced form relationship between cohort-level consump-

[10]Our definition of stock holders includes consumer units whose market value of stocks and bonds are equal to one dollar. As Vissing-Jorgensen (2002) reports, there are many $1 holding in the data but its reliability is questionable.

tion and individual household-level consumption in the cohort as follows:

$$\log(c_{ict}) = \log(c_{ct}) + \varepsilon_{ict}, \qquad \varepsilon_{ict} \sim i.i.d.(0, \sigma_{ct}^2) \tag{9}$$

where c_{ict} is consumption level of household i in cohort c at time t, c_{ct} is cohort-level consumption for cohort c at time t, and ε_{ict} is a household-specific idiosyncratic shock at time t, which has mean zero and variance σ_{ct}^2. That is, we model log of individual consumption as a random draw from a distribution with mean $\log(c_{ct})$ and variance σ_{ct}^2.

In this reduced form model, the simple average of $\log(c_{ict})$ over households in cohort c at time t is a consistent estimate of $\log(c_{ct})$ by the law of large numbers. Since the CEX is a random sample from U.S. population, we use the CEX sample weights in taking the average. We interpret the CEX sample weights as the number of off-sample households who are represented by the consumer unit in the sample. Namely, we consider that there are ω_{ict} households who are similar to household i, and hence whose consumptions are equal to c_{ict}. Therefore, our estimate of cohort-level logged consumption is the weighted average of logged consumption expenditures over households in the cohort, using the CEX sample weights. That is,

$$\widehat{\log(c_{ct})} := \frac{1}{\omega_{ct}} \sum_{i \in I_{ct}} \omega_{ict} \log(c_{ict}), \tag{10}$$

where I_{ct} is the set of households in cohort c at time t, ω_{ict} is the CEX sample weights, and $\omega_{ct} := \sum_{i \in I_{ct}} \omega_{ict}$.[11]

We construct annual estimates by aggregating monthly cohort-level consumptions from the MTAB file in the CEX. We choose the annual frequency, rather than quarterly frequency, because risk-sharing can be tested more accurately in the low frequency data as discussed by Hayashi et al. (1996).

The number of CUs in a cohort varies across cohorts. For example, there are few CUs with stockholding and business ownership, whereas there are many CUs with non-stockholding and non-business-ownership. This variation in the number of CUs in synthetic cohorts can be problematic because

[11] In order to obtain $\widehat{c_{ct}}$ from $\widehat{\log(c_{ct})}$, we need an adjustment based on asymptotic normality under the Central Limit Theorem. Specifically, we use the following formula:

$$\widehat{c_{ct}} = \exp(\widehat{\log(c_{ct})}) \times \exp(-\hat{\sigma}_{ct}^2 / 2N_{ct})$$

where N_{ct} is the number of sample for cohort c at time t and $\hat{\sigma}_{ct}^2$ is the estimated variance.

time-series data of synthetic cohorts with few CUs tend to be much volatile than that of synthetic cohorts with many CUs. This leads to high standard errors for synthetic cohorts with few CUs.

We restrict the data as follows. First, we exclude the samples with incomplete response on income variables since it may be a signal of larger measurement errors in the response. Next, we drop the synthetic cohorts whose number of CUs is strictly less than 5. Finally, we exclude the synthetic cohorts whose observations are strictly less than 10 years. We end up with 202 synthetic cohorts and 4168 cohort-year observations in our sample.

We estimate cohort-level income in the same way as cohort-level consumption. That is, our estimate of cohort-level logged income is the weighted average of logged income over households in the cohort, using the CEX sample weights.

We obtain the standard errors of the risk-sharing measures using a bootstrap method. For each bootstrap replication, we resample CUs within a cohort for every period from the empirical distribution weighted by the CEX sample weights. Thus, each consumer unit is chosen with probability ω_{ict}/ω_{ct} at time t. This resampling procedure is mechanically equivalent to re-weighting the sample differently from the CEX sample weights, keeping the total sample weights in the cohort unchanged.[12] The number of replications is shown below tables.

We apply our theory of risk-sharing measures to the estimated cohort-level consumption and income paths. The results with standard errors are summarized in the next section.

