Trade and Within-Country Consumption Inequality*

Job Market Paper

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Abstract

I study the effect of trade on the consumption by different income groups within a country. Using a model of trade with product differentiation and pricing-to-market, I show that trade changes consumption inequality even if income inequality is unchanged, a departure from previous trade models. Trade liberalization affects the pricing mark-ups and income groups firms sell to, changing consumption inequality by changing variety and prices. I show that when the trade partner is an identical country trade liberalization decreases consumption inequality. Trade with a country with different income inequality decreases consumption inequality.

JEL Classification: F16, L11

Keywords: inequality, variable mark-ups, nonhomothetic preferences, heterogeneous firms

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1 Introduction

How does trade affect inequality within a country? The classic approach within the trade literature, based upon the Hecksher-Ohlin model, examines the effect of trade on the relative wages of income groups, and then maps those changes to changes in relative consumption. This approach implies that if trade doesn't change relative earnings then relative consumption doesn't change either. However, recent trade literature has established that cross country price levels of traded goods are systematically positively correlated with per capita income: firms charge higher prices in countries with higher GDP per capita[1], and that this effect is particularly strong in tradeable consumption goods.[2] As traded goods are part of the consumers consumption bundle, these pricing practices can exacerbate or alleviate the consumption inequality - the difference in aggregate consumption of different income groups - even if income inequality is unchanged. In order to understand how trade changes consumption inequality, it is therefore important to include this underlying price mechanism in a model of trade: that is the goal of this paper.

I introduce two income groups into a general equilibrium trade model with nonhomothetic preferences. The non-homothetic utility in this model results in bounded marginal utility for goods so that there exists a choke price above which consumers are unwilling to buy a good even if they have access to it on the market. Consumers therefore endogenously choose not only the quantity but also the variety of goods to consume. Firms are heterogeneous in productivity and monopolistic price setters, as per Melitz [2003] and Chaney [2008], but cannot perfectly price discriminate between income groups and must therefore choose whether to make their good available at a low price (such that both consumers can afford the good), or a high price (such that it is exclusively affordable to wealthy consumers).

Previous trade models use homothetic preferences, which limits both income groups to consume the same variety of goods[3], and consumers of lower income groups consume a constant fraction of the high income group's consumption. Therefore, under the assumption that trade liberalization does not affect the *relative* incomes, relative consumption inequality is unaffected. With per-capita and inequality dependent mark-ups that exist in the data and in my model, I show that this is no longer true; even if incomes are unaffected by a trade liberalization it is still possible that

[1] For example, see Fieler [2011], Hummels [2009], Simonovska [2011]

[2] Hsieh [2007]

[3] Unless explicit assumptions are made that restrict market access of certain income groups, a case I ignore.

consumption profiles are changed. A decrease in the cost of trade changes the firms optimal price, and may change which income group market they serve (high income, or both type). This changes the variety and quantity of goods consumers buy. I find that the effect of trade on consumption inequality depends crucially on the characteristics on the trade partner.

The resurgence of nonhomothetic demand in trade has resulted in various possible specifications of demand structures. The interested reader should refer to Markusen [2010] for a comprehensive discussion of the usefulness non-homothetic preferences in trade. Bekkers [2011] compares the predictions of hierarchic demand, demand for quality and demand finickyness, all of which have been used in the trade literature to explain the positive correlation between prices and income, and find that only models with hierarchic demand systems can explain the negative correlation between prices and inequality. Simonovska [2011] compares hierarchic demand structures with models of search costs and find that these models fail to capture the negative relationship between prices and market size that can be matched by hierarchic demand. Conversely, she finds that the nonhomothetic demand presented by Feenstra [2003] and Melitz [2008] capture the negative relationship between prices and market size but do not capture per-capita income effects on prices. With these results in mind, I use a hierarchic demand structure as specified in Simonovska [2011] to generate nonhomotheticity in my demand function.

To summarize, in this paper I present a heterogenous-firm model of international trade with two income groups with identical nonhomothetic demand residing in each country. I find that even if trade does not change relative incomes, the optimal pricing strategies of firms change as a result of trade liberalization which in turns changes relative consumption inequality. The layout of the paper is as follows: In section 2, I introduce my model. In section 3, I define consumption inequality, in section 4 I present simulation results.

2 Model

I consider a static environment with a finite number of countries, i $\in \{1, ..., I\}$, trading varieties of final goods. Let s denote the exporter ("source") and d the importer ("destination").

2.1 Consumer

Country $d \in \{1,..,I\}$ has two consumer types, $k = \{H, L\}$, each of population size M_d^k such that $\sum_k M_d^k = M_d$, where M_d denotes the total population. Consumers types differ only by their effective labor, θ_d^k. Both consumer types supply 1 unit of labor inelastically, so that the total effective labor supply is $L_d = M_d^H \theta_d^H + M_d^L \theta_d^L$. Let w_d denote income per unit of effective labor, then income for type k is given by $w_d^k = \theta_d^k w_d$. As $w_d^H = \frac{\theta_d^H}{\theta_d^L} w_d^L$, the assumption that $\theta_d^H > \theta_d^L$ is sufficient to ensure that H-type has higher income than L-type. All consumers have the identical utility function over the available goods, G_s, from countries s={1,...,I}.

$$U_d^k = \sum_s \int_0^{G_s} \log(c_{sd}^k(i) + q) di \tag{1}$$

where q>0 and is assumed to be the same across all goods and countries.

Given income, w_d^k and prices, $p_{sd}(i)$, each type of consumer chooses $c_{sd}^k(z)$ to solve

$$\max_{c_{sd}^k(i)} \quad \textstyle\sum_s \int_0^{G_s^k} \log(c_{sd}^k(i) + q) di \tag{2}$$

$$\text{s.t} \quad \textstyle\sum_s \int_0^{G_s} p_{sd}(i) c_{sd}^k(i) di \leq w_d^k \tag{3}$$

$$c_{sd}^k(i) \geq 0 \tag{4}$$

The solution to the consumer's problem is

$$c_{sd}^k(i) = \max\{0, \frac{w_d^k + q P_d^k}{p_{sd}(i) N_d^k} - q\} \tag{5}$$

where

$$N_d^k = \sum_s \int_0^{G_s^k} 1 di \tag{6}$$

is the number of products purchased by consumer k and

$$P_d^k = \sum_s \int_0^{G_s^k} p_{sd}(i) di \tag{7}$$

is the pricing aggregator over the goods purchased by consumer k.

Note that the only variety-specific variable in (5) is $p_{sd}^k(i)$; all the other terms are aggregates. It is then obvious that consumers have a bounded marginal utility for any good i: if $p^k_{sd}(i) > \frac{w^k_d + qP^k_d}{qN^k_d}$ (i.e. the price for a specific good i is "too high") the consumer will choose the optimal boundary solution of $c^k_{sd}(i) = 0$. As consumers have different incomes, it is possible for the variety and price aggregator of each consumer to be different. For future use, define the following terms,

$$\text{Truncated Demand of type } k: \qquad Q^k_d = \frac{w^k_d + qP^k_d}{N^k_d} \tag{8}$$

$$\text{Aggregate Truncated Demand:} \qquad S_d = M^H_d Q^H_d + M^L_d Q^L_d \tag{9}$$

Proposition 2.1. *If $Q^H_d \geq Q^L_d$ then $\forall z$, $c^H_d(z) \geq c^L_d(z)$*

Proof: Rewrite equation (5) as $c^H_{sd}(i) = \max\{0, \frac{Q^H_d}{p_{sd}(i)} - q\}$ and $c^L_{sd}(i) = \max\{0, \frac{Q^L_d}{p_{sd}(i)} - q\}$. The difference can be written as $c^H_{sd}(i) - c^L_{sd}(i) = \frac{1}{p_{sd}(i)}\left(\max\{0, Q^H_d - p_{sd}(i)q\} - \max\{0, Q^L_d - p_{sd}(i)q\}\right)$. As both consumers face the same price, $Q^H_d \geq Q^L_d$ is sufficient to give the result $c^H_d(z) \geq c^L_d(z)$. □
Proposition (2.1) implies that it will never be true that firms only sell the good to L-type; they will either sell to just the H-type or to both types.

