A Direct Approach to Measuring the Degree of Partial Insurance to Income Risk

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Research Question

Household's Risk Sharing

Many studies reject the hypothesis of full insurance to household's income shocks.

Research Question

How large and how important are departures from full insurance?

Approach

- We propose measures of
  1. Welfare Gain from Full Insurance, WG_i,
  2. Size of Income Risk, IR_i, and
  3. Degree of Partial Insurance, PI_i.
- We apply them to the US data.

Findings

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1. U.S. households insured 37.03% of their cohort-level risk, and would gain 1.83% of their annual expenditure under full insurance.

2. Households who faced a higher risk tended to insure a larger portion of their risk.

3. Stockholders and business owners hedged a larger portion of their income risk, but faced a higher risk, ending up with more welfare cost than other households.

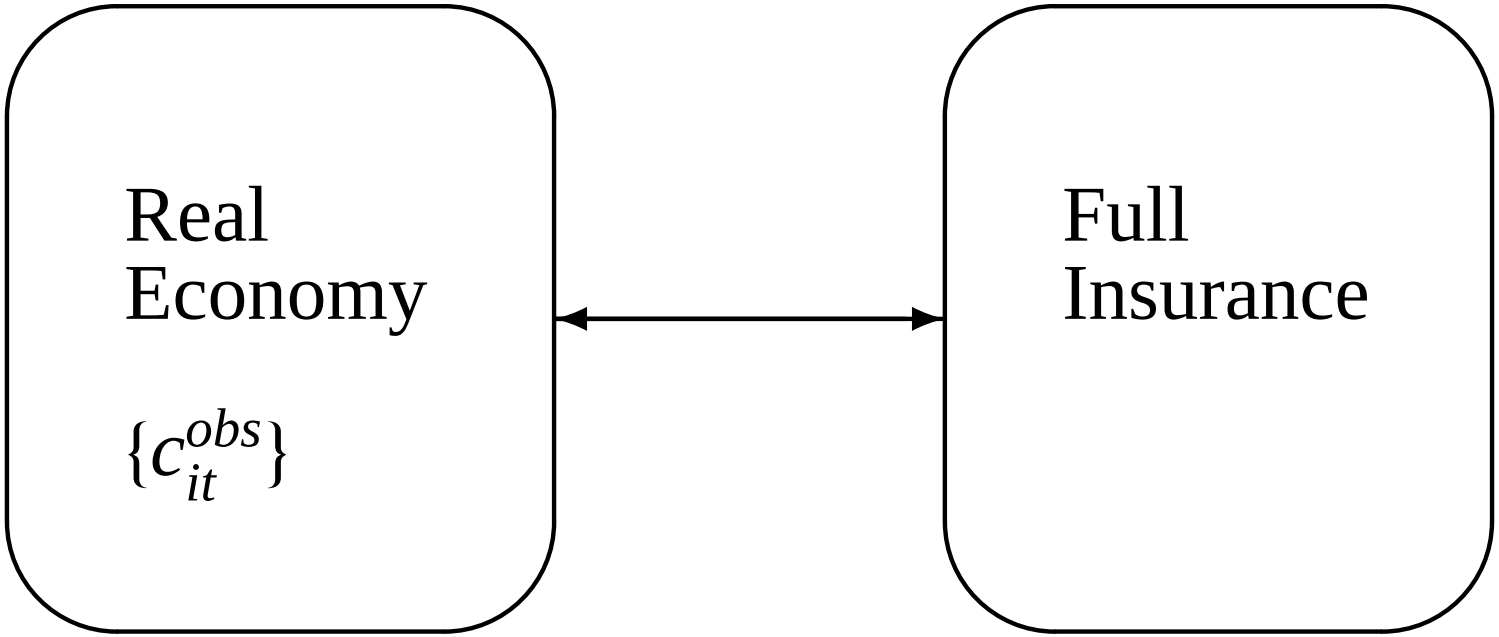
Literature Review

1. Hypothesis testing of full insurance.
   - Many studies have rejected full insurance hypothesis.
   - Cochrane (1991), Mace (1991), Nelson (1994), Attanasio and Davis (1996), Brav et al. (2002), and Guvenen (2007).

