Learning about Growth

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Abstract: We introduce a model in which consumers must learn whether a country's growth rate shock is permanent or transitory. We document that for developing countries TFP growth rates have been highly volatile, while for developed country the growth rate is comparatively stable. This difference is sufficient to explain the allocation puzzle, the observation that developing countries with higher growth experience higher capital outflow. In addition, we document that the allocation puzzle decreases over time - in agreement with our model predictions. We interpret this as evidence that the learning mechanism underlying our model is relevant to understanding the differences in behavior of developed and developing countries.

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1 Introduction

We introduce a model in which agents learn the composition of growth of TFP. Agents don't know how much of the observed growth is permanent or transitory. The permanent income hypothesis implies agents who believe growth is mostly transitory follow different consumption and investment paths than those who believe shocks are permanent. In particular, agents in a country in which, historically, the growth rate is mostly permanent would believe most of the currently observed growth is permanent. This implies that they would decrease their savings (foreign bond holdings in an SOE economy) in response to an increase in growth (a positive relationship between growth and the capital account). Agents in countries in which growth rates are highly volatile however, would view current growth to be mostly transitory and would increase their savings. This results in a negative relationship between growth and the capital account. However, even in countries with a historically highly volatile growth rates, should TFP continued to grow (as would be the case in an emerging economy), agents will update their beliefs about the composition of growth to reflect a permanent growth of TFP, endogenously leading to behavior that matches stable growth regime.

Aguiar and Gopinath [2007] show most of the volatility at the business cycle level of Mexico's TFP comes from transitory shocks to the trend growth (i.e. volatile growth), whereas for Canada most of the volatility at the business cycle level comes from transitory shocks around a stable growth trend. Instead of looking only at Canada and Mexico we generalize our sample to Non-OECD and OECD countries, and look at long-run TFP growth rates instead of business cycle level. We use GMM estimation on TFP data to obtain the relative contributions of the permanent and transitory components in our sample of countries, and find that non-OECD countries experience volatile long run trend growth, while OECD countries have stable growth rates. We do not intend to address the causes for the differences in volatility of TFP growth rates in this paper.

We apply our model to the allocation puzzle documented by Gourinchas and Jeanne [2009] (GJ). According to standard neoclassical growth theory, a country that experiences relative productivity growth should receive an inflow of capital to equate marginal productivity and an inflow of savings for consumption smoothing - a positive correlation between productivity growth and capital inflows. GJ document that amongst developing countries this correlation is not strongly positive - and may even be negative.[1] This has been termed "the allocation puzzle" - the *allocation* of the flows amongst DVC's is inconsistent with theory.[2]

This project introduces the method of kalman filtering, used within other branches of economics (i.e. Guvenen [2007]), to models of trade and development. It is important to notice that this model induces different behavior in developing countries not because of an exogenous parameter (i.e. corruption), but because of easily measurable difference in the historical TFP processes (which informs the initial beliefs). This means that this model is equally applicable to both undeveloped and developed countries. Boz et al. [2008] imbed kalman filtering in the Aguair-Gopinath Mexico model and find that the model can reproduce some of the business-cycle statistics without the need for excessive volatility in the transitory shocks.[3]

A few other papers have proposed solutions to the allocation puzzle. Sandri [2009] models the behavior of entrepreneurs facing incomplete financial markets and risky investment; poor financial markets lead to entrepreneurs to increase saving to self-finance investment for firm growth, causing capital outflows. Buera and Shin [2009] also consider entrepreneurial funding, but through a different mechanism: Heterogeneous agents have the choice of being either an entrepreneur or a worker; capital outflows follow liberalization due to the rapid disinvestment of less productive entrepreneurs and slow capital absorption by new entrepreneurs due to borrowing constraints. Aghion et al. [2009] describes a model in which local firms familiar

[1] This pattern remains even after adjusting for foreign aid inflows.Rajan et al. [2007] investigate further and show that the puzzle is driven by savings outflows exceeding any additional investment inflows after a growth episode.

[2] It is different from the Lucas puzzle, which states that the *level* of flows from developed (DC) to developing (DVC) countries is too low

[3] Specifically, the model predicts a higher variability of consumption relative to output and a strongly negative correlation between the trade balance and output

with domestic monitoring technology accumulate assets to attract foreign investors who are familiar with frontier technology and are willing to fund projects co-financed and monitored by local firms.

In contrast with this strand of the literature, we do not require that the institutions within the developing countries be significantly worse than those of developed countries, though adding these sort of frictions would amplify our results. Even with the same financial institutions, we would still see capital outflows due to the uncertainty of TFP growth. While we do not intend to address the cause of the difference in TFP processes in this paper, it suggests a potentially fertile line of future research. Additionally, in our model TFP growth is exogenous, and causes the savings outflows, while in the above papers savings outflows causes TFP growth. Adding any of the above mechanisms would only strengthen our results.

In section 2 we introduce our model. In section 3 we provide analytical intuition for a simplified version of our model that shows how kalman filtering works in this class of models, and discuss limitations. In section 4 we estimate the TFP process for developed and developing countries, and in section 5 we discuss how to derive the allocation puzzle from the data and provide an analysis of the time series evolution of the correlation between capital flows and growth - an aspect of the puzzle that has not been studied before. In section 6 we compare numerical results from our model to the data, and section 7 concludes.

2 Model

Our model is a small open economy (SOE) with neoclassical production. Time is discrete and infinite. The only shock is to labor augmenting TFP, where agents have uncertainty about the composition of a given shock. We restrict uncertainty to the first T periods, after which every country grows at the same, known rate without any further shocks.