2.2 Firm Problem

I follow Melitz [2003] and assume a continuum of firms in each country using labor-only technology, differentiated by only their productivity. I use the simplification of Chaney [2008] and assume that firms can pay a fixed cost, f_c, to draw a productivity z from a Pareto distribution with support $[b_i, \infty]$ where $b_i \geq 1$ denotes the minimum productivity level of the draw in country i.[4] Productivity and country uniquely identify each firm. Firms pay a tariff cost, $\tau_{sd} \geq 1$, where $\forall s \quad \tau_{ss} = 1$, and a fixed cost of export, f_{sd}, where $f_{ss} = 0$ and $f_{sd} = f_x$ for $s \neq d$. The fixed cost is a one-time cost incurred to establish trade with each trading partner - it can be thought of as the cost to updating packaging for sales in foreign countries, establishing stores or undertaking initial market research. τ_{sd} is a cost that is paid for each good imported, and can be thought of as representing transportation costs or tariffs. As each firm sells only one good, and goods from different countries are not substitutable, a good i from country s is equivalent to a firm z from country s. I will

[4]The Pareto cdf is $F(z) = 1 - \left(\frac{b_i}{z}\right)^\gamma$, the pdf is $f(z) = \frac{\gamma b^\gamma}{z^{\gamma+1}}$, where the shape parameter $\gamma > 0$. Different b's denote countries varying levels of productivity, with a higher b denoting a country with a higher average productivity. I will set $b_i = 1$ for all countries, and capture different country-level productivities by allowing different average labor productivities.

assume a no-arbitrage constraint across borders - the wages of consumers in neighboring countries do not enter the firms problem - and that firms cannot price discriminate between agents of different income groups: regardless of income all consumers face the same prices.[5]

As firms cannot price discriminate between income groups, each firm can set a low price such that their good is affordable to both consumers and earn a resulting profit of $\pi_{sd}^{B}(z)$, or it can set a high price such that its good is only affordable to the rich consumer-type earn profit $\pi_{sd}^{H}(z)$. As I assume a linear production function with only labor and a no-arbitrage condition, I can examine the firms production decision for each destination country, d, separately. The firm's problem then, is to choose the pricing plan that results in

$$\pi_{sd}(z) = \max\{\pi_{sd}^{B}(z), \pi_{sd}^{H}(z), 0\} \tag{10}$$

where a choice of 0 indicates that the firm has drawn a productivity that is too inefficient to produce under either pricing plan.

Given the consumers indirect demand functions $x^k(p_{sd}(z); P_d^k, N_d^k, w_d^k) = M_d^k c^k(z)$[6], a firm z that sells to both types in country d solves

$$\pi_{sd}^{B}(z) = \max_{p_{sd}(z)\ell_{sd}(z)} \quad p_{sd}(z)x_{sd}^{H}(z) + p_{sd}(z)x_{sd}^{L}(z) - w_s\ell_s(z) \text{ s.t} \tag{11}$$

$$x_{sd}^{H}(z) + x_{sd}^{L}(z) = \tfrac{z}{\tau_{sd}}(\ell_s(z) - f_{sd}) \tag{12}$$

where $\ell_{sd}(z) = \theta_s^H \ell_s^H(z) + \theta_s^L \ell_s^L(z)$ represents the effective labor hired by the firm, where market clearing imposes that $\sum_d \int \ell_{sd}(z)dz = M_s^H\theta_s^H + M_s^L\theta_s^L$. The assumption of a linear production technology ensures that firms are indifferent between the two types of labor.

[5] It is possible to add a constraint such that $p_{sd}^H(z) \leq (1+r)p_{sd}^L(z)$, where r>0. For example, a firm could choose to locate stores with the lower price goods close to poor neighborhoods but far from wealthy neighborhoods. Poor consumers could then buy the good at the low price, travel to the rich neighborhoods and sell it at a markup of r. A necessary condition for this constraint to bind is $(1+r)^2 < \left(\frac{w_j^H + qP_j^H}{N_j^H}\right)\left(\frac{N_j^L}{w_j^L + qP_j^L}\right)$. This additional constraint complicates the model considerably by adding another equilibrium configuration for cutoffs without considerably changing the results.

[6] I abbreviate this as $x_d^k(z)$.

A firm that sells only to the H-type of consumers solves

$$\pi^H_{sd}(z) = \max_{p_{sd}(z),\ell_{sd}(z)} \quad p_{sd}(z)x^H_{sd}(z) - w_s\ell_{sd}(z) \quad \text{s.t.} \tag{13}$$

$$x^H_{sd}(z) = \tfrac{z}{\tau_{sd}}(\ell_{sd}(z) - f_{sd}) \tag{14}$$

Denote by $p^H_{sd}(z)$ the price a firm would charge if the sell only to the H-type, and $p^B_{sd}(z)$ the price a firm would charge if selling to both types. Solving the firms problem we find that the optimal price for a firm selling only to the H-type consumer is

$$p^H_{sd}(z) = \underbrace{\left[\frac{\tau_{sd}w_s}{z}\right]^{\frac{1}{2}}}_{\text{Marginal Cost}} \underbrace{\left[\frac{Q^H_d}{q}\right]^{\frac{1}{2}}}_{\text{Mark-up}} \tag{15}$$

which results in a profit

$$\pi^H_{sd} \;=\; q^{\frac{1}{2}}M^H_d\left[\left(\frac{Q^H_d}{q}\right)^{\frac{1}{2}} - \left(\frac{\tau_{sd}w_s}{z}\right)^{\frac{1}{2}}\right]^2 - w_s f_{sd} \tag{16}$$

If the firm sells to both types of consumers optimal price is

$$p^B_{sd}(z) \;=\; \underbrace{\left[\frac{\tau_{sd}w_s}{z}\right]^{\frac{1}{2}}}_{\text{Marginal Cost}} \underbrace{\left[\frac{S_d}{qM_d}\right]^{\frac{1}{2}}}_{\text{Mark-Up}} \tag{17}$$

which results in a profit

$$\pi^B_{sd}(z) \;=\; q^{\frac{1}{2}}M_d\left[\left(\frac{S_d}{qM_d}\right)^{\frac{1}{2}} - \left(\frac{w_s\tau_{sd}}{z}\right)^{\frac{1}{2}}\right]^2 - w_s f_{sd} \tag{18}$$

Notice that all firms serving destination d have the same mark-up choices, with variations in the prices of firms with different productivities driven by differing marginal costs of serving market d, or by firms choosing different market segments.

Proposition 2.2. *If $Q^H_d \geq Q^L_d$ then $\forall z$ $p^H_{sd}(z) \geq p^L_{sd}(z)$.*

Proof: Rearrange pricing equations (15) and (17) to find that it is sufficient to show that the markup for $p^H_{sd}(z)$ is greater than the markup for $p^L_{sd}(z)$. The required condition for this to hold is

$M_d Q_d^H > M_d^H Q_d^H + M_d^L Q_d^L$, which is true if $Q_d^H > Q_d^L$. $\square$

2.3 Cutoff Productivity

Unlike standard Melitz models in which profit is a linear functions of firm productivity, profits in this model are quadratic and can have multiple cutoff configurations. There are up to three productivities of potential interest: the productivity at which the profit from selling exclusively to the H-type is zero, z^{H0}, the productivity at which the profit from selling to the both types is zero, z^{B0}, and the productivity at which firms are indifferent between selling to the H-type and L-type, z^{HB}.

z_{sd}^{H0} is found by solving $\pi_{sd}^H(z_{sd}^{H0}) = 0$ to obtain

$$z_{sd}^{H0} = \tau_{sd} w_s \frac{q M_d^H}{\left[\left(M_d^H Q_d^H\right)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}\right]^2} \tag{19}$$

Similarly, z_{sd}^{B0} is found by solving $\pi_{sd}^B(z_{sd}^{B0}) = 0$

$$z_{sd}^{B0} = \tau_{sd} w_s \frac{q M_d}{\left[S_d^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}\right]^2} \tag{20}$$

Finally, the productivity at which firms are indifferent between selling to the H-type and the L-type, z_{sd}^{HB} is given by solving $\pi_{sd}^B(z) = \pi_{sd}^H(z)$,

$$z_{sd}^{HB} = \tau_{sd} w_s \frac{q \left[M_d^{\frac{1}{2}} - M_d^{H^{\frac{1}{2}}}\right]^2}{\left[S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right]^2} \tag{21}$$

With three cutoffs there are 10 possible cutoff configurations that begin with a zero profit productivity. The purpose of this section is to show that only one of these configurations is an equilibrium.