4 Results

We calculate our measures of risk-sharing using the CEX synthetic panel data. The first subsection documents the results for the overall economy. We find a small welfare gain on average from full insurance, and a positive correlation between the magnitude of risk and the degree of partial insurance. The second subsection investigates how stockholding and business ownership affect risk-sharing performance. We find that they led to a high ability to

[12]We take a shortcut in this resampling procedure by setting a unit of resample at 50,000. That is, we resample 50,000 CUs at once in terms of the CEX sample weights. Since the total CEX sample weights is about 6,000,000 for each month, we need about 120 random draws for each month, in making one bootstrap replication.

Table 1: Measures of Efficiency in Risk Sharing (Weighted Average)

Welfare Gain	**Income Risk**	**Partial Insurance**
1.83	3.69	37.03
(0.04)	(0.14)	(2.44)

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. Bootstrapped standard errors are reported in parentheses. The number of bootstrap replications is 20.

hedge risk, although they also increased income risk at the same time. The third subsection shows the results under various values of parameters and various hypothetical sequences of the Arrow-Debreu prices, to investigate the sensitivity of the measures. The measures continuously change in parameter values, but largely respond to aggregate consumption growth rate.

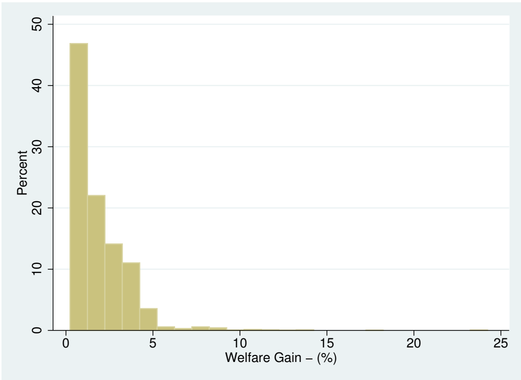
4.1 Aggregate Results

We begin with the discussion on risk-sharing performance in the whole economy. There are two main findings. First, the estimated possible welfare gain from perfect insurance to cohort-specific income risk was small on average: 1.83% of the total expenditure on nondurables and services. Second, those who faced a higher risk tended to insure a larger portion of their risk.

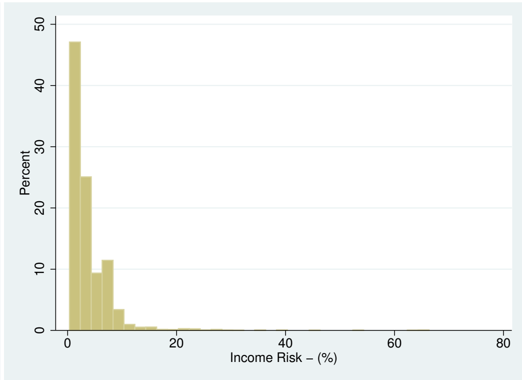
All the results in this subsection are estimated with one specific set of parameters of utility function and with the baseline estimates of Arrow-Debreu prices. The discount factor, β, is set at 0.98 and the coefficient of relative risk aversion, γ, is set at 2. The Arrow-Debreu prices are estimated from the CEX aggregate estimates of the consumption measure that we define in the data section.

Table 1 shows the economy-wide average of the three measures: the welfare gain from perfect insurance (WG), the magnitude of income risk (IR), and the effectiveness of partial insurance (PI). WG shows that U.S. households would gain 1.83% of their actual consumption expenditure on average if they could fully insure their cohort-specific income risk. In other words, the welfare cost due to the lack of perfect insurance would be \$183 per year if the annual expenditure on nondurables and services was \$10,000.