2. The degree of insurance and possible welfare gain.
   - Other measures are proposed in the previous literature.
   - Attanasio and Davis (1996), Schulhofer-Wohl (2007), and Blundell et al. (2008).

$\Rightarrow$ We propose a simple way of measuring them, and provide new findings.

Welfare Gain: Idea



Measure of Welfare Gain

Definition

Given estimated Arrow-Debreu prices, $\{P_t\}$,

- Observed cost: $\rho_i^{obs} := \sum_{t=1}^{T} P_t c_{it}^{obs}$.
- Minimum cost:

$$\rho_i^{min} := \min_{\{c_{it}\}_{t=1}^{T}} \sum_{t=1}^{T} P_t c_{it} \qquad s.t. \quad \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}) \geq \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}^{obs}).$$

- Welfare gain is defined as

$$WG_i := \frac{\rho_i^{obs} - \rho_i^{min}}{\rho_i^{obs}}.$$

Proposition 1

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$$\text{Full insurance} \quad \Leftrightarrow \quad WG_i = 0 \quad {}^{\forall} i \in I \ \text{ and } \ {}^{\forall} s \in S.$$

$$\Rightarrow \quad WG_i = 0 \quad {}^{\forall} i \in I$$

$$\text{for the realized } \hat{s} \in S.$$

Key to the Proof

Separability of preferences over states of the economy.

P_t under Identical Preferences

Assumptions

(A1) Market Clearing for c_{it}^{obs}.

$$\sum_{i \in I} c_{it}^{obs} = \sum_{i \in I} y_{it} = \sum_{i \in I} c_{it}^{*}, \qquad {}^{\forall} t = 1, \cdots, T.$$

(A2) Identical Homothetic Preferences

$$\beta_i = \beta, \qquad u_i(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \qquad {}^{\forall} i \in I.$$

Under (A1) and (A2), we obtain

$$\frac{P_t(s)}{P_1(s)} = \beta^{t-1} \left(\frac{\sum_{i \in I} y_{it}}{\sum_{i \in I} y_{i1}} \right)^{-\gamma} = \beta^{t-1} \left(\frac{\sum_{i \in I} c_{it}^{obs}}{\sum_{i \in I} c_{i1}^{obs}} \right)^{-\gamma},$$

for all $t = 1, \cdots, T$, and $s \in S$.

Decomposition of the Welfare Gain

Welfare gain depends on two factors:

1. **Size of income risk**
   - $WG_i \uparrow$ if the size of risk is large.
2. **Degree of insurance against income risk**
   - $WG_i \downarrow$ if they are close to full insurance.

Measure of Income Risk

Definition

Apply WG_i to autarky consumption path, i.e., income path, $\{y_{it}\}$.

- Observed cost: $\rho_i^{obs,y} := \sum_{t=1}^{T} P_t y_{it}$.
- Minimum cost:

$$\rho_i^{min,y} := \min_{\{c_{it}\}_{t=1}^{T}} \sum_{t=1}^{T} P_t c_{it} \quad s.t. \quad \sum_{t=1}^{T} \beta_i^{t-1} u_i(c_{it}) \geq \sum_{t=1}^{T} \beta_i^{t-1} u_i(y_{it}).$$

- Income risk is defined as

$$IR_i := \frac{\rho_i^{obs,y} - \rho_i^{min,y}}{\rho_i^{obs,y}}.$$

Measure of Partial Insurance

Definition

$$PI_i := \frac{IR_i - WG_i}{IR_i}, \qquad \Leftrightarrow \; WG_i = IR_i \times (1 - PI_i).$$

- $PI_i = 1$ when $WG_i = 0$: Full Insurance.
- $PI_i = 0$ when $WG_i = IR_i$: No Insurance.

