Economics 8101 **Exercises**

Fall Semester 2007, Session I

1. Consider Walrasian demand function $x^*(p,w)$ for $p \gg 0$ and $w \geq 0$ generated by a continuous and l.n.s. utility function $u\colon \Re^L_+ \to \Re$ which is homogeneous of degree one (so that the induced preference is homothetic). Show that $x^*(p,w)$ is monotone in p, that is

$$[x^*(p',w) - x^*(p,w)](p'-p) \leq 0$$

for every p', p, w. You may assume that x^* is differentiable.

2. Consider the utility function $u(x_1,x_2,x_3) = x_1 + v(x_2,x_3)$ for some function $v : \Re^2_+ \to \Re$ with $D^2 v$ negative definite. Let $x_i^*(p_1,p_2,p_3,M)$ be the utility maximizing demand function, $i = 1,2,3$. Show that the income effect on consumption of commodities 2 and 3 is zero whenever the demand for all three commodities is interior. That is, show that $\dfrac{\partial x_i^*}{\partial M} = 0$ for $i = 2,3$. Derive $\dfrac{\partial x_1^*}{\partial M}$.

3. Consider the demand function of two goods $d(p_1,p_2,w) = \left(\dfrac{2w - p_2}{p_1}, \dfrac{p_2 - w}{p_2}\right)$ for $p_1 > 0$, $p_2 > 0$ and $2w > p_2 > w$.
   (a) Is any of the goods a Giffen good or an inferior good?
   (b) Find the Slutsky matrix of d and verify whether it is negative semi-definite and symmetric. Is this demand function a Walrasian demand function (i.e., is it rationalizable by a utility function)?

4. Consider a reflexive, transitive, and complete preference relation on $\Re^n_+$. For every consumption bundle $x \in \Re^n_+$, let $u(x) \in \Re_+$ be a number (if it exists) such that x is indifferent to $u(x)\mathbf{e}$, where $\mathbf{e}$ denotes the unit vector in $\Re^n_+$.
   (i) Give an example of a preference relation that is continuous but not strictly increasing, and for which so constructed u does not constitute a utility representation. Justify your answer.
   (ii) Give an example of a preference relation that is strictly increasing but not continuous for which u does not constitute a utility representation. Justify your answer.

5. Consider preference relation on $\Re^n_+$, where $n > 1$, defined by

$$x \succeq x' \text{ if and only if } x \geq x',$$

for every $x, x' \in \Re^n_+$.

(i) Show that this preference relation does not have a utility representation.

(ii) Show that this preference relation is locally non-satiated.

(iii) Describe the demand at a price vector $p \in \Re^n_{++}$ and income $w > 0$, i.e., the set of preference maximizing consumption bundles in the budget set at (p, w).