

Non Cooperative Game Theory

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As in the homeworks, *prove or disprove* means that either you prove the statement or you provide a counterexample to it.

Question 1. The normal form game G_2 is obtained from the normal form game G_1 by elimination of some pure strategies that are **strictly** dominated in G_1 . Let S_k^i be the set of mixed strategies in the game G_k , $k = 1, 2$.

1. Define a natural map e from mixed strategies in the game G_2 to mixed strategies in G_1 .
2. Prove that a strategy profile s_2 is a Nash equilibrium of G_2 if and only if $s_1 \equiv e(s_2)$ is a Nash equilibrium of G_1 .

Question 2. The normal form game G_2 is obtained from the normal form game G_1 by elimination of some pure strategies that are **weakly** dominated in G_1 .

1. Prove or disprove: a strategy profile s_2 is a Nash equilibrium of G_2 if $s_1 \equiv e(s_2)$ is a Nash equilibrium of G_1 .
2. Prove or disprove: if $s_1 = e(s_2)$ is a Nash equilibrium of G_1 then s_2 is a Nash equilibrium of G_2 .
3. Prove or disprove: if s_1 is a Nash equilibrium of G_1 then there exists an s_2 Nash equilibrium of G_2 such that $s_1 = e(s_2)$.

Question 3. Let G be a finite normal form game. Define the best response map of player i , and prove that it is upper-hemicontinuous.

Question 4. Let G be a zero-sum normal form game, and let $s^i, i = 1, 2$ denote a mixed strategy of player i . Prove or disprove: the set

$$\{s^1 : s \text{ is an equilibrium of } G \text{ for some } s = (s^1, s^2)\}$$

is a convex set.

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Final Examination

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Question 1.

1. Consider an extensive form game. Prove that for any behavioral strategy of a player i there is a general strategy that induces the same probability distribution on final nodes, for any given general strategy of the other players. Do the same when "behavioral" is replaced with "mixed".
2. Give an example of a game that has no Subgame Perfect Equilibrium in pure strategies.

Question 2. Two players play the following game

	l	r
T	4,4	1,7
B	7,1	2,2

for two periods. In each period the two players choose the action simultaneously, and then the two actions are announced to the two players. The final payoff is the sum of the payoff in each period.

1. Write the extensive form game of the game that has been described above. You can draw the diagram or use a tree to describe the game.
2. Find the Nash equilibria of the extensive form game
3. Find the Subgame Perfect equilibria of the extensive form game