Proposition 2.3. *There is only one zero crossing cutoff in the equilibrium cutoff configuration*

Proof: Let z_{sd}^0 denote the productivity such that $\max\{\pi_{sd}^H(z_{sd}^0), \pi_{sd}^B(z_{sd}^0)\} = 0$. As profit equations (16) and (18) increase in z, $\forall z > z_{sd}^0$, $\max\{\pi_{sd}^H(z), \pi_{sd}^B(z)\} > \max\{\pi_{sd}^H(z_{sd}^0), \pi_{sd}^B(z_{sd}^0)\} = 0$. $\square$

Assumption 2.4. *Fixed costs don't prevent trade:* $M_d^H Q_d^H > w_s f_{sd}$ *and* $S_d > w_s f_{sd}$ $\forall d$

Proposition 2.5. *If assumption (2.4) is satisfied, then in the limit all firms choose to sell to both consumer groups.*

Proof: $lim_{z\to\infty}\pi_{sd}^H(z) \to \left(M_d^H Q_d^H - f_{sd}\right)$ and $lim_{z\to\infty}\pi_{sd}^B(z) \to (S_d - f_{sd})$. With assumption 2.4 in the limit $\pi_{sd}^B > 0$ and firms enter and sell in the market. Since $Q_d^L > 0$ in the limit $\pi_{sd}^B > \pi_{sd}^H$, and firms find it more profitable to sell to both types when they enter. □

Theorem 2.6. *If propositions 2.3 and 2.5 are satisfied, then there are only 2 potential equilibrium cutoff configurations:* $\{z_{sd}^{B0}\}$ *and* $\{z_{sd}^{H0}, z_{sd}^{HB}\}$.

Proposition 2.3 restricts attention to cutoff configurations with only one zero profit cutoff, reducing the possible cutoff configurations from 10 possible sets to 4: $\{z_{sd}^{H0}\}$; $\{z_{sd}^{B0}\}$; $\{z_{sd}^{H0}, z_{sd}^{HB}\}$; $\{z_{sd}^{B0}, z_{sd}^{HB}\}$. Given that profit functions are increasing and continuously differentiable on the support of productivities, any crossing indicates a switch in market structure. Proposition 2.5 further reduces these sets by limiting attention to only sets in which all firms will sell to both groups in the limit, of which there are 2: $\{z_{sd}^{B0}\}$; $\{z_{sd}^{H0}, z_{sd}^{HB}\}$.[7] □

Assumption 2.7. *Fixed costs of trade are small enough that it is profitable for some foreign firms to sell only to H-type:*$\forall s,d$ $f_x < \frac{M_d^{H\frac{1}{2}}}{M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}}} \frac{\left(M_d Q_d^H\right)^{\frac{1}{2}} - S_d^{\frac{1}{2}}}{w_s^{\frac{1}{2}}}$

Proposition 2.8. *If* $Q_d^H > Q_d^L$ *then the only potential domestic equilibrium cutoff configuration is* $\{z_{dd}^{H0}, z_{dd}^{HB}\}$. *In addition, if the fixed cost of export satisfy assumption (2.7), then the only equilibrium cutoff configuration is* $\{z_{sd}^{H0}, z_{sd}^{HB}\}$ $\forall s,d$.

Proof: Suppose that $\{z_{dd}^{B0}\}$ is an equilibrium: then it must be that $z_{dd}^{H0} > z_{dd}^{B0}$. Rearrange equations (19) and equations (20)to find that this implies

$$S_d^{\frac{1}{2}} > \left(M_d Q_d^H\right)^{\frac{1}{2}} \tag{22}$$

which cannot be satisfied if $Q_d^H > Q_d^L$. The proof for traded goods cutoffs is identical, but requires the use of the limit of fixed cost of exports. The only remaining potential equilibrium cutoff is

[7] For example, the configuration $\{z^{B0}, z^{HB}\}$ implies that firms initially sell to both groups, then switch to selling to just the H-group. This is inconsistent with 2.5.

$\{z_{dd}^{H0}, z_{dd}^{HB}\}$. □

The upper bound on the fixed cost of exports is needed because f_x causes the less productive firms that sell only to the H-types, to drop out of the export market. The upper bound on f_x ensures that some firms earn enough positive profit from export even if they can only sell to the H-type.

Proposition 2.9. *If $Q_d^H < Q_d^L$, the only potential domestic equilibrium cutoff configuration is $\{z_{dd}^{B0}\}$.*

Proof: Follows as prop (2.8). □

Notice that, unlike proposition 2.8, proposition 2.9 does not require a limit on the fixed cost of exports.

Theorem 2.10. *If $w_d^H > w_d^L$ and f_x satisfies assumption (2.7), then $\{z_{sd}^{B0}\}$ is not an equilibrium cutoff configuration.*

Proof: Suppose that $\{z_{sd}^{B0}\}$ is the equilibrium for some source-destination pair. By propositions 2.8 and 2.9, this can only be true if $Q_d^H < Q_d^L$, therefore $\{z_{sd}^{B0}\}$ is the equilibrium for all firms selling in d. This implies that all firms sell to both consumers, so that $N_d^H = N_d^L = N_d$ and $P_d^H = P_d^L = P_d$. We can therefore simplify $Q_d^H = \frac{w_d^H + qP_d}{N_d}$ and $Q_d^L = \frac{w_d^L + qP_d}{N_d}$. If $w_d^H > w_d^L$ however, then $Q_d^H > Q_d^L$, which leads to a contradiction with proposition (2.8). □

In this paper I assume that $w_d^H > w_d^L$ and assumptions (2.4) and (2.7) are satisfied, therefore the only possible equilibrium configuration is $\{z_{sd}^{H0}, z_{sd}^{HB}\}$, and therefore in equilibrium $Q_d^H \geq Q_d^L$.

Rearrange equations (19), and (21) and use $f_{dd} = 0$ and $\tau_{dd} = 1$ to equate the cutoffs for each source country with the equivalent cutoff for goods sourced from domestic firms,

$$z_{sd}^{H0^{-\frac{1}{2}}} = \left(\frac{w_d}{w_s \tau_{sd}}\right)^{\frac{1}{2}} z_{dd}^{H0^{-\frac{1}{2}}} - \left(\frac{f_{sd}}{qM_d^H \tau_{sd}}\right)^{\frac{1}{2}} \tag{23}$$

$$z_{sd}^{HB^{-\frac{1}{2}}} = \left(\frac{w_d}{\tau_{sd} w_s}\right)^{\frac{1}{2}} z_{dd}^{HB^{-\frac{1}{2}}} \tag{24}$$

In a homothetic preference Melitz model domestic and foreign cutoffs are be expressed in a manner similar to z_{sd}^{HB}: foreign cutoffs are simply a scaled up (or down) version of the domestic cutoff. This scaling rule is not true for z_{sd}^{H0}. Even though z_{sd}^{H0} and z_{dd}^{H0} comove[8], the degree of that comovement

[8] Obviously, comovements with changes related to w_d, w_s or τ_{sd} are more complicated

ranges from almost zero to $\frac{w_s \tau_{sd}}{w_d}$, depending on the profitability in the destination H-type market, as captured by the value of z_{dd}^{H0}, relative to the fixed export cost, f_x. Profitability in the H-type market matters because the firms that are most affected by f_x are low productivity ones: the ones that sell exclusively to the H-type consumers. Because the cutoffs movements in z_{sd}^{H0} and z_{sd}^{HB} are not perfectly correlated with each other it is possible for any change in cutoffs to increase (or decrease) consumption inequality.