Data

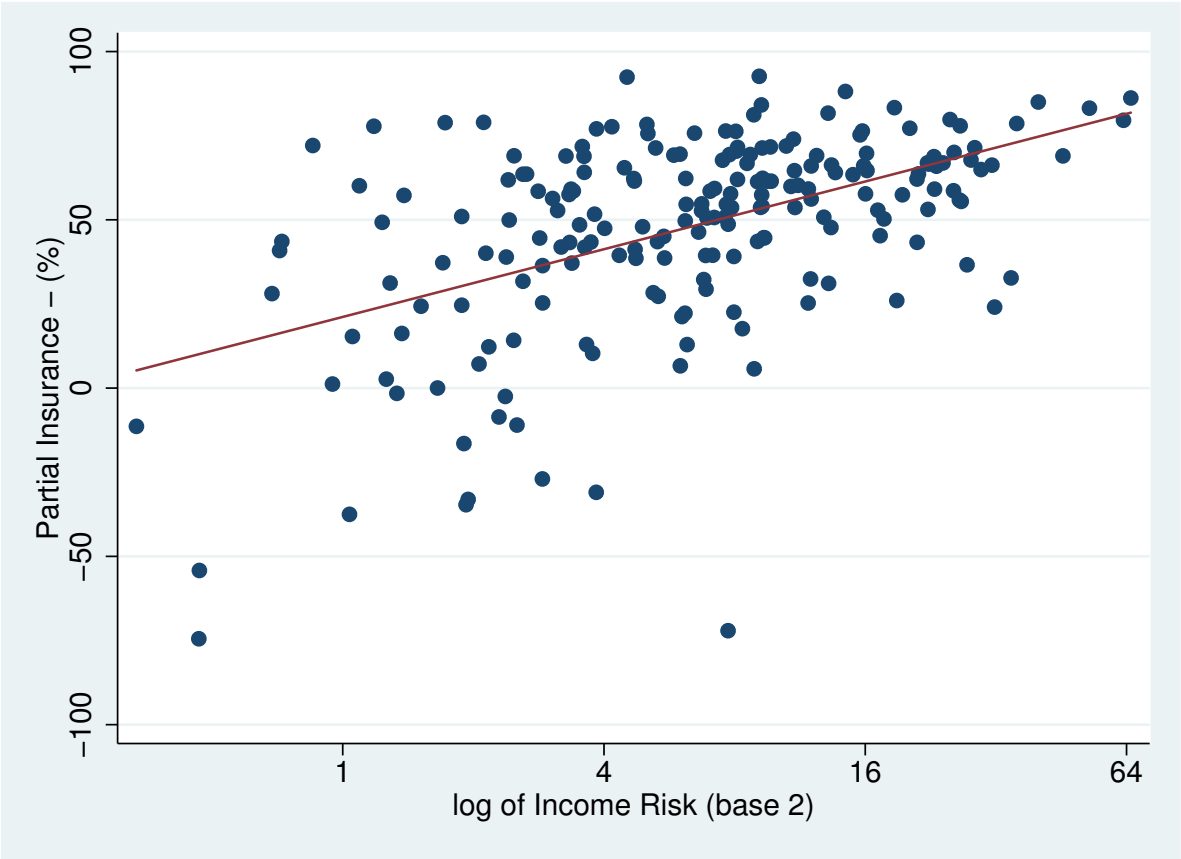
1. Annual synthetic panel data from the CEX (1980-2006).
   - Birth Cohort, Education, Business ownership, Homeownership, and Stockholding.
2. Consumption: expenditures on nondurables and services.
3. Income: “nondurable” Income, i.e.,
   - After-tax income including lump sum income,
   - minus expenditure on durables.
4. Square root equivalence scales.

Risk Sharing Measures

Welfare Gain	Income Risk	Partial Insurance
1.83	3.69	37.03
(0.04)	(0.14)	(2.44)

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $\beta = 0.98$, $\gamma = 2$, and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. Bootstrapped standard errors are reported in parentheses. The number of bootstrap replications is 20.