2.1 TFP Process

We specify a process of TFP in which the average growth rate of TFP over the first T periods is heterogeneous across countries. At $t = 0$, each country draws an average, unobserved growth rate, denoted η^i. A country's actual growth rate deviates from η^i due to a unobserved shock which we denote by $\epsilon_t^i \sim N(0, \sigma_\epsilon^2)$, where σ_ϵ^2 is known.

$$A_{t+1}^i = e^{\eta^i + \epsilon_{t+1}^i} A_t^i$$

After T periods, where T is known, every country's TFP grows at the same uniform rate γ. Thus for $t \geq T$:

$$A_{t+1}^i = \gamma^{T-t} A_T^i$$

For future reference, note that the process for the growth rate is simply:

$$g_t^i = \eta^i + \epsilon_t^i$$

Denote $\hat{x} = \log(x)$ and write the TFP process for t=0,...,T-1 in logs as:

$$\hat{A}_{t+1}^i = (\eta^i + \epsilon_{t+1}^i) + \hat{A}_t^i$$

2.2 Initial Conditions and Information

We set initial conditions so that all countries are ex-ante identical, hence $k_0^i = k_0^j$ and $A_0^i = A_0^j$, and we normalize $k_0^i = 1$ and $A_0^i = 1$. We assume that countries draw their average growth rate, η^i, and their initial belief regarding the average growth rate, $\tilde{\mu}_0^i$ from:

$$\begin{bmatrix} \eta^i \\ \tilde{\mu}_0^i \end{bmatrix} \sim N\left(\begin{bmatrix} \mu \\ \mu \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right) \tag{1}$$

where we assume that countries know the values of μ, σ^2, ρ. Making this belief dogmatic corresponds to the assumption that there is no relationship between initial realized growth rates and beliefs, and also implies that a country cannot learn about the true correlation through cross-sectional information.

2.3 Learning Process

A country faces a signal extraction problem in determining how much growth is due to their mean growth rate and how much is due to transitory shocks. Since the initial belief, mean growth rates and their errors are normal, the Kalman Filter can be applied.

For $0 < t \leq T$ agents are learning about the TFP growth rate - trying to separate the two unobserved components (η_t^i and ϵ^i) using the sequence of observed growth rates $\{g_n^i\}_{n=0}^{t-1}$ to make a prediction about next periods TFP. Let $(\tilde{\mu}_t^i, \tilde{\sigma}_t^{i^2})$ be the forecast mean and variance of growth before the realization of g_t^i. We get the recursive filtering equations:

$$\underbrace{\tilde{\mu}_{t+1}^i}_{\text{New Prediction}} = \underbrace{\tilde{\mu}_t^i}_{\text{Current Prediction}} + \underbrace{\kappa_t^i(g_t^i - \tilde{\mu}_t^i)}_{\text{Weighted realized error in current prediction}} \tag{2}$$

$$\tilde{\sigma}_{t+1}^{i^2} = (1 - \kappa_t^i)\tilde{\sigma}_t^{i^2} \tag{3}$$

$$\text{where the Kalman gain is } \kappa_t^i \equiv \frac{1}{1 + \frac{\sigma_\epsilon^2}{\tilde{\sigma}_t^{i^2}}} \tag{4}$$

Note the role of the Kalman gain κ. If agents are very sure about the growth rate estimate ($\tilde{\sigma}_t^i \approx 0$) or if the variance of the transitory shocks (σ_ϵ) is very large then $\kappa_t^i \approx 0$ and agents will make only very small adjustments in their predictions if today realizations vary greatly from their predictions.

Using the definition of g_t^i, we can relate this equation back to TFP. The mean and variance of the forecast of $t+1$ log TFP conditional on information at time t, $(A_t^i, g_t^i, \tilde{\mu}_t^i, \tilde{\sigma}_t^i)$, are:

$$\tilde{\mathbb{E}}\hat{A}_{t+1}^i = \tilde{\mu}_{t+1}^i + \hat{A}_t^i \tag{5}$$

$$\tilde{\mathbb{V}}_t\hat{A}_{t+1}^i = \tilde{\sigma}_{t+1}^{i2} + \sigma_\epsilon^2 \tag{6}$$

Thus the forecast of the level of TFP, A_{t+1}^i, conditional on information available at time t is log-normal. Using equations 5 and 6 we can summarize the forecasting distribution with the first two moments:

$$\begin{aligned}
\tilde{\mathbb{E}}_t A_{t+1}^i &= A_t^i \exp\left(\tilde{\mu}_t^i + \kappa_t^i(g_t^i - \tilde{\mu}_t^i) + \frac{1}{2}(\tilde{\sigma}_{t+1}^{i2} + \sigma_\epsilon^2)\right) \\
\tilde{\mathbb{V}}_t A_{t+1}^i &= \left(\mathrm{e}^{\tilde{\sigma}_{t+1}^{i2}+\sigma_\epsilon^2} - 1\right)\exp\left(2(\tilde{\mu}_t^i + \kappa_t^i(g_t^i - \tilde{\mu}_t^i)) + (\tilde{\sigma}_{t+1}^{i2} + \sigma_\epsilon^2)\right)(A_t^i)^2
\end{aligned}$$

For future reference, it will be useful to note that a s period ahead forecast of log TFP conditional on information at time t has mean and variance:

$$\tilde{\mathbb{E}}_t\hat{A}_{t+s}^i = \hat{A}_t^i + s\tilde{\mu}_{t+1}^i \tag{7}$$

$$\tilde{\mathbb{V}}_t\hat{A}_{t+s}^i = s^2\tilde{\sigma}_t^2 + s\sigma_\epsilon^2 \tag{8}$$

$$\tag{9}$$

Hence the forecast mean of the level of TFP is simply:

$$\tilde{\mathbb{E}}_t A_{t+s}^i = A_t^i \exp\left(s\tilde{\mu}_{t+1} + \frac{s^2\tilde{\sigma}_t^2 + s\sigma_\epsilon^2}{2}\right)$$