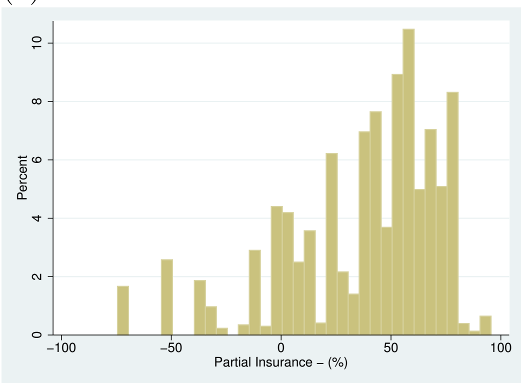
IR and PI provide an insight for WG by considering the amount of original income risk and the percentage of hedged income risk. IR shows that income



(a) Distribution of Welfare Gain



(b) Distribution of Income Risk



(c) Distribution of Partial Insurance

Figure 1: Distributions of Risk Sharing Measures

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$.

risk of a U.S. household would cost 3.69% of annual consumption expenditure on average if they did not have any instrument to hedge their income risk. PI shows that U.S. households insured 37.03% of their income risk on average, under insurance available in the real economy.

Since the distributions of these measures were highly skewed, the means of the distributions convey partial information on characteristics of population. Figure 1 shows the histograms of WG, IR, and PI. The distribution of WG exhibits right skewness, which means that most people had a small welfare gain from full insurance. More than 90 % of households would have less than 5% of their actual consumption expenditure as a benefit from full insurance to cohort-specific shocks. On the other hand, there were also households who had a huge welfare gain from full insurance. IR also has a right skewed distribution. That is, most households faced to a small amount of income risk. The distribution of PI is, on the other hand, skewed to the left. Households typically hedged a sizable portion of their income risk, but some households had a negative PI.

Interestingly, we find a positive correlation between IR and PI: the amount of risk that households face and the degree of partial insurance.[13] Figure 2 is a scatter plot of PI and log of IR in base 2. The degree of partial insurance and the size of income risk were positively correlated with coefficient 0.50. The solid line in Figure 2 is the fitted line of the regression model shown at the bottom of Figure 2. The regression result implies that doubling IR is associated with 10.1% increase in PI.

This result implies that people with a considerable amount of income risk may tend to make more of an effort in hedging risk. This is qualitatively consistent with a prediction of models with limited enforcement. In particular, Krueger and Perri (2006) argue that households facing higher income risk have a smaller incentive to deviate from the contract and hence the constrained optimal contract of those households can attain better risk-sharing. Therefore, the observed positive correlation between IR and PI is consistent with the allocation under a market implementation of such a contract under limited enforcement.

[13] This is why the relationship (3) does not hold for the means in Table 1.