Partial Insurance v.s. Income Risk



The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006.

$$PI_i = \underset{(5.1)}{21.1} + \underset{(1.5)}{10.1}\ \log_2(IR_i), \qquad R^2 = 0.25.$$

The t-value of $\log_2(IR_i)$ is 6.81. The correlation of PI_i and $\log_2(IR_i)$ is 0.50.

Stockholders v.s. Non-Stockholders

	Stockholders	Non-Stockholders
WG_i	3.22	1.71
	(0.29)	(0.03)
IR_i	7.60	3.36
	(0.64)	(0.12)
PI_i	43.76	36.46
	(6.16)	(2.55)

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Business Owners v.s. Non-Business Owners

	Business Owners	Non-Business Owners
WG_i	2.98	1.70
	(0.19)	(0.03)
IR_i	7.97	3.20
	(0.80)	(0.09)
PI_i	57.31	34.72
	(2.13)	(2.64)

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Sensitivity of Welfare Gain, *WG*

	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\beta = 0.90$	0.42	0.85	1.74	2.66	4.55	8.84	13.16
$\beta = 0.95$	0.44	0.89	1.81	2.77	4.76	9.49	14.28
$\beta = 0.98$	0.45	0.90	1.83	2.80	4.81	9.72	14.92
$\beta = 1.0$	0.45	0.91	1.84	2.80	4.79	9.78	15.32
$\beta = 1.1$	0.43	0.85	1.71	2.57	4.32	8.94	16.46
$\beta = 1.2$	0.37	0.74	1.49	2.23	3.72	7.50	15.41
$\beta = 1.5$	0.24	0.49	1.00	1.52	2.58	5.26	10.31

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ or $u(c) = log(c)$ if $\gamma = 1$. Our baseline is $\beta = 0.98$ and $\gamma = 2$.

- Monotonically increasing in γ.
- First increasing and then decreasing in β.

Sensitivity of Income Risk, *IR*

	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\beta = 0.90$	0.83	1.66	3.34	4.97	8.03	14.02	19.14
$\beta = 0.95$	0.89	1.79	3.59	5.34	8.57	14.83	20.12
$\beta = 0.98$	0.92	1.84	3.69	5.47	8.74	15.13	20.65
$\beta = 1.0$	0.93	1.86	3.71	5.50	8.77	15.22	20.97
$\beta = 1.1$	0.88	1.75	3.47	5.13	8.15	14.47	21.91
$\beta = 1.2$	0.77	1.54	3.04	4.50	7.18	12.82	21.28
$\beta = 1.5$	0.63	1.25	2.46	3.63	5.83	10.35	16.64

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- Monotonically increasing in γ.
- First increasing and then decreasing in β.

Sensitivity of Partial Insurance, *PI*

	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
$\beta = 0.90$	35.88	35.31	34.09	32.78	30.02	23.22	9.99
$\beta = 0.95$	37.74	37.30	36.27	35.04	32.15	24.22	10.80
$\beta = 0.98$	38.15	37.80	37.03	35.86	33.11	24.86	11.01
$\beta = 1.0$	38.22	37.93	37.19	36.21	33.65	25.39	11.09
$\beta = 1.1$	37.72	37.55	37.15	36.64	35.28	29.52	12.45
$\beta = 1.2$	37.56	37.27	36.71	36.18	35.15	32.20	18.52
$\beta = 1.5$	43.61	43.22	42.40	41.56	39.85	35.63	28.14

The data include 4168 observations for 202 synthetic cohorts from 1980 to 2006. Weighted by the CEX sample weights. Use $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ or $u(c) = log(c)$ if $\gamma = 1$. Our baseline is $\beta = 0.98$ and $\gamma = 2$.

- Monotonically decreasing in γ.
- No clear pattern for changes in β.

Conclusion

Summary

1. Propose a new framework for assessing the realized performance of partial insurance.
2. Apply it to the U.S. data.

Findings

1. U.S. households insured 37.03% of their cohort-level risk, and would gain 1.83% of their annual expenditure under full insurance.
2. Households who faced a higher risk tended to insure a larger portion of their risk.
3. Stockholders and business owners hedged a larger portion of their income risk, but faced a higher risk, ending up with more welfare cost than other households.