2.4 Equilibrium of the World Economy

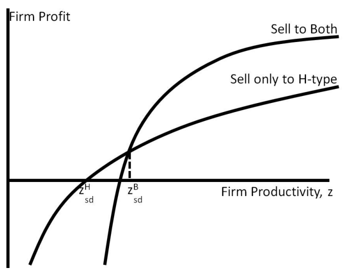

Figure 1: Cutoff

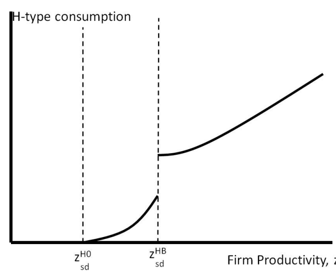

Figure 2: H-consumption

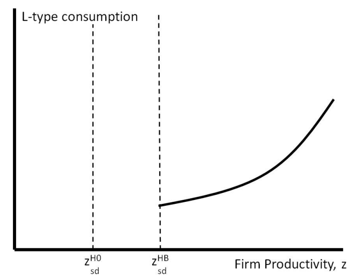

Figure 3: L-consumption

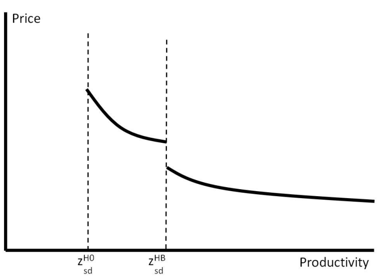

Figure 4: Price

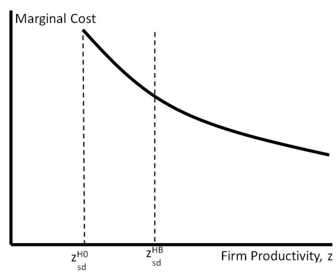

Figure 5: Marginal Cost

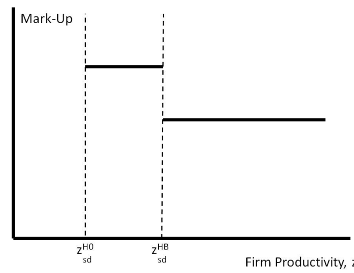

Figure 6: Mark Up

Figure (1) plots the profit earned from selling just to H-type, $\pi_{sd}^{H}(z)$, the profit from selling to both types, $\pi_{sd}^{B}(z)$, and the zero profit line as a function of productivity. For any given productivity z, if $\pi_{sd}^{H}(z) > \pi_{sd}^{B}(z)$ the firm sells to H-type consumers, if $\pi_{sd}^{B}(z) > \pi_{sd}^{H}(z)$ the firm sells to both types, and if both profit options are less than zero, the firm choose not to produce. As shown in theorem 2.6 and proposition 2.8 the only configuration of an economy's cutoffs is as shown: $\forall z \geq z_{sd}^{HB}$ firms sell to both consumer types, while for $z_{sd}^{H0} \leq z < z_{sd}^{HB}$ firms sell only to H-type. Figures (2) and (3) plots consumption for each group: H and L. All firms with productivity below the productivity threshold do not produce. Firms with $z_{sd}^{H0} \leq z < z_{sd}^{HB}$ are profitable enough to produce, but sell only to H-type consumers, therefore L-type consumers do not consume. For

$z \geq z_{sd}^{HB}$ both H and L consume goods. As proven in proposition 2.1 for any z, $c_{sd}^{H}(z) > c_{sd}^{L}(z)$. Figure (4) plots price against productivity. As proved in proposition 2.2, prices charged in the H-region, $z_{sd}^{H0} \leq z < z_{sd}^{HB}$, are greater than those charged when the firms are selling to both. The marginal cost component of price, figure (5), is not affected by the cutoff, and decreases steadily with increased productivity. Figure (6) plots mark-ups. Mark-ups are dependent on cutoffs and destination but not productivity: Firms selling to just the H-type charge a high markup, while firms selling to both groups the charge a lower mark-up.

Let the number of firms in s who have paid the fixed cost to draw a productivity be denoted by J_s. Of the firms that have drawn the productivity, the number serving H in destination d from source s, N_{sd}^{H}, is equal to the number of firms who have drawn productivities equal to or greater than z_{sd}^{H0}. With a Pareto distribution of productivities this is given by

$$N_{sd}^{H} = J_s b_s^{\gamma} z_{sd}^{H0^{-\gamma}} \tag{25}$$

while the number of firms producing for the L-type in d from country s is given by the firms who drew productivities greater than z_{sd}^{HB}

$$N_{sd}^{L} = J_s b_s^{\gamma} z_{sd}^{HB^{-\gamma}} \tag{26}$$

The total number of firms from which each type in d purchase goods is simply given by

$$N_d^k = \sum_s N_{sd}^k \tag{27}$$

The pricing aggregator for the L-type, P_d^L is given by

$$\begin{aligned} P_d^L &= \sum_s J_s \int_{z_{sd}^{HB}}^{\infty} p_{sd}^{B}(z) \gamma b_s^{\gamma} z^{-(\gamma+1)} dz \\ &= \frac{\gamma w_d^L}{q} \frac{\left[M_d^{\frac{1}{2}} + M_d^{H^{\frac{1}{2}}} \right]}{\left[\frac{1}{2} M_d^{\frac{1}{2}} - \gamma M_d^{H^{\frac{1}{2}}} \right]} - \frac{w_d \gamma N_d^L}{\left(z_{dd}^{H0} z_{dd}^{HB} \right)^{\frac{1}{2}}} \frac{M_d^{H^{\frac{1}{2}}}}{\left[\frac{1}{2} M_d^{\frac{1}{2}} - \gamma M_d^{H^{\frac{1}{2}}} \right]} \end{aligned} \tag{28}$$

while the pricing aggregator for the H-type is given by

$$
\begin{aligned}
P_d^H &= P_d^L + \sum_s J_s \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} p_{sd}^H(z)\gamma b_s^\gamma z^{-(\gamma+1)} dz \qquad (29) \\
&= \left[\frac{M_d^{H^{\frac{1}{2}}} + M_d^{\frac{1}{2}}}{M_d^{H^{\frac{1}{2}}}}\right] P_d^L + \frac{2\gamma w_d^H}{q} - \frac{2\gamma w_d^L}{q}\left[\frac{M_d^{H^{\frac{1}{2}}} + M_d^{\frac{1}{2}}}{M_d^{H^{\frac{1}{2}}}}\right] - \frac{2\gamma}{q}\left(\frac{w_d q}{M_d^H z_{dd}^{H0}}\right)^{\frac{1}{2}} \sum_s b_s^\gamma J_s z_{sd}^{H0^{-\gamma}} (w_s f_{sd})^{\frac{1}{2}}
\end{aligned}
$$

Wage income for consumer of type k is given by

$$
w_d^k = \theta_d^k w_s \tag{30}
$$

Firms will draw productivities until expected profit for firms in s equal the fixed cost of entry

$$
w_s f_c = \sum_d \int_{z_{sd}^H}^{z_{sd}^B} \pi_{sd}^H(z)\gamma b_s^\gamma z^{-\gamma+1} dz + \int_{z_{sd}^B}^{\infty} \pi_{sd}^B(z)\gamma b_s^\gamma z^{-\gamma+1} dz \tag{31}
$$

Equation (31) implies that expected profit is zero. Since the law of large numbers is satisfied in this economy expected profit is equal to average profit, and therefore aggregate profit, which is just average profit multiplied by number of firms, is zero. This means that there is no rebated profit and aggregate consumer income is simply the earned wages. Consumers spend all their income, and income spent on goods has to equal the total sales by $\forall s$.