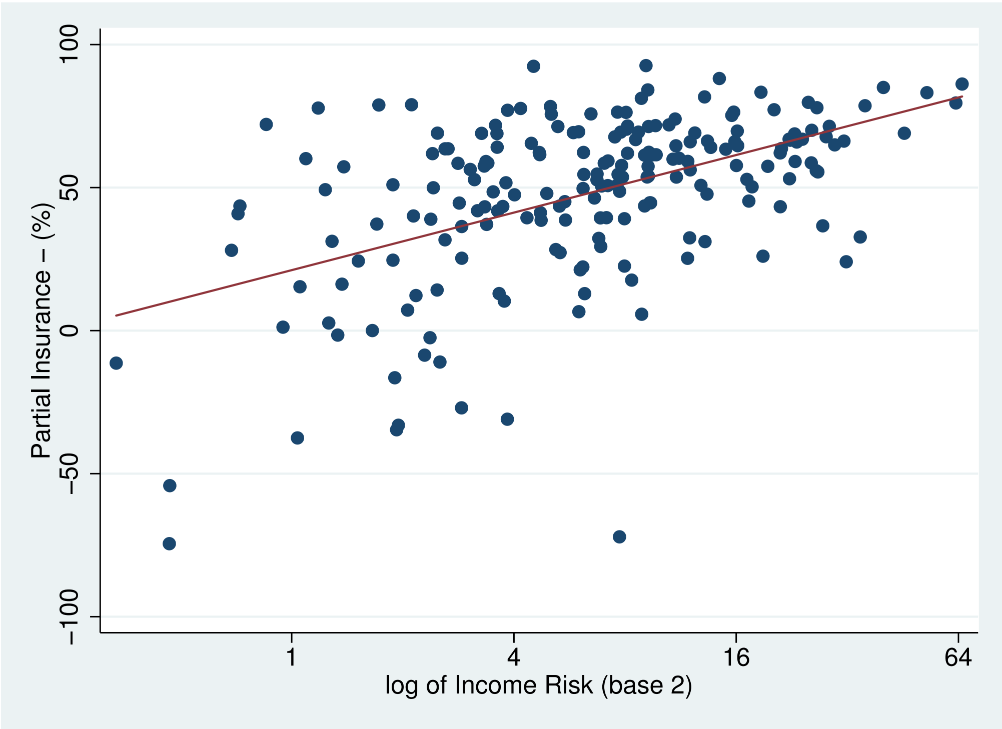


Figure 2: Partial Insurance v.s. Income Risk

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use a log scale (base 2) for x-axis. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The fitted line is

$$
\begin{array}{ccccc} PI_i & = & 21.1 & + & 10.1 \ \log_2(IR_i), \qquad R^2 = 0.25, \\ & & (5.1) & & (1.5) \end{array}
$$

with the standard errors shown in parentheses. The coefficient on $\log_2(IR_i)$ is statistically positive with t-value = 6.81. The correlation of PI_i and $\log_2(IR_i)$ is 0.50.

Table 2: Population Distribution (% of the Sum of the Sample Weights)

	Stockholders	**Non-Stockholders**	**Both**
Business Owners	0.9	9.3	10.2
Non-Business Owners	6.8	82.9	89.8
Both	7.8	92.2	100.0

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights.

4.2 Stockholders and Business Owners

In this subsection, we investigate how stockholding and business ownership affect the performance of household's risk-sharing. To do so, we classify synthetic cohorts into four groups; cohorts who own both business and stocks (B,S), those who own their business but do not hold stocks (B,NS), those who do not own their business but hold stocks (NB,S), and those who own neither business nor stocks (NB,NS). We use the same parameters of utility function and the estimates of Arrow-Debreu prices as in the previous subsection.

Table 2 shows the population distribution over these four groups. Each cell of the table shows the weighted sum of the number of cohorts in the group, using the CEX sample weight. Therefore, Table 2 reports the U.S. population distribution over the four categories during the period of 1980-2006. The largest group was (NB,NS) with 82.9% and the smallest group was (B,S) with 0.9%.

Table 3 - 5 show WG, IR, and PI for each of the four groups, respectively. The (3,3) cell of each table shows the economy-wide average. It is obtained by taking the weighted average over the four groups.

Table 3 shows the welfare gain from full insurance for each group. Under perfect insurance, households in the (B,S) group would gain the most (5.32%) and those in the (NB,NS) group would gain the least (1.60%). The third row of Table 3 shows that stockholders would gain more than non-stockholders (3.22% vs 1.71%). The third column shows that business owners would gain more than people who do not own business (2.98% vs 1.70%).

The higher WG seems to imply that stockholders and business owners are doing worse job in hedging risk, although they have more ways of hedging risk. However, without distinguishing the amount of risk and the degree of partial insurance, we cannot tell whether stocks and businesses are serving as means of hedging risk.

Table 3: Welfare Gain $\left(\frac{\rho_i^{obs}-\rho_i^{min}}{\rho_i^{obs}}\right)$ - Decomposition (Weighted Average)

	Stockholders	**Non-Stockholders**	**Both**
Business Owners	5.32	2.75	2.98
	(1.36)	(0.15)	(0.19)
Non-Business Owners	2.94	1.60	1.70
	(0.21)	(0.03)	(0.03)
Both	3.22	1.71	1.83
	(0.29)	(0.03)	(0.04)

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. Bootstrapped standard errors are reported in parentheses. The number of bootstrap replications is 20.

This argument highlights the importance of considering both of the amount of risk held by a household and the degree of insurance attained by the household. IR and PI are the measures of the size of income risk and the degree of partial insurance. They are shown in Table 4 and 5, respectively.

Table 4 shows IR for each of the four groups. On average, a household in the (B,S) group had the largest amount of income risk (13.34%). Also, we find that business ownership increases income risk; IR is 7.97% for business owners and 3.20% for non-business owners. As it is argued in the literature, owning business makes people face more risk in their income. Similarly, stockholding increases the amount of income risk sizably; IR is 7.60% for stockholders and 3.36% for non-stockholders.