2.4 Consumer Problem

For what follows we suppress the country specific superscripts. We assume a small open economy with immobile capital. For $t \geq T$ no further learning takes place and the problem takes the form of a standard growth model with a growth rate of γ:

$$V_T(A,b,k) = \max_{c,b',k'} u(c) + \beta V_T(\gamma A, b', k') \tag{10}$$

$$c + b' + k' = A^{1-\alpha}k^{\alpha} + (1-\delta)k + (1+r)b \tag{11}$$

while for $t < T$ the agents are learning η

$$V_t(A,b,k,g,\tilde{\mu},\tilde{\sigma}) = \max_{c,b',k'} u(c) + \beta \int V_{t+1}\left(A', b', k', \hat{A}' - \hat{A}, \tilde{\mu}', \tilde{\sigma}'\right) dF(A';\tilde{\mu}',\tilde{\sigma}') \tag{12}$$

$$\text{s.t.} \tag{13}$$

$$c + b' + k' = A^{1-\alpha}k^{\alpha} + (1-\delta)k + (1+r)b \tag{14}$$

where $\tilde{\mu}'$ and $\tilde{\sigma}'$ are given by equations (2)-(4)

In our model, b_t is the capital flows into and out of the country. $b_t > 0$ indicates savings, and therefore a capital outflow, while $b_t < 0$ indicates borrowing - a capital inflow. The allocation puzzle couched in the terms of our model is therefore $\text{corr}(b_t, g_t) \geq 0$ - growth does not induce agents to borrow.

3 Analysis

3.1 Solution After Uncertainty is Realized

Proposition 3.1. *Assume the utility function has the two properties that $u(\gamma^k x) = h(\gamma)^k u(x)$ and $u'(\gamma^k x) = g(\gamma)^k u'(x)$. If $\beta h(\gamma) < 1$ and $1 = \beta(1+r)g(\gamma)$, then there is a real valued function $V_T(A_T, k_T, b_T)$ that solves the recursive problem for $t \geq T$ and there are some functions:*

$$\bar{c}(A_T, b_T, k_T), \bar{k}(A_T, b_T, k_T), \bar{b}(A_T, b_T, k_T)$$

such that, for $t \geq T$:

- $c_t = \gamma^{t-T}\bar{c}(A_T, b_T, k_T)$

- $k_{t+1} = \gamma^{t+1-T}\bar{k}(A_T, b_T, k_T)$

- $b_{t+1} = \gamma^{t+1-T}\bar{b}(A_T, b_T, k_T)$

To see why proposition 3.1 is true, note that the first order conditions for $t \geq T$ are given by:

$$u'(c_t) = \beta u'(c_{t+1}) \left((\gamma^{t+1}A_T)^{1-\alpha}k_{t+1}^{\alpha} + 1 - \delta\right) \tag{15}$$

$$u'(c_t) = \beta u'(c_{t+1})(1+r) \tag{16}$$

Under our assumptions, we can replace each variable with the appropriate version from the theorem to get:

$$1 = \beta g(\gamma) \left(A_T^{1-\alpha}\bar{k}^{\alpha} + 1 - \delta\right) \tag{17}$$

$$1 = \beta g(\gamma)(1+r) \tag{18}$$

The importance of the assumption on $g(x)$ can be seen immediately from the first order condition on bonds, equation(18). Once we have verified that these conditions can be met we can compute the value as:

$$V_T(A_T, k_T, b_T) = \frac{1}{1-\beta h(\gamma)} u(\bar{c}(A_T, k_T, b_T))$$

which is real valued so long as $\beta h(\gamma) < 1$.

3.2 Analytical Example

In general, learning makes the problem analytically intractable before uncertainty is resolved. However, if we make the assumption that utility is quadratic and time variant, and abstract from production, an analytical solution is possible.

Proposition 3.2. *Assume that* $u_t(x) = F_1\gamma^t x - F_2 x^2, \alpha = 0, 1 = \beta\gamma(1+r),$ *and* $1 > \beta\gamma^2$. *Then the policy functions that solve the maximization problem of a country are given by: For* $t \geq T$*:*

$$c_t(A_T, b_T) = \gamma^{t-T}\left(A_T + (1+r-\gamma)b_T\right) \tag{19}$$

$$b_{t+1}(A_T, b_T) = \gamma^{t+1-T} b_T \tag{20}$$

$$\tag{21}$$

For $t < T$*:*

$$\begin{aligned} c_{T-t} &= \frac{1+r-\gamma}{1+r}A_{T-t} + (1+r-\gamma)b_{T-t} + \frac{1+r-\gamma}{1+r}\tilde{\mathbb{E}}_{T-t}\sum_{j=1}^{t-1}\frac{A_{T-t+j}}{(1+r)^j} + \tilde{\mathbb{E}}_{T-t}\frac{A_T}{(1+r)^t} \\ b_{T-t+1} &= \frac{\gamma}{1+r}A_{T-t} + \gamma b_{T-t} - \frac{1+r-\gamma}{1+r}\tilde{\mathbb{E}}_{T-t}\sum_{j=1}^{t-1}\frac{A_{T-t+j}}{(1+r)^j} - \tilde{\mathbb{E}}_{T-t}\frac{A_T}{(1+r)^t} \end{aligned}$$

The formulas for $\bar{c}$ and $\bar{b}$ are found by using the budget constraint for $t > T$ and for $t = T$. Once the endowment process is deterministic, the country simply consumes it's endowment plus the growth adjusted savings at T. Given this consumption policy, the sequential problem (in appendix for reference), is used to find Euler Equations for $t < T$ and we conduct backwards iteration to get the expression for c_{T-t}. The consumption function can be understood as a growth and belief-augmented version of what would occur with quadratic preferences under uncertainty, in which consumption at t is just the present value of expected future income.

It is instructive to consider the bond policy function. Define $B_{t+1} = \frac{b_{t+1}-\gamma b_t}{A_t}$. Then the covariance between B_{t+1} and g_t will clearly take the opposite sign as the correlation between the beliefs about growth and true growth rates. Therefore once this covariance is positive (and it must be eventually, no matter how negatively beliefs and reality are assumed to initially covary), the savings and growth must become negatively correlated as in the neoclassical model. We conjecture, but cannot prove, that the correlation is monotonic, in the sense that with each passing year the model would generate a correlation between savings and growth that is closer to that of the full information version. In absense of a general proof, we provide an example in which the covariance between beliefs and reality changes sign after one period, as does the covariance between savings and growth.

