Economics 8101 **Midterm Exam**
Fall Semester 2006, Session I **Jan Werner**

1. (40 points) Consider a reflexive, transitive, and complete preference relation on $\mathbb{R}^L_+$.

   a. Suppose that the preference relation is also continuous and strictly increasing (or strongly monotone). For every consumption bundle $x \in \mathbb{R}^L_+$, let $\alpha_x \in \mathbb{R}_+$ be a number such that x is indifferent to $\alpha_x \mathbf{e}$, where $\mathbf{e}$ denotes the unit vector in $\mathbb{R}^L_+$. Show that α_x exists and is unique, for every x. Further, show that function u defined by $u(x) = \alpha_x$ is a utility representation of the preference relation.

   b. Let $L = 2$ and let the preference relation be lexicographic with the first preference for total consumption of the two goods (i.e., for $x_1 + x_2$) and the second preference for consumption of good 2 (i.e., for x_2.) Explain why the construction of part (a) does not lead to a utility representation of this lexicographic preference. Which property of preference relation used in part (a) does not hold for the lexicographic preference? [A proof for your answer to this last question is not asked for.]

2. (25 points) Consider the following supply function of a firm

$$\psi(p_1, p_2) = \left(-\frac{2p_2}{p_1}, \frac{p_2}{p_1}\right) \quad \text{for} \quad p_1 > 0, p_2 > 0.$$

The firm produces good 2 using good 1 as input. Show that this supply function cannot result from profit maximization on any production set.

3. (25 points) Let $e(p, \bar{u})$ be the expenditure function of a consumer whose preferences on the consumption set $\mathbb{R}^L_+$ are described by a continuous and

locally non-satiated utility function $u\colon \mathbb{R}^L_+ \to \mathbb{R}$. Show that e is a concave function of prices.

4. (10 points) Consider a list of observations $(p^1, x^1), \ldots, (p^T, x^T)$ where $p^t \in \mathbb{R}^L_+$ and $x^t \in \mathbb{R}^L_+$ are price vector and consumption plan of a consumer, respectively, for every $t = 1, \ldots, T$.

a. State the Weak and the Strong Axioms of Revealed Preference (or alternatively, the Generalized Weak and the Generalized Axioms) for these observations.

b. Give an example of two observations with two goods (i.e., $L = 2$ and $T = 2$) that do not satisfy either of the axioms you stated in (a).