Table 5 shows PI, which captures the fraction of insured risk to total risk. We find that households in the (B,S) group hedged risk the most and those in the (NB,NS) group hedged risk the least. On average, a household in the (B,S) group insured 58.72% of its income variation while a household in the (NB,NS) group insured 34.14% of its income variation. Since the majority of people in the U.S. was in the (NB,NS) group as reported in Table 2, PI for the whole economy was close to the value for the (NB,NS) group.

Stockholders insured more risk than non-stockholders (43.76% vs 36.46%). This result is consistent with a theory. Stock market participation does improve the degree of partial insurance. In other words, having a diversified portfolio helps households smooth out their consumption paths.[14]

[14]Even in the subgroups of business owners and non-business owners, PI was higher for

Table 4: Income Risk $\left(\frac{\rho_i^{aut}-\rho_i^{min}}{\rho_i^{obs}}\right)$ - Decomposition (Weighted Average)

	Stockholders	**Non-Stockholders**	**Both**
Business Owners	13.34	7.43	7.97
	(2.10)	(0.88)	(0.80)
Non-Business Owners	6.82	2.90	3.20
	(0.57)	(0.07)	(0.09)
Both	7.60	3.36	3.69
	(0.64)	(0.12)	(0.14)

Table 5: Partial Insurance $\left(\frac{\rho_i^{aut}-\rho_i^{obs}}{\rho_i^{aut}-\rho_i^{min}}\right)$ - Decomposition (Weighted Average)

	Stockholders	**Non-Stockholders**	**Both**
Business Owners	58.72	57.17	57.31
	(8.65)	(2.17)	(2.13)
Non-Business Owners	41.72	34.14	34.72
	(6.63)	(2.80)	(2.64)
Both	43.76	36.46	37.03
	(6.16)	(2.55)	(2.44)

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. Bootstrapped standard errors are reported in parentheses. The number of bootstrap replications is 20.

Business ownership also increases the degree of insurance that the household attains. On average, business owners insured 57.31% of their income risk, while non-business owners insured 34.72% of their income risk. This result implies that more than a half of the risk that business owners face is insurable, although several researchers argued that their risk may be uninsurable through financial market (Heaton and Lucas (2000) and Guvenen (2007)). At least, the uninsurable risk due to asymmetric information does not seem to be quite large quantitatively.

Analyzing Table 4 and 5 together provides an explanation why WG was higher for stockholders. Although stockholding strengthened the ability of

stockholders than for non-stockholders. Although business income is usually positively correlated with common stock returns (Heaton and Lucas (2000)), stockholding is still useful in hedging income risks.

Figure 3: Distribution of Welfare Gain Measure, $\frac{\rho_i^{obs}-\rho_i^{min}}{\rho_i^{obs}}$ - Decomposition

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$.

risk hedge (Table 5), it made households bear a larger amount of income risk (Table 4). The increase in the amount of income risk overwhelmed the increase in the risk-hedging ability, and therefore, WG was higher for stockholders than for non-stockholders. The same argument applies to business owners and non-business owners.

Similarly, PI and IR provides why households in the (NB,NS) group had the lowest welfare gain from full insurance. It was because of the small amount of their income risk. Although they hedged the least portion of their income risk, the uninsured risk did not cost much.

Now, we look at the conditional distributions of each measure by group. We can see how the shapes of the distributions differ across groups. Figure 3 shows the WG distributions for the four groups. The WG distribution in every group is skewed to the right, but the degree of skewness differs across groups. The degree of skewness is highest for the (NB,NS) group and lowest for the (B,S) group. That is, households in the (B,S) group had more various levels of welfare gains than those in other groups.