Proposition 3.3. *Continue making the assumptions from propositions 3.1 and 3.2. Also assume the alternate TFP process of* $A^i_{t+1} = (\eta^i + \epsilon^i_{t+1})A^i_t$ *and put* $T = 2$. *Assume that* $\rho = 0$ *Then:*

$$COV(b_1, A_1) = -\frac{1+r-\gamma}{(1+r)^2}\rho\sigma^2 = 0$$

but

$$COV(B_2, \frac{A_2}{A_1}) = -\frac{\sigma^2}{1+r}\left(\kappa_0 + (1-\kappa_0)\rho\right) < 0$$

This is just algebra and computing covariances, but it is illustrative of the prior point. Note that the correlations themselves are much more difficult to compute, which is why we have found conditions where the covariance changes signs so that we can say something definite about the correlations, in particular that is possible to generate a zero-correlation between growth and capital flows in the short run, but in the long run the correlation will become positive. In section 5.3, we study the time series behaviour of the allocation puzzle and show that this is consistent with the data.

4 TFP Estimation

4.1 Data Methodology

We set initial capital stock to $k_0 = I_o/(g_i + \delta)$ where g_i is growth rate of real investment for the first 10 years of data, and I_0 is the initial level of real investment from version 6.1 of the Penn World Tables (PWT Alan Heston and Aten [2002]).[4] Future capital stock is

[4]PWT 7.0 is slated for release in the last quarter of 2009. Among other things, it will include an update to ICP 2005, the pricing survey data that comprises the bulk of the data that PWT uses to construct the international dollar estimate. We will update all our data work to PWT 7.0 once it is released.

constructed using $k_t = (1-\delta)k_{t-1} + I_t$, where $\delta = 0.06$.

This method of calculating initial capital stock comes from Caselli [2004]. On a balanced growth path, $\bar{x} = (\delta + \gamma)\bar{k}$, where δ is capital depreciation and γ is the growth rate. If a country is on a balanced growth path and we have information on investment data at date T, we can calculate k_T using $x_T = (\delta + g_i)k_T$, where g_i is the mean growth rate which we take to be the growth rate of real investment for the next ten years. Clearly, this requires that k_0 *and* the 10 years of investment data we use to calculate g_i do not overlap the dates of interest, as the learning mechanism implies that the country is no longer on the balanced growth path. As can be seen in the appendix, k_0 starts in 1950 for most countries, (i.e. we assume balanced growth path from 1950-1960), well outside the date range we are considering for capital flows. Caselli [2004] examines the affect of different ways of estimating initial capital stock and concludes that "improv[ments to] the initial capital stock estimates [by using different methods of construction] is not likely to lead to major revisions to the baseline result".

We construct labor-augmenting TFP using RGDP in Laspeyres chained International Dollars and population from PWT, fraction of population aged 15-64 ("working age") from WDI, and the constructed series for capital stock:

$$A_t = \left(\frac{y_t}{k_t^{\alpha}}\right)^{1/(1-\alpha)} \frac{1}{L_t}$$

where $\alpha = 0.3$ and L_T, the size of the workforce, is population multiplied by fraction of population that is working age. The growth of TFP between $t = 0$ and T is given by

$$\Delta A_0^T = \left(\frac{A_T}{A_0}\right)^{1/T} - 1 \qquad (22)$$

4.2 TFP Process Estimation

We estimate σ_ϵ^2, μ_η, σ_η^2 for growth process using our constructed TFP process, A_t.
Let $\hat{A}_t = log(A_t)$ Then $\Delta\hat{A}_t = \hat{A}_t - \hat{A}_{t-1} = \eta^i + \epsilon_t^i$ where η^i is the mean growth rate for country i and ϵ_t^i is the transitory shock. We choose our our parameters to match the following moments:

Moment	Equation	DVC Estimation	DC Estimation
μ	$\frac{1}{N}\sum_{i=1}^{N}(\eta^i)$	0.020	0.012
σ	$\frac{1}{N}\sum_{i=1}^{N}(\eta^i - \mu)^2$	0.003	0.0001
σ_ϵ	$\frac{1}{T}\sum_{t=0}^{T}\frac{1}{N}\sum_{i=1}^{N}(\Delta\hat{A}_t - \eta^i)^2$	0.008	0.001

Using these equations and our constructed TFP series we estimate the following moments. σ for developed countries is one-sixth than that of developing countries, showing developed countries growth rates are more similar to each other than what we observe for developing countries. σ_ϵ is more than four times larger for developing countries, so that developing countries have growth rates that are more than four times as volatile as that of developed countries. We interpret this as developed countries having stable growth trend compared to developing countries.

5 The Allocation Puzzle

5.1 Constructing Change in Net External Position

We use the Net International Investment Position (NIIP) and cumulated errors and omissions (CEO) from Lane and Milesi-Ferretti [2007] to construct the net external position (a stock variable) in year t[5]

$$\text{NEP}_t = \text{CEO}_t - \text{NIIP}_t \tag{23}$$

Next, we need to convert from current US dollars to constant international dollars. We use the PPP adjustment method in Hsieh and Klenow [2007]. We divide by the PPP Adjustor, Q_t,

$$Q_t = P_t \frac{CGDP_t}{RGDP_t} \tag{24}$$

where $CGDP_t$ ($RGDP_t$) is GDP in current (constant) international dollars and P_t is the price of investment goods in PWT. As described in GJ, the PPP adjustment method may decrease our estimates of the volume capital flows, but will not cause a change in signs. Our methodology for constructing the change in net external position is slightly different than GJ. While GJ adds the sum of the (PPP adjusted) yearly capital flows from the IFS and CEO_t from LMF to NEP_{1980} to obtain an estimate for the net external position in 2000, we repeat the procedure in equation (23). While these two approaches are theoretically equivalent, there is a difference in the way the data is constructed: Lanes-Milesi-Feretti take valuation effects (due to asset price and currency movements) into account, while the data from the IFS does not. Given that our primary motivator is that developing countries face more uncertainty - which includes behavior of asset prices and currency movements - taking valuation effects into account is more consistent with our model. The change in net external position from t=0 to T is

$$\Delta\text{NEP}_0^T = \frac{\text{NEP}_T}{Q_T} - \frac{\text{NEP}_0}{Q_0} \tag{25}$$

If $\Delta\text{NEP}_0^T < 0$ then, on net, capital has left the country, if $\Delta\text{NEP}_0^T > 0$, capital has entered the country. Finally, we adjust for country size for dividing by initial RGDP.