$$
M_d^H w_d^H + M_d^L w_d^L = \sum_s R_{sd} \tag{32}
$$

Total sales revenue of all firms in s selling to d is given by

$$
R_{sd} = \gamma b_s^\gamma J_s \left[\int_{z_{sd}^{H0}}^{z_{sd}^{HB}} p_{sd}^H(z) M_d^H c_d^H(z) z^{-(\gamma+1)} dz + \int_{z_{sd}^{HB}}^{\infty} p_{sd}^B(z)\left(\frac{S_d}{p_{sd}^B(z)} - M_d q\right) z^{-(\gamma+1)} dz\right] \tag{33}
$$

Equations (32)-(33) can be combined to find a more compact expression for the number of firms

$$
J_s = \frac{M_s^H \theta_s^H + M_s^L \theta_s^L}{(\gamma+1) f_c + b_s^\gamma \sum_d f_{sd} z_{sd}^{H0^{-\gamma}}} \tag{34}
$$

With a homothetic Melitz model, the final summation term in the denominator would not exist: number of firms would be purely a function of source country characteristics. In this model, the cutoffs in *any* destination affects the number of firms: any movement in H-type cutoffs will change the number of firms in all source countries. The reason for this can be seen in equation (31): Firms expected profit over all destinations are considered against the cost of taking the draw. The H-type market is a source of profit, though the ability of low productivity firm to access it is dependent upon the fixed export cost relative to the size of profit than can be earned from serving that destination. A change in H-cutoff will change expected profits and induce entry or exit as the expected profits of firms change.

Definition 2.11. *For s,d=1,..I, given τ_{sd}, M_d, M_d^k, $F(z)$, f_c, f_{sd}, q, w_1, and b_s an equilibrium consists of productivity thresholds $\{\hat{z}_{sd}^{H0}, \hat{z}_{sd}^{HB}\}$; measure of entrants, $\hat{J}_s$; a measure of firms from country s serving k-type market d, $\hat{N}_{sd}^k$; the total measure of firms serving type k in market d, $\hat{N}_d^k$; a aggregate price statistic for each k-type, $\hat{P}_d^k$; wages for each type, $\hat{w}_d^k$; per consumer consumption, $\hat{c}_{sd}^k(z)$; total consumer-K allocation, $\hat{C}_{sd}^K$, firm pricing rules $\hat{p}_{sd}^k(z)$, production plans, $\hat{x}_{sd}^k$ and firm profits $\hat{\pi}_{sd}(z)$ such that: (i) Given P_d, w_d^k, p_{sd}, $\hat{c}_{sd}^K(z)$ satisfies (5) and solves the individual consumers problem, (ii) $\hat{C}_{sd}^K$ is given by $M_d^k c_{sd}^k(z)$ and satisfied aggregate demand for goods by type k, (iii) $\hat{p}_{sd}^H(z)$ is given by (15) and solves firm's problem is selling only to H, (iv) $\hat{p}_{sd}^B(z)$ is given by (17) and solves firm's problem if selling to both types, (v) $\hat{x}_{sd}^k(s)$ satisfies goods market condition: $\hat{x}_{sd}^k(s) = C_{sd}^K(z)$, (vi) $\hat{\pi}_{sd}^H(s)$ is given by (13), (vii) $\hat{\pi}_{sd}^B(s)$ is given by (11), (vii) $\hat{w}_d^k$ is given by (78), (viii) $\hat{z}_{sd}^{H0}; \hat{z}_{sd}^{HB}; \hat{N}_d^k; \hat{P}_d^H; \hat{P}_d^L; \hat{w}_d; \hat{J}_s$ jointly satisfy (19),(21),(27),(28),(29),(32),(34)*

3 Consumption Inequality and Trade

When defined by data, consumption inequality reflects the difference in the aggregate quantity of goods purchased by each consumer group. Within the model, the aggregate quantity of consumption goods each consumer purchases is simply the integral over the appropriate interval of $c_{sd}^H(z)$ and $c_{sd}^L(z)$, given by

$$C_d^L = \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^{\infty} \frac{Q_d^L(z)}{p_{sd}^B(z)} z^{-(\gamma+1)} dz - \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^{\infty} q z^{-(\gamma+1)} dz \quad (35)$$

$$C_d^H = \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} \frac{Q_d^H(z)}{p_{sd}^H(z)} z^{-(\gamma+1)} dz + J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^{\infty} \frac{Q_d^H(z)}{p_{sd}^B(z)} z^{-(\gamma+1)} dz - J_s b_s^\gamma \gamma \int_{z_{sd}^{H0}}^{\infty} q z^{-(\gamma+1)} dz$$

It is useful to note that there are two components to changes in welfare inequality: A change in the intensive margin - the change in quantity of a good consumed due to the change in prices - and an extensive margin - the change in variety or the number of firms selling to the income group.

Given the nonlinearity of the model, log-linearization is required to understand the mechanisms involved in changing consumption inequality. Let $\hat{x} = \log(x') - log(x)$, where x denotes the original value, and x' denotes new value. $\hat{x}$ therefore approximately denotes the percent change in variable. Log-linearizing the equations for variety we obtain

$$\hat{N}_{sd}^{H} = \hat{J}_s - \gamma \hat{z}_{sd}^{H0} \tag{36}$$

$$\hat{N}_{sd}^{L} = \hat{J}_s - \gamma \hat{z}_{sd}^{HB} \tag{37}$$

For prices we obtain

$$\hat{p}_{sd}^{H}(z) = \frac{1}{2} \underbrace{\left(\hat{\tau}_{sd} + \hat{w}_s\right)}_{\text{Marginal Cost}} + \frac{1}{2} \underbrace{\left(\hat{Q}_d^H - \hat{q}\right)}_{\text{Mark-Up}} \tag{38}$$

$$\hat{p}_{sd}^{B}(z) = \frac{1}{2} \underbrace{\left(\hat{\tau}_{sd} + \hat{w}_s\right)}_{\text{Marginal Cost}} + \frac{1}{2} \underbrace{\left(\frac{M_d^H\left(Q_d^H - Q_d^L\right)}{S_d}\hat{M}_d^H - \frac{S_d - M_d Q_d^L}{S_d}\hat{M}_d + \frac{M_d^L Q_d^L}{S_d}\hat{Q}_d^L + \frac{M_d^H Q_d^H}{S_d}\hat{Q}_d^H - \hat{q}\right)}_{\text{Mark-Up}}$$

Notice that if cutoffs and/or mark-ups faced by H and L types will have change differently, consumption inequality will change. For variety inequality to change the cutoffs faced by the two groups needs to change by different amount.

$$\hat{N}_{sd}^{H} - \hat{N}_{sd}^{L} = \gamma\left(\hat{z}_{sd}^{H0} - \hat{z}_{sd}^{HB}\right) \tag{39}$$

If equation (39) is positive, then the H-types gain more varieties than the L-types and variety inequality increases.

$$\hat{p}_{sd}^{B}(z) - \hat{p}_{sd}^{H}(z) = \frac{1}{2}\left[\frac{M_d^H\left(Q_d^H - Q_d^L\right)}{S_d}\left(\hat{M}_d^H - \hat{M}_d\right) + \frac{S_d - M_d^H Q_d^H}{S_d}\left(\hat{Q}_d^L - \hat{Q}_d^H\right)\right] \tag{40}$$

If equation (40) is positive, then the mark-up on goods sold to both types change increases (decreases) by more than (less than) the mark-up on goods sold to the just the high types.

In a model with homothetic preferences both consumers consume the same variety of goods (so cutoffs experience the same change) and mark-ups are constant, and therefore it doesn't generate a change in consumption inequality.