Figure 4 shows the IR distribution for each group. Right skewness of the IR distribution is most pronounced in the (NB,NS) group and least pronounced in the (B,S) group. It implies that households in the (B,S) group

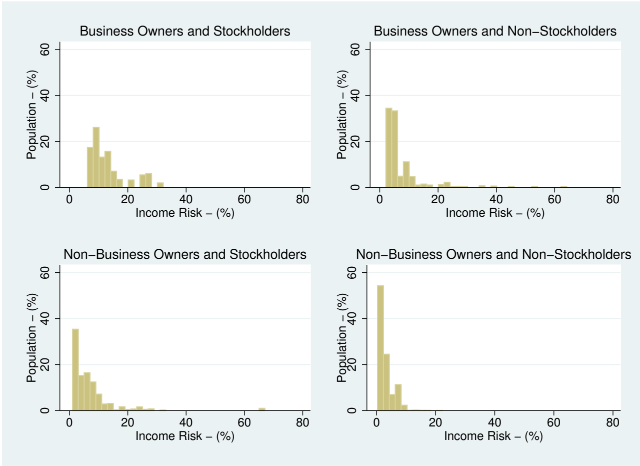


Figure 4: Distribution of Income Risk Measure, $\frac{\rho_i^{aut}-\rho_i^{min}}{\rho_i^{obs}}$ - Decomposition

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$.

were more likely to face a higher income risk than those in the (NB,NS) group.

Figure 5 shows the PI distribution by group. It is noticeable that every household in the (B,S) group had a positive PI, while some households in other subgroups had a negative PI. Also, we find the range of the PI distribution for the (NB,NS) group was the largest among the four groups. These results also imply that people who face a large amount of income risk tend to insure a larger portion of their risk.

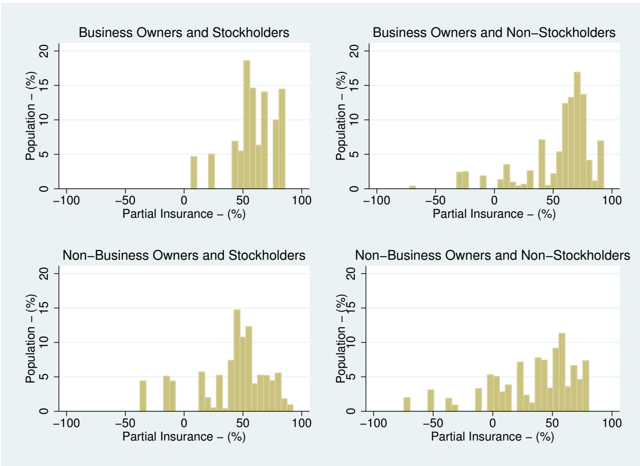


Figure 5: Distribution of Partial Insurance Measure, $\frac{\rho_i^{aut}-\rho_i^{obs}}{\rho_i^{aut}-\rho_i^{min}}$ - Decomposition

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$.

Table 6: Welfare Gain $\left(\frac{\rho_i^{obs}-\rho_i^{min}}{\rho_i^{obs}}\right)$ under various values of (β, γ)

	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\beta = 0.90$	0.42	0.85	1.74	2.66	4.55	8.84	13.16
$\beta = 0.95$	0.44	0.89	1.81	2.77	4.76	9.49	14.28
$\beta = 0.98$	0.45	0.90	1.83	2.80	4.81	9.72	14.92
$\beta = 1.0$	0.45	0.91	1.84	2.80	4.79	9.78	15.32
$\beta = 1.1$	0.43	0.85	1.71	2.57	4.32	8.94	16.46
$\beta = 1.2$	0.37	0.74	1.49	2.23	3.72	7.50	15.41
$\beta = 1.5$	0.24	0.49	1.00	1.52	2.58	5.26	10.31

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ or $u(c) = log(c)$ if $\gamma = 1$. Our baseline is $\beta = 0.98$ and $\gamma = 2$.

4.3 Sensitivity Analysis

In this subsection, we show the results with different parameter values of the utility function and with various sequences of the Arrow-Debreu prices. Tables 6 - 8 respectively show WG, IR, and PI for different levels of the discount factor, β, and the risk aversion coefficient, γ. The risk-sharing measures continuously respond to changes in the discount factor and the risk aversion coefficient. The measures monotonically change in most cases, whereas WG and IR show a hump shape in response to changes in the discount factor.