[5]The sign convention is tricky: NIIP uses current account sign convention, so a negative NIIP indicates foreign capital inflow. Following standard practice, cumulative errors and omissions (CEO) are considered unreported capital flows and carries capital account sign conventions - foreign capital inflow in CEO is a positive entry. NEP_t is capital account units.

Figure 1: 1980-2000 avg. TFP growth: (L) All countries, (M) Foreign Aid Excluded, (R) No extreme aid

5.2 The Allocation Puzzle

To establish the allocation puzzle, we take corr$(\frac{\Delta NCF_0^T}{Y_0}, \Delta\text{TFP})$ across the 66 countries in our sample for the years 1980-2000. The year 1980 is chosen as a result of Chinn and Ito [2008], who find that countries in our sample had completed their financial liberalization by 1980. This means that the capital outflows we measure aren't caused by the relaxation of international capital mobility constraints. We find that the correlation is -0.2311, with a significance level of 0.0620. This negative correlation between capital flows and TFP growth is the allocation puzzle.

Foreign financial aid is included in capital flows, and this may skew results. We use net disbursements (ND_t)[6] from the UN Development Assistance Committee - Overseas Development Assistance to construct net foreign aid inflows, ΔB_0^T.

$$\Delta B_0^T = \sum_{t=0}^{T-1} \frac{ND_t}{Q_t} \tag{26}$$

The change in capital flows adjusted for foreign aid becomes:

$$\Delta D_0^T = \Delta \text{NCF}_0^T - \Delta B_0^T \tag{27}$$

[6]Net disbursements are defined in the data as total grants + concessionnal development loans - principal repayments on loans

	Non Aid-Adjusted	Aid Adjusted	Drop Extreme Aid
1980-2000	-0.2311	0.2025	0.0799
	(0.0620)	(0.1030)	(0.5622)

Table 1: Allocation Puzzle

where we have assumed that all foreign aid remains within the country and does not leave as private capital outflows. Since $\Delta B_0^T > 0$ for all countries in our sample (all countries are, on net, recipients of foreign aid) equation (27) clearly shows that $\Delta D_0^T < \Delta \mathrm{NCF}_0^T$. In other words, by excluding foreign aid implies we are overestimating capital outflows. Furthermore, within our sample of countries the correlation of 1980 - 2000 avg. TFP growth and net foreign aid inflows is negative and significant[7], so the fastest growing countries receive the least foreign aid inflows. The combination of these effect implies that our foreign aid adjusted numbers are strongly biased *against* the correlation puzzle. However, when we remove foreign aid, we find a positive but insignificant correlation of 0.2025 (0.1030). This positive correlation arises from the assumption that all aid remains within the country, with the resulting overestimation of capital outflows effects mostly the slow growing economies. Examination of figure 2 reveals that using this method of adjusting foreign aid some countries experienced net capital outflows greater than their 1980 GDP. If we drop all countries for which net capital outflows between 1980 and 2000 exceeds their 1980's GDP (i.e. $\frac{\Delta D_0^T}{Y_{1980}} < -1$) we find a correlation of 0.0799 (0.5622).We will refer to these as extreme aid countries. Extreme aid countries are Botswana, Benin, Burkina Faso, Ethiopia, Jordan, Kenya, Malawi, Mali, Nepal, Niger, Papua New Guinea, Senegal, Tanzania, Togo and Uganda. Our model has strong implications for the evolution of the correlation of TFP growth and capital flows through time, a dimension of the allocation puzzle which has not been studied before. Intuitively, our model predicts that as time passes agents learn the true growth rate, therefore we should observe the correlation between TFP growth and capital outflows become less negative.

[7]We find a correlation coefficient of -0.3137, with a significance level of 0.0092, or significance at 1%.

We choose 10 years as our smallest window to avoid business cycle effects and compare correlations from 1980-1990 and 1980-2000. We observe an increase in the correlation from -0.2607 to -0.2311. While each individual correlation is still negative, the correlation becomes more positive as we increase the length of time. Dropping Jordan (JOR) and Congo (COG), potentially biasing outliers, changes the 1980-1990 correlation to -0.2451 (0.0509) and the 1980-2000 correlation to -0.2193 (0.0817), so that our result still holds.

	1980-1990	1980-2000
All countries	-0.2607	-0.2311
	(0.0345)	(0.0620)
Aid Adjusted	-0.0405	0.2025
	(0.7506)	(0.1030)

Table 2: 10 year to 20 year window

6 Numerical Results

[TO BE COMPLETED]

[CURRENTLY, THIS SECTION CONSIDERS ONLY THE ANALYTICAL EXAMPLE SOLUTION]

Using the consumption and savings equations derived under quadratic utility in the endowment economy, we compute the time series of bond holdings. We follow Gourinchas and Jeanne by putting $\beta = 0.96$, which along with our estimate for the growth rate of developed countries implies that $r = 2.98\%$. We compute the correlation of the endogenous varabiables of interest over a large cross section of countries. Fortran code with details can be found on our websites. We are able to match the correlation between debt and growth over a twenty year period, and find that the correlation implied over the first ten years is very close (but slightly higher) than in the data. While we are forced to impose an extremely unrealistic initial correlation between the mean beliefs and and reality of $\rho = -0.602$, we see some evidence that learning is a relevant friction.