4 Numerical Example: Consumption Inequality

Parameter		Value
N	# of countries	2
γ	Pareto Shape	4.5
M_d	Population	1
M_d^H	20% of population	0.2
b_s	Minimum Productivity	1
θ_d^H	Effective labor of H-type	2
θ_d^L	Effective labor of L-type	1.5
f_c	Fixed entry cost	1e-3
f_x	Fixed cost of export	1e-4
q	Non homothetic parameter	2

Table 1: Parameters used in Numerical Exercise

In this section I examine the effect that a change in trade cost has on the relative variety, quantity consumed and utilities of consumer groups, H and L, in a 2 country trade example. I then compare the model's predictions regarding each group's gain with different kinds trade partners. In all exercises the destination country has the parameters indicated in table 4, which imply an upper 20% income share of 25%.

4.1 Changing Trade Barrier

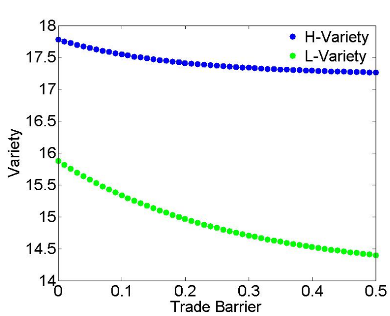

Figure 7: Variety

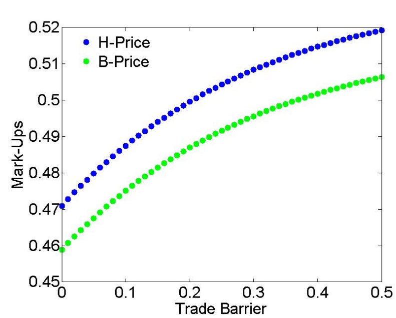

Figure 8: Mark-Ups

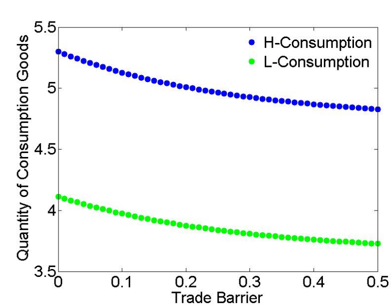

Figure 9: Consumption

In the following exercise I change trade barrier ($\tau_{sd} - 1$) from zero (free trade) to 0.5 (a 50% cost for each good imported) for two identical countries to examine the effect of trade cost on consumption inequality.

Figure 7 shows the varieties purchased by each type of consumer with respect to trade cost. For both consumers the variety, N_d^k, decreases with higher trade barriers. Figure 8 shows the mark-up component of prices indicated in equations (15) and (17). As the variety decreases, the mark-ups charged by the remaining firms increase for all firms, resulting in an increase in prices of the remaining goods - in addition to the increase that results from the increase of marginal cost. Finally, figure 9 the quantity of consumption goods, C_d^k purchased by consumers at different trade costs. For both consumers C_d^k increases with lower trade barriers. Figure 9 shows the quantity of consumption goods, C_d^k purchased by consumers at different trade costs. For both consumers C_d^k the decrease in variety and increase in prices combine to decrease aggregate consumption.

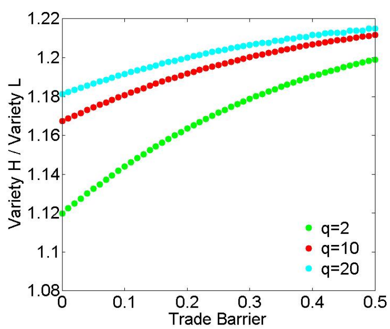

Figure 10: $\frac{N_d^H}{N_d^L}$

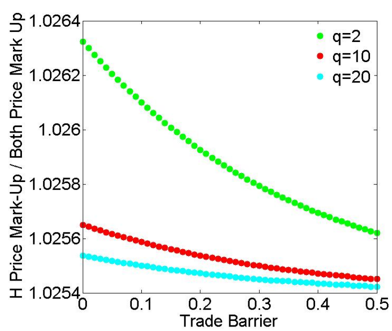

Figure 11: $Q_d^H \frac{M_d}{S_d}$

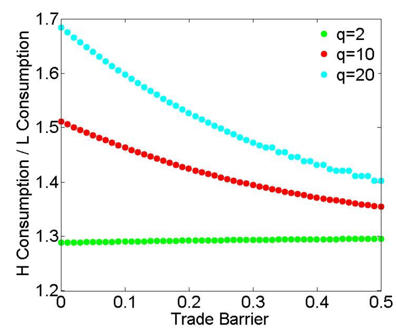

Figure 12: $\frac{C_d^H}{C_d^L}$

The focus of this paper however, is on relative number of goods purchased. In a model with homothetic preferences the ratio of varieties would be constant and equal to 1, $\frac{N_d^H}{N_d^L} = 1$, as all consumers will buy all the goods available to them on the market. With non-homothetic preferences it is no longer true that this ratio is equal to one, yet it is still possible that the ratio is constant. Figure 10 shows that this is not the case. As the trade cost increases, the ratio of goods the H consumer relative to that that both consume increases. A decrease in trade cost therefore decreases variety inequality. Figure (11) shows that the markup component on the price of goods sold to both consumers increases by slightly more than the price of goods sold to just the rich - but the increase is only slight. Figure 12 shows that a a decrease in trade cost decreases quantity of consumption goods purchased by the H consumer relative to that purchased by the L-consumer increases.

Figures (10)-(12) also show the change for different values of the non-homothetic parameter. As the non-homothetic parameters increases, the effects of changing the trading decreases.

4.2 Changing Source Country Income Inequality

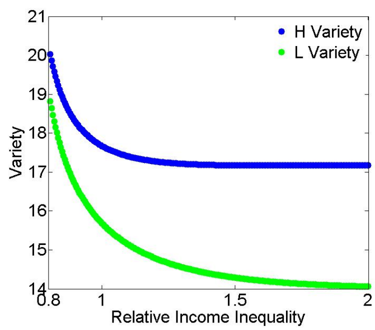

Figure 13: Variety

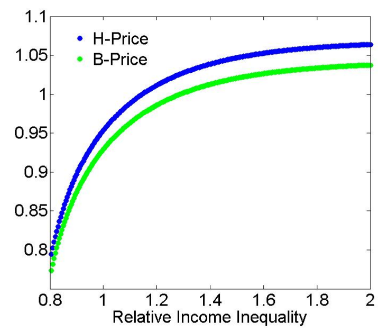

Figure 14: Mark-Ups

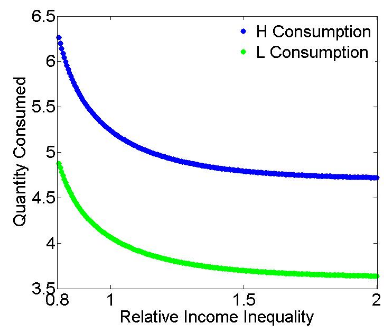

Figure 15: Consumption

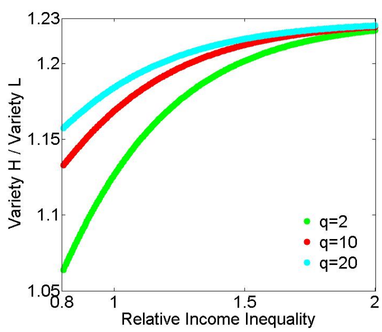

Figure 16: $\frac{N_d^H}{N_d^L}$

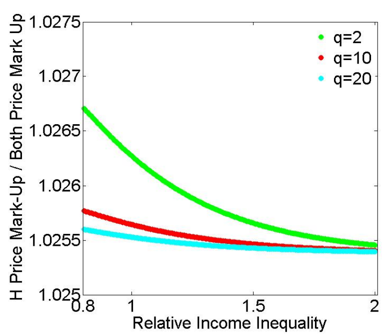

Figure 17: $Q_d^H \frac{M_d}{S_d}$

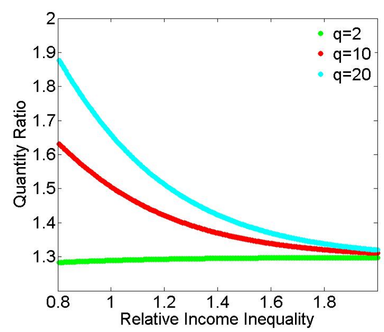

Figure 18: $\frac{C_d^H}{C_d^L}$

In this exercise I study the effect of a change in income inequality of a source country. To accomplish this I keep all destination parameters fixed, while changing the productivity each labor type in the source country to keep aggregate labor supply constant and equal to the destination country, $M_d^H\theta_d^H + M_d^L\theta_d^L = M_s^H\theta_s^H + M_s^L\theta_s^L$. Income inequality, measured as the income share of the upper 20th percentile, increases from 20% (equality) to 50%. In all figures, the vertical line shows the income inequality in the destination country (fixed and equal to 25%).