As the risk aversion coefficient increases, WG and IR increase but PI decreases. This result is consistent with theory. As people become risk averse, they evaluate more the utility cost from variation in consumption. Therefore, they would anticipate more cost from income variation, and thus, gain more from full insurance. The degree of partial insurance decreases because their risk-sharing performance is devaluated as they become more risk averse given the same observed variations in consumption and income.

In response to changes in the discount factor, WG and IR show a hump-shaped pattern, while PI does not show a clear pattern in our choice of β's. WG initially increases but starts decreasing at around $\beta = 1$. For IR, the turning point is higher than that for WG, and at around $\beta = 1.2$. These patterns depend on observed variations in consumption and income.

Table 9 shows the result with hypothetical sequences of the Arrow-Debreu prices, keeping $\beta = 0.98$ and $\gamma = 2$ as in our baseline case. We consider two

Table 7: Income Risk $\left(\frac{\rho_i^{aut}-\rho_i^{min}}{\rho_i^{obs}}\right)$ under various values of (β, γ)

	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\beta = 0.90$	0.83	1.66	3.34	4.97	8.03	14.02	19.14
$\beta = 0.95$	0.89	1.79	3.59	5.34	8.57	14.83	20.12
$\beta = 0.98$	0.92	1.84	3.69	5.47	8.74	15.13	20.65
$\beta = 1.0$	0.93	1.86	3.71	5.50	8.77	15.22	20.97
$\beta = 1.1$	0.88	1.75	3.47	5.13	8.15	14.47	21.91
$\beta = 1.2$	0.77	1.54	3.04	4.50	7.18	12.82	21.28
$\beta = 1.5$	0.63	1.25	2.46	3.63	5.83	10.35	16.64

Table 8: Partial Insurance $\left(\frac{\rho_i^{aut}-\rho_i^{obs}}{\rho_i^{aut}-\rho_i^{min}}\right)$ under various values of (β, γ)

	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\beta = 0.90$	35.88	35.31	34.09	32.78	30.02	23.22	9.99
$\beta = 0.95$	37.74	37.30	36.27	35.04	32.15	24.22	10.80
$\beta = 0.98$	38.15	37.80	37.03	35.86	33.11	24.86	11.01
$\beta = 1.0$	38.22	37.93	37.19	36.21	33.65	25.39	11.09
$\beta = 1.1$	37.72	37.55	37.15	36.64	35.28	29.52	12.45
$\beta = 1.2$	37.56	37.27	36.71	36.18	35.15	32.20	18.52
$\beta = 1.5$	43.61	43.22	42.40	41.56	39.85	35.63	28.14

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ or $u(c) = log(c)$ if $\gamma = 1$. Our baseline is $\beta = 0.98$ and $\gamma = 2$.

types of sequences associated with the following two types of aggregate consumption path. One steadily grows at a constant rate without fluctuations, and the other is constant except for a surge in one particular year and a fall to the previous level in the following year. The risk-sharing measures largely change in response to changes in aggregate consumption growth rate.

Rows 2 - 5 show the results under smoothed aggregate consumption paths at different growth rates. We find that WG and IR increase as the growth rate increases. On the other hand, PI dramatically decreases as the growth rate increases. PI falls to -56.18% if aggregate consumption growth rate is 3%, which is around the average growth rate of aggregate consumption expenditure on nondurables and services in NIPA. Note that there is a concern on aggregate consumption pattern between CEX and NIPA as argued in Slesnick (2001). The CEX expenditure does not show a steady growth, whereas NIPA consumption shows around 3% growth.

The rest of rows show the results with the Arrow-Debreu prices that has a jump in one particular year. Such Arrow-Debreu prices are associated with aggregate consumption paths that are constant except for a surge in one year and a fall to the previous level in the following year. The risk-sharing measures change a little in response to the one-year jump in the Arrow-Debreu prices.