	1980-1990	1980-2000
Data	-0.261	-0.231
Model	-0.274	-0.231

Table 3: 10 year to 20 year window

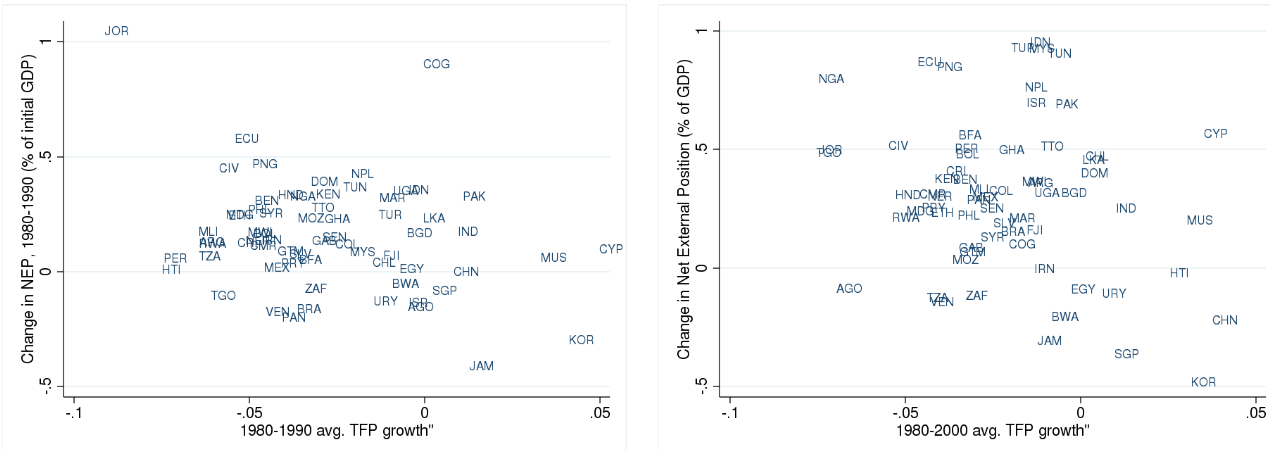

Figure 2: 10 year to 20 year window

7 Time Series Behavior

Given our model, a theory about learning TFP growth, 1980 may seem arbitrary year to use as t=0. To address this concern we redo the data work using three different events as our initial dates: First, we use the date of the beginning of a growth acceleration episode (12 countries). Next, we use as our initial date the year of a stable regime change (16 countries). Finally, we consider the date of joining WTO as our initial year (9 countries). For each of these three experiments we find the same behaviour: There is a negative correlation between inflows and TFP growth for the first five years after the event, which becomes less negative (and in some cases positive) as we increase the length of time to 10 years after the event.[8] It is also noteworthy all of these occurrences, with the exception of the regime change with foreign aid included, seem to trigger a capital outflow, as is evidenced by the larger negative

[8]We would have preferred to make the smallest window 10 years and then scale up to 20 years, but doing so would have reduced our sample size to 0, 5 and 1 country respectively.

correlation we find when we compare net capital flows for the first five years after the event (T to T+5) to the net capital flows for the five years before the event (T-5 to T). Unfortunately, we also find that almost all of these correlations are not significant, with or without foreign aid. This insignificance is almost certainly due to the small sample sizes, and we hope that when we update to PWT 7.0 (which extends the data a further 8 years) the increase in sample size will result in significance for these correlations. Figures associated with each of the reported correlations are available in the appendix.

First, we use as our initial year is the beginning of a growth acceleration episode as identified by Hausmann et al. [2005], in which a growth acceleration episode is identified as a date t such that the growth rate of real gdp per capita from t to t+n ($g_{t,t+n}$, n=[0,1,..,7]) satisfies the following three criteria

1. $g_{t,t+n} \geq 3.5\%$
2. $\Delta g_{t-n,t+n} \geq 0.2\%$
3. $y_{t+n} \geq \max y_i \quad \forall i \leq t+n$

Condition 1 is simply that the growth rate exceeds that of the OECD's. Condition 2 states that growth needs to accelerate. Condition 3 removes economic recovery episodes by requiring that rgdp per cap needs to be higher than any prior year on record.

	T-5	T+5	T+10
Non Aid-Adjusted	0.0421	-0.3547	-0.2840
	(0.8967)	(0.2580)	(0.3709)
Aid-Adjusted	0.1502	0.0680	0.2363
	(0.6413)	(0.8336)	(0.4597)

Table 4: Growth Acceleration Episodes

The correlations evolve as one would expect given the existence of the allocation puzzle - the growth acceleration episode triggers a capital outflow. However, none of these correlation are clearly significant.

Next, We use as our initial date for each country the year of a stable regime change - a regime change that is neither preceded or anteceded by another regime change for a period of 10 years between the years 1975 and 1990. We find 17 countries within our sample. To determine the date of the regime change, we use Marshall and Jaggers [2007] Polity IV database, which tracks all political regime changes (positive or negative) in independent countries with total population greater than 500,000 in 2007.

As table 4 shows, there is a net outflow of capital after a regime change, and this outflow decreases the longer the regime is in power. It seems reasonable that countries that are politically unstable, or seem on the brink of a political regime change, may experience a surge in foreign aid, for either humanitarian or political motives. When we remove foreign aid flows we still have the same behavior, though once again the correlations are not significant.

	T-5	T+5	T+10
Non Aid-Adjusted	-0.1542	-0.0491	0.0037
	(0.5686)	(0.8516)	(0.9888)
Foreign Aid Removed	-0.0362	0.0548	0.1758
	(0.8941)	(0.8402)	(0.5148)
Foreign Aid Removed, Drop Chile	0.0644	-0.4473	-0.0766
	(0.8196)	(0.0823)	(0.7780)

Table 5: Regime Change

Finally, we use the date of joining WTO as t=0. Nine countries in our sample joined GATT/WTO between 1975 and 1990. Once again we find the same result both with and without foreign aid flows - the negative correlation between capital outflows and TFP growth becomes less negative the longer the country has been a member.