As inequality in the source country increases the variety enjoyed by both groups in the destination country decreases. This decrease is less for the H-type and variety inequality increases. Quantity consumed decreases with increases inequality, while consumption inequality decreases. In summary, as the inequality in the source country increases, the consumption inequality decreases.

5 Conclusion

In this paper I have studied the effect of trade on consumption inequality, using the channel of market and price discrimination. I show that even in the absence of income changes, trade liberalization changes the optimal pricing decision of firms which may cause changes in consumption inequality. I find that for identical countries trade liberalization decreases consumption inequality. Furthermore, I find that trade with a country with different income inequality decreases consumption inequality.

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A Detailed Derivations

A.1 Model

To find P_d^L begin with

$$P_d^L = \sum_s J_s \int_{z_{sd}^{HB}}^{\infty} p_{sd}^B(z)\gamma b_s^\gamma z^{-(\gamma+1)}dz \tag{41}$$

This evaluates to

$$P_d^L = \frac{\gamma}{\gamma+\frac{1}{2}}\left[\frac{S_d w_d}{qM_d}\right]^{\frac{1}{2}} z_{dd}^{HB^{-\frac{1}{2}}} \sum_s J_s b_s^\gamma z_{sd}^{HB^{-\gamma}} \tag{42}$$

Use the definition of N_d^L and z_{dd}^{HB} to obtain

$$P_d^L = \frac{\gamma}{\gamma+\frac{1}{2}}\frac{N_d^L}{q}\left[\frac{S_d}{M_d}\right]^{\frac{1}{2}}\left[\frac{S_d^{\frac{1}{2}}-\left(M_d^H Q_d^H\right)^{\frac{1}{2}}}{M_d^{\frac{1}{2}}-M_d^{H^{\frac{1}{2}}}}\right] \tag{43}$$

Next, P_d^H is given by

$$P_d^H = P_d^L + \sum_s J_s \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} p_{sd}^H(z)\gamma b_s^\gamma z^{-(\gamma+1)}dz \tag{44}$$

which evaluates to

$$\begin{aligned} P_d^H &= P_d^L + \frac{\gamma}{\gamma+\frac{1}{2}}\left[\frac{Q_d^H}{q}\right]^{\frac{1}{2}}\sum_s J_s\left[\frac{\tau_{sd}w_s}{z_{sd}^{H0}}\right]^{\frac{1}{2}} z_{sd}^{H0^{-\gamma}} - \frac{\gamma}{\gamma+\frac{1}{2}}\left[\frac{Q_d^H}{q}\right]^{\frac{1}{2}}\sum_s J_s\left[\frac{\tau_{sd}w_s}{z_{sd}^{HB}}\right]^{\frac{1}{2}} z_{sd}^{HB^{-\gamma}} \\ &= P_d^L + \frac{\gamma}{\gamma+\frac{1}{2}}\frac{N_d^H Q_d^H}{q} - \frac{\gamma}{\gamma+\frac{1}{2}}\frac{N_d^L}{q}Q_d^{H^{\frac{1}{2}}}\left[\frac{S_d^{\frac{1}{2}}-\left(M_d^H Q_d^H\right)^{\frac{1}{2}}}{M_d^{\frac{1}{2}}-M_d^{H^{\frac{1}{2}}}}\right] - \frac{\gamma}{\gamma+\frac{1}{2}}\frac{1}{q}\left[\frac{Q_d^H}{M_d^H}\right]^{\frac{1}{2}}\sum_s b_s^\gamma J_s z_{sd}^{H0^{-\gamma}}\left[w_s f_{sd}\right]^{\frac{1}{2}} \end{aligned} \tag{45}$$

Next, substitute $S_d = M_d^H Q_d^H + M_d^L Q_d^L$, and use $Q_d^L = \frac{w_d^L + qP_d^L}{N_d^L}$ and $M_d^L = \left(M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}\right)\left(M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}}\right)$ into the expression for P_d^L to obtain

$$P_d^L = \frac{\gamma w_d^L}{q} \frac{\left[M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}}\right]}{\left[\frac{1}{2}M_d^{\frac{1}{2}} - \gamma M_d^{H\frac{1}{2}}\right]} - \frac{\gamma N_d^L}{q} \frac{\left(M_d^H Q_d^H\right)^{\frac{1}{2}} \left[S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right]}{\left[\frac{1}{2}M_d^{\frac{1}{2}} - \gamma M_d^{H\frac{1}{2}}\right]\left[M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}\right]} \tag{46}$$

Use the definition of Q_d^H and P_d^L to simplify P_d^H and obtain

$$P_d^H = \left[\frac{M_d^{H\frac{1}{2}} + M_d^{\frac{1}{2}}}{M_d^{H\frac{1}{2}}}\right] P_d^L + \frac{2\gamma w_d^H}{q} - \frac{2\gamma w_d^L}{q}\left[\frac{M_d^{H\frac{1}{2}} + M_d^{\frac{1}{2}}}{M_d^{H\frac{1}{2}}}\right] - \frac{2\gamma}{q} \frac{Q_d^{H\frac{1}{2}}}{M_d^{H\frac{1}{2}}} \sum_s b_s^{\gamma} J_s z_{sd}^{H0-\gamma} \left(w_s f_{sd}\right)^{\frac{1}{2}} \tag{47}$$

Finally, substitute the definition of the cutoffs into these expressions to obtain equations (28) and (29).

Firms will continue to draw productivities until the expected profit equals the cost of taking the productivity draw

$$w_s f_c = \sum_d \int_{z_{sd}^H}^{z_{sd}^B} \pi_{sd}^H(z)dz + \sum_d \int_{z_{sd}^B}^{\infty} \pi_{sd}^B(z)dz$$

Expected profit, $\int \pi_{sd}^k(z)dz$ has two components, expected revenue, R_{sd}^k and expected cost, TC_{sd}^k.

$$\begin{aligned} w_s f_c &= \sum_d E(\pi_{sd}) && (48) \\ &= \sum_d E(R_{sd}) - E(T_{sd}) && (49) \end{aligned}$$

Total cost, TC is $J_s \int_{z^*} b^\gamma w_s \ell_s z^{-(\gamma+1)} dz$, where the cost for firm z is $w_s \ell_s(z) = \frac{w_s \tau_{sd}}{z} M_d^H c_d^H(z) + \frac{\tau_{sd} w_s}{z} M_d^L c_d^L(z) + w_s f_{sd}$.

