Economics 8101 **Final Exam**
Fall Semester 2006, Session I **Jan Werner**

1. (30 points) Consider a consumer whose preferences over risky consumption plans have an expected utility representation $E[v(c)]$, where the von Neumann-Morgenstern utility function $v: \Re \to \Re$ is strictly increasing and continuous. Prove that the consumer is risk averse if and only if, for every $x \in \Re$ and random variables y and z, with $E(y) = E(z)$, it holds

$$\text{if } y \text{ is more risky than } z, \text{ then } \rho(x, y) \geq \rho(x, z).$$

2. (25 points) Consider a consumer with expected utility function $E[v(c)]$, where the von Neumann-Morgenstern utility function v is given by

$$v(x) = -e^{-\alpha x},$$

for some parameter $\alpha > 0$.

   (i) Show that utility function v has constant absolute risk aversion. Further, show that every utility function with constant absolute risk aversion must be a linear transformation of function v for some $\alpha > 0$.

   (ii) Show that risk compensation of v is an increasing function of α, for every x and z.

   In answering (i) and (ii), you may use the Theorem of Pratt. However, if you use a corollary, you need to show how it follows from the theorem.

3. (25 points) Consider a consumer with consumption set $\Re^L_+$ and utility function $u : \Re^L_+ \to \Re$, assumed locally non-satiated, continuous, and strictly concave. Let $x^*(p, w)$ be the Walrasian demand function of price vector $p \in \Re^L_{++}$ and income $w \in \Re_+$. Assume that x^* is differentiable.
   State a definition of the Slutsky matrix of demand function x^*. Show that the Slutsky matrix is negative semi-definite. You may use properties of the Hicksian demand in your answer without proofs, but you need to state them clearly.

4. (20 points) Let y and z be two arbitrary random variables (on some finite state space). Consider the following statement: If $E(z) = 0$, then $y + z$ is more risky than y. Show that this statement is false.