8 Conclusion

We introduce learning into a standard neoclassical growth model and use it to explain the allocation puzzle. While we have limited success - we need to impose a significant negative

	T-5	T+5	T+10
Non Aid-Adjusted	-0.2512	-0.4513	0.2179
	(0.5145)	(0.2228)	(0.5733)
Foreign Aid Removed	-0.3697	-0.6362	0.-0.0234
	(0.3274)	(0.0655)	(0.9524)

Table 6: GATT/WTO Membership

relationship between reality and initial beliefs - we are capable of matching both the initial correlation and the evolution of the allocation puzzle.

As a next step, we plan to solve the model with production and precautionary savings in hopes that this will allow for a more reasonable initial correlation, and to see if we can solve the Feldstein-Horoika puzzle using the same model.

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A Sequential Problem

We write the sequential problem for completeness. A country chooses functions in $(c_t, k_{t+1}, b_{t+1})_{t=0}^{\infty}$ such that $(c_t, k_{t+1}, b_{t+1}) : \mathbb{R}^t \to \mathbb{R}^3_+$ for $0 \leq t \leq T$ and $(c_t, k_{t+1}, b_{t+1}) : \mathbb{R}^T \to \mathbb{R}^3_+$ for $t \geq T$. These functions must solve the maximization problem:

$$\max \sum_{t=0}^{T} \beta^t \int u\left(c_t(A^t)\right) d\tilde{F}_t(A^t; \tilde{\mu}_0, \tilde{\sigma}_0^2) + \beta^T \int \sum_{t=T}^{\infty} u\left(c_t(A^T)\right) d\tilde{F}_T(A^T, \tilde{\mu}_0, \tilde{\sigma}_0^2)$$

subject to

$$\begin{aligned}
\text{For } t &= 0, ..., T-1 \\
c_t + k_{t+1} + b_{t+1} &= A_t^{1-\alpha} k_t^{\alpha} + (1-\delta)k_t + (1+r)b_t \\
\text{For } t &\geq T \\
c_t + k_{t+1} + b_{t+1} &= (\gamma^{t-T} A_T)^{(1-\alpha)} k_t^{\alpha} + (1-\delta)k_t + (1+r)b_t
\end{aligned}$$

B Tables

B.1 Developing Countries

Country	Isocode	k_0 Date	TFP Start	TFP End	NCF Start	NCF End	For. Aid Start	For. Aid End
Angola	(AGO)	1960	1960	1996	1985	2000	1970	1996
Argentina	(ARG)	1950	1960	2000	1970	2000	1970	2000
Bangladesh	(BGD)	1959	1960	2000	1973	2000	1971	2000
Benin	(BEN)	1959	1960	2000	1970	2000	1970	2000
Bolivia	(BOL)	1950	1960	2000	1970	2000	1970	2000

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Country	Isocode	k_0 Date	TFP Start	TFP End	NCF Start	NCF End	For. Aid Start	For. Aid End
Botswana	(BWA)	1960	1960	1999	1974	1999	1970	2000
Brazil	(BRA)	1950	1960	2000	1970	2000	1970	2000
Burkina Faso	(BFA)	1959	1960	2000	1974	2000	1970	2000
Cameroon	(CMR)	1960	1960	2000	1970	2000	1970	2000
Chile	(CHL)	1951	1960	2000	1970	2000	1971	2000
China, P.R.	(CHN)	1952	1960	2000	1981	2000	1979	2000
Colombia	(COL)	1950	1960	2000	1970	2000	1970	2000
Congo, Rep.	(COG)	1960	1960	2000	1970	2000	1970	2000
Costa Rica	(CRI)	1950	1960	2000	1970	2000	1970	2000
Cote d'Ivoire	(CIV)	1960	1960	2000	1970	2000	1970	2000
Cyprus	(CYP)	1950	1960	1996	1973	1996	1970	1996
Dominican Rep.	(DOM)	1951	1960	2000	1970	2000	1970	2000
Ecuador	(ECU)	1951	1960	2000	1970	2000	1970	2000
Egypt, Arab Rep.	(EGY)	1950	1960	2000	1970	2000	1970	2000
El Salvador	(SLV)	1950	1960	2000	1970	2000	1970	2000
Ethiopia	(ETH)	1950	1960	2000	1970	2000	1970	2000
Fiji	(FJI)	1960	1960	2000	1977	2000	1970	2000
Gabon	(GAB)	1960	1960	2000	1970	2000	1970	2000
Ghana	(GHA)	1955	1960	2000	1970	2000	1970	2000
Guatemala	(GTM)	1950	1960	2000	1970	2000	1970	2000

Continued on Next Page...

Country	Isocode	k_0 Date	TFP Start	TFP End	NCF Start	NCF End	For. Aid Start	For. Aid End
Haiti	(HTI)	1967	1967	1998	1970	1998	1970	2000
Honduras	(HND)	1950	1960	2000	1970	2000	1970	2000
India	(IND)	1950	1960	2000	1970	2000	1970	2000
Indonesia	(IDN)	1960	1960	2000	1981	2000	1970	2000
Iran, Islamic Rep	(IRN)	1955	1960	2000	1970	2000	1970	2000
Israel	(ISR)	1950	1960	2000	1970	2000	1970	1996
Jamaica	(JAM)	1953	1960	2000	1970	2000	1970	2000
Jordan	(JOR)	1954	1960	2000	1970	2000	1970	2000
Kenya	(KEN)	1950	1960	2000	1970	2000	1970	2000
Korea, Rep.	(KOR)	1953	1960	2000	1970	2000	1970	1999
Madagascar	(MDG)	1960	1960	2000	1970	2000	1970	2000
Malawi	(MWI)	1954	1960	2000	1970	2000	1970	2000
Malaysia	(MYS)	1955	1960	2000	1970	2000	1970	2000
Mali	(MLI)	1960	1960	2000	1970	2000	1970	2000
Mauritius	(MUS)	1950	1960	2000	1970	2000	1970	2000
Mexico	(MEX)	1950	1960	2000	1970	2000	1970	2000
Morocco	(MAR)	1950	1960	2000	1970	2000	1970	2000
Mozambique	(MOZ)	1960	1960	2000	1980	2000	1970	2000
Nepal	(NPL)	1960	1960	2000	1970	2000	1970	2000
Niger	(NER)	1960	1960	2000	1970	2000	1970	2000