Table 9: Sensitivity to Estimates of Arrow-Debreu Prices - Weighted Average

	Welfare Gain (%)	Income Risk (%)	Partial Insurance (%)
Baseline	1.83	3.69	37.03
3% growth	4.61	5.39	-56.18
2% growth	2.75	4.00	-43.01
1% growth	1.47	3.22	37.93
No growth	0.89	3.14	67.85
Jump in 1980	1.21	3.51	62.79
Jump in 1981	1.28	3.51	59.19
Jump in 1982	1.19	3.46	61.52
Jump in 1983	1.14	3.43	63.24
Jump in 1984	1.18	3.56	65.15
Jump in 1985	1.27	3.61	60.52
Jump in 1986	1.30	3.59	58.70
Jump in 1987	1.28	3.52	59.73
Jump in 1988	1.20	3.49	61.04
Jump in 1989	1.18	3.48	59.68
Jump in 1990	1.27	3.61	58.26
Jump in 1991	1.32	3.61	55.81
Jump in 1992	1.34	3.67	57.33
Jump in 1993	1.37	3.68	56.05
Jump in 1994	1.35	3.64	56.56
Jump in 1995	1.39	3.72	54.37
Jump in 1996	1.38	3.75	55.33
Jump in 1997	1.35	3.69	55.11
Jump in 1998	1.39	3.65	53.25
Jump in 1999	1.38	3.60	52.65
Jump in 2000	1.32	3.54	53.88
Jump in 2001	1.33	3.43	49.49
Jump in 2002	1.31	3.40	50.89
Jump in 2003	1.35	3.35	46.87
Jump in 2004	1.30	3.29	50.24
Jump in 2005	1.18	3.22	53.58
Jump in 2006	1.10	3.32	60.24

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The first row shows the results with our baseline estimates of hypothetical Arrow-Debreu prices. Rows 2 - 5 show the results with aggregate consumption paths that grow at a constant rate without fluctuations. The rest of rows show the results with aggregate consumption paths that are constant except for a surge in one year and fall to the previous level in the next year.

5 Conclusion

We propose a simple framework for assessing the realized performance of partial insurance available in the real economy, compared to hypothetical perfect insurance. This framework consists of three measures: the welfare cost of uninsured risk (WG), the size of household's income risk (IR), and the percentage of risk that households insure under partial insurance (PI). We apply the framework to the U.S. economy by constructing a synthetic panel data set from the Consumer Expenditure Survey (CEX) for the period of 1980-2006. We find that, on average, 37% of cohort-specific income risk was insured, while the remaining uninsured risk costed only 1.8% of the annual expenditure on nondurables and services.

This paper has two contributions to the literature. The first is to provide a comprehensive framework for characterizing the welfare gain from perfect insurance as a composite of the magnitude of income risk and the degree of partial insurance. This decomposition gives an insight to the relationship between consumption smoothing and risk-sharing. For example, it seems puzzling that, although stockholders have more options to smooth out their consumption path by holding a diversified portfolio, the stockholder's consumption path typically fluctuates more than the non-stockholder's one. The framework shows that this is because stockholders face more income risk and still bear a larger amount of uninsured risk than non-stockholders.

The second contribution is to provide new empirical evidence on the relationship between the magnitude of income risk and the degree of partial insurance. There was a positive correlation between them for U.S. households; households bearing a higher income risk tended to hedge a larger portion of their risk. This observation provides a qualitative support for the models of risk-sharing under limited enforcement. Further research to fully reconcile this positive correlation using a model will be insightful to household's risk-sharing behaviors.

Although it is important to consider heterogeneous preferences in assessing risk-sharing performances, it is beyond the scope of this paper. It requires a way to estimate the Arrow-Debreu prices in an economy with different types of agents. The estimation in this paper assumes the identical homothetic preferences.

Since a synthetic cohort approach is employed, household's idiosyncratic risk is not examined in this paper. Although the analysis on cohort-specific risk has several advantages, as addressed by Attanasio and Davis (1996), it is also interesting to investigate how important the idiosyncratic risk is. Such research can be done by applying the framework to a panel data on consumption and income. In the case of the U.S., the PSID panel data can be used, after it is combined with the CEX to obtain an estimate of expenditure on nondurables and services, as conducted by Blundell, Pistaferri, and Preston (2008).

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