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Country	Isocode	k_0 Date	TFP Start	TFP End	NCF Start	NCF End	For. Aid Start	For. Aid End
Nigeria	(NGA)	1950	1960	2000	1970	2000	1970	2000
Pakistan	(PAK)	1950	1960	2000	1970	2000	1970	2000
Panama	(PAN)	1950	1960	2000	1970	2000	1971	2000
Papua New Guinea	(PNG)	1960	1960	1999	1973	1999	1970	2000
Paraguay	(PRY)	1951	1960	2000	1970	2000	1970	2000
Peru	(PER)	1950	1960	2000	1970	2000	1970	2000
Philippines	(PHL)	1950	1960	2000	1970	2000	1970	2000
Rwanda	(RWA)	1960	1960	2000	1970	2000	1970	2000
Senegal	(SEN)	1960	1960	2000	1970	2000	1970	2000
Singapore	(SGP)	1960	1960	1996	1970	1996	1970	1995
South Africa	(ZAF)	1950	1960	2000	1970	2000	1993	2000
Sri Lanka	(LKA)	1950	1960	2000	1970	2000	1970	2000
Syrian Arab Rep	(SYR)	1960	1960	2000	1970	2000	1970	2000
Tanzania	(TZA)	1960	1960	2000	1970	2000	1970	2000
Togo	(TGO)	1960	1960	2000	1970	2000	1970	2000
Trinidad & Tobago	(TTO)	1950	1960	2000	1970	2000	1970	2000
Tunisia	(TUN)	1961	1961	2000	1970	2000	1970	2000
Turkey	(TUR)	1950	1960	2000	1970	2000	1970	2000
Uganda	(UGA)	1950	1960	2000	1970	2000	1970	2000
Uruguay	(URY)	1950	1960	2000	1970	2000	1970	2000

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Country	Isocode	k_0 Date	TFP Start	TFP End	NCF Start	NCF End	For. Aid Start	For. Aid End
Venezuela, RB	(VEN)	1950	1960	2000	1970	2000	1970	2000

Table 7: DVC Country list (66)

B.2 Developed Countries

Country	Isocode	k_0 start	TFP Start	TFP End
Australia	(AUS)	1950	1960	2000
Austria	(AUT)	1950	1960	2000
Belgium	(BEL)	1950	1960	2000
Canada	(CAN)	1950	1960	2000
Denmark	(DNK)	1950	1960	2000
Finland	(FIN)	1950	1960	2000
France	(FRA)	1950	1960	2000
Germany	(GER)	1970	1970	2000
Greece	(GRC)	1950	1960	2000
Iceland	(ISL)	1950	1960	2000
Ireland	(IRL)	1950	1960	2000
Italy	(ITA)	1950	1960	2000

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Country	Isocode	k_0 start	TFP Start	TFP End
Japan	(JPN)	1950	1960	2000
Luxembourg	(LUX)	1950	1960	2000
Netherlands	(NLD)	1950	1960	2000
New Zealand	(NZL)	1950	1960	2000
Norway	(NOR)	1950	1960	2000
Portugal	(PRT)	1950	1960	2000
Spain	(ESP)	1950	1960	2000
Sweden	(SWE)	1951	1960	2000
Switzerland	(CHE)	1950	1960	2000
United Kingdom	(GBR)	1950	1960	2000
United States	(USA)	1950	1960	2000

Table 8: Developed Countries list (23)

B.3 Time Series Behavior, Initial Dates

Country	Growth Acceleration	Regime Change	GATT/WTO Membership
Argentina	1990		
Bolivia			1990
Botswana			1987
Brazil		1985	
Chile	1986	1989	
China	1987		
Colombia			1981

Costa Rica			1990
Ecuador		1979	
El Salvador		1984	
Fiji		1990	
Honduras		1982	
India	1982		
Indonesia	1987		
Iran		1982	
Jordan		1989	
Korea	1984		
Malaysia	1988		
Mauritius	1983		
Mexico		1977	1986
Morocco			1987
Nigeria		1979	
Panama		1989	
Papua New Guinea	1987	1975	
Peru		1980	
Philippines			1979
Senegal		1978	
Syrian Arab Republic	1989		
Tunisia		1987	1990
Uganda	1989	1980	
Uruguay	1989	1985	
Venezuela			1990

Table 9: Initial Years

C Aid-Adjusted Figures

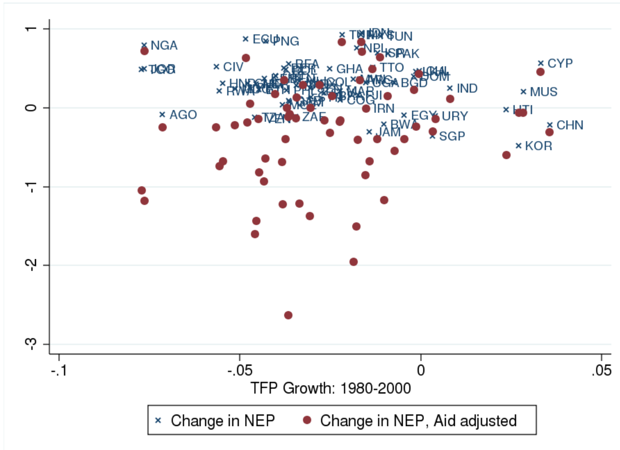

Figure 3: Comparing No-Aid Adjusted and Aid-Adjusted changes in Net External Position

Note that the Aid-Adjusted case is simply the downward vertical shift of the Aid-included case for all countries except Korea and Singapore. For example, Angola (AGO) aid adjusted capital flows (the circle) is beneath the non-aid-adjusted net capital flows (the x). Korea and Singapore have a shift in TFP growth as well because the foreign aid data series ends before the capital flows does, meaning that we need to recalculate TFP growth for the shorter time span.