$$\begin{aligned} TC_{sd} &= \gamma J_s b_s^\gamma w_s \tau_{sd} \int_{z^{H0}}^{z^{HB}} M_d^H c_d^H(z) J_s z^{-(\gamma+2)} dz + \int_{z^{H0}}^{z^{HB}} \left(M_d^H c_d^H + M_d^L c_d^L(z) \right) z^{-(\gamma+2)} dz \quad (50) \\ &+ \gamma J_s b_s^\gamma \int_{z_{sd}^H}^{\infty} w_s f_{sd} z^{-(\gamma+1)} dz \\ &= \frac{\gamma N_{sd}^H M_d^H Q_d^H}{2(\gamma+1)\left(\gamma+\frac{1}{2}\right)} + \frac{\gamma^2 J_s b_s^\gamma z_{sd}^{H0-\gamma}}{(\gamma+1)\left(\gamma+\frac{1}{2}\right)} \left[M_d^H Q_d^H w_s f_{sd} \right]^{\frac{1}{2}} + \frac{J_s b_s^\gamma z_{sd}^{H0-\gamma} (w_s f_{sd})}{\gamma+1} \quad (51) \\ &+ \frac{\gamma N_{sd}^L \left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}} \right)}{2(\gamma+1)\left(\gamma+\frac{1}{2}\right)} \left[S_d^{\frac{1}{2}} + \left(M_d^H Q_d^H\right)^{\frac{1}{2}} + \frac{2\gamma M_d^{H\frac{1}{2}}}{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}} \left(\left(M_d Q_d^H\right)^{\frac{1}{2}} - S_d^{\frac{1}{2}} \right) \right] \end{aligned}$$

Trade value ,R, is given by $\sum_k J_s b_s^\gamma \int_z M_d^k p_{sd}^k(z) c_{sd}^k(z) z^{-(\gamma+1)} dz$

$$\begin{aligned} R_{sd} &= \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} \gamma J_s b_s^\gamma p_{sd}^H(z) M_d^H c_d^H(z) z^{-(\gamma+1)} dz + \int_{z_{sd}^{HB}}^{\infty} \gamma b_s^\gamma J_s p_{sd}^B(z) \left(M_d^H c_d^H(z) + M_d^L c_d^L(z) \right) z^{-(\gamma+1)} dz \\ &= \frac{N_{sd}^H M_d^H Q_d^H}{2\left(\gamma+\frac{1}{2}\right)} + \frac{\gamma J_s b_s^\gamma z_{sd}^{H0-\gamma}}{\left(\gamma+\frac{1}{2}\right)} \left[M_d^H Q_d^H w_s f_{sd} \right]^{\frac{1}{2}} \quad (52) \\ &+ \frac{N_{sd}^L \left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}} \right)}{2\left(\gamma+\frac{1}{2}\right)} \left[S_d^{\frac{1}{2}} + \left(M_d^H Q_d^H\right)^{\frac{1}{2}} + \frac{2\gamma M_d^{H\frac{1}{2}}}{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}} \left(\left(M_d Q_d^H\right)^{\frac{1}{2}} - S_d^{\frac{1}{2}} \right) \right] \end{aligned}$$

Equations (51) and (52) can be rearranged to show that

$$TC_{sd} = \frac{\gamma}{\gamma+1} R_{sd} + \frac{w_s f_{sd}}{\gamma+1} b_s^\gamma J_s z_{sd}^{H0-\gamma} \quad (53)$$

Under the assumption of balanced trade, sales to the rest of the world equal purchases from the rest of the world, $\sum_{d \neq s} R_{sd} = \sum_{s \neq d} \sum_k p_{sd}^k C_{sd}^k$. Aggregate income is $\sum w_s^k \ell_s^k$ and is equal to total spending, $\sum_s \sum_k p_{sd}^k C_{sd}^k$.

$$\begin{aligned} \sum_{d \neq s} R_{sd} &= \sum_k w_s^k \ell_s^k - p_{ss}^k C_{ss}^k \quad (54) \\ \sum_d R_{sd} &= \sum_k w_s^k \ell_s^k = Y_s \quad (55) \end{aligned}$$

Because of the large number of firms, expected sales equal average sales and total sales equal the average sales multiplied by number of firms, J_s.

$$\sum_d R_{sd} = J_s \sum_d E(R_{sd}) \tag{56}$$

$$\Rightarrow \sum_d E(R_{sd}) = \frac{\sum_d R_{sd}}{J_s} \tag{57}$$

$$\sum_d E(R_{sd}) = \frac{\sum_k w_s^k \ell_s^k}{J_s} \tag{58}$$

We can use this expression to simplify free entry as

$$w_s f_c = \sum_d \frac{\sum_k w_s \ell_s^k}{J_s} - \frac{\gamma}{\gamma+1} \frac{\sum_k w_s \ell_s^k}{J_s} - \frac{b_s^\gamma w_s f_{sd} z_{sd}^{H0-\gamma}}{\gamma+1} \tag{59}$$

$$= \frac{Y_s}{J_s(\gamma+1)} - \sum_d \frac{b_s^\gamma w_s f_{sd} z_{sd}^{H0-\gamma}}{\gamma+1} \tag{60}$$

$$\Rightarrow J_s = \frac{Y_s}{(\gamma+1) w_s f_c + w_s b_s^\gamma \sum_d f_{sd} z_{sd}^{H0-\gamma}} \tag{61}$$

A.2 Consumption Inequality and Trade

Within the model, the aggregate quantity of consumption goods each consumer purchases is given by

$$C_d^L = \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^\infty \frac{Q_d^L(z)}{p_{sd}^B(z)} z^{-(\gamma+1)} dz - \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^\infty q z^{-(\gamma+1)} dz \tag{62}$$

$$C_d^H = \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} \frac{Q_d^H(z)}{p_{sd}^H(z)} z^{-(\gamma+1)} dz + J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^\infty \frac{Q_d^H(z)}{p_{sd}^B(z)} z^{-(\gamma+1)} dz - J_s b_s^\gamma \gamma \int_{z_{sd}^{H0}}^\infty q z^{-(\gamma+1)} dz$$

Rearranging z_{dd}^{HB} to give $Q_d^L = \frac{w_d q}{z_{dd}^{HB}} \frac{M_d^{H\frac{1}{2}}}{M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}}} \left[\frac{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}}{M_d^{H\frac{1}{2}}} + 2 \left(\frac{z_{dd}^{HB}}{z_{dd}^{H0}} \right)^{\frac{1}{2}} \right]$ and remembering $\sum_s J_s b_s^\gamma z_{sd}^{HB^{-\gamma}} = N_d^L$ and $\sum_s J_s b_s^\gamma z_{sd}^{H0^{-\gamma}} = N_d^H$, this evaluates to

$$
\begin{aligned}
C_d^L &= \frac{\gamma q N_d^L}{\gamma - \frac{1}{2}} \frac{M_d^{\frac{1}{2}}}{M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}}} \frac{\left(M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right) z_{dd}^{H0^{\frac{1}{2}}} + 2 \left(M_d^H z_{dd}^{HB} \right)^{\frac{1}{2}}}{\left(M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right) z_{dd}^{H0^{\frac{1}{2}}} + \left(M_d^H z_{dd}^{HB} \right)^{\frac{1}{2}}} - q N_d^L \qquad (63) \\
C_d^H &= \frac{\gamma q N_d^L}{\gamma - \frac{1}{2}} \left(\frac{z_{dd}^{HB}}{z_{dd}^{H0}} \right)^{\frac{1}{2}} \left[\frac{\left(M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right) \left(z_{dd}^{HB^{\frac{1}{2}}} - z_{dd}^{H0^{\frac{1}{2}}} \right)}{\left(M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right) z_{dd}^{H0^{\frac{1}{2}}} + \left(M_d^H z_{dd}^{HB} \right)^{\frac{1}{2}}} \right] + \frac{\gamma q}{\gamma - \frac{1}{2}} \sum_s \frac{J_s b_s^\gamma z_{sd}^{H0^{-\gamma}}}{1 - \left(\frac{w_s f_{sd} z_{dd}^{H0}}{w_d q M_d^H} \right)^{\frac{1}{2}}} - q N_d^H
\end{aligned}
$$

I can also use consumer utility function

$$
\begin{aligned}
U_d^L &= \sum_s \gamma b_s^\gamma J_s \int_{z_{sd}^{HB}}^{\infty} z^{-(\gamma+1)} \log \left(\frac{Q_d^L}{p_{sd}^B(z)} \right) dz \qquad (64) \\
U_d^H &= \sum_s \gamma b_s^\gamma J_s \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} z^{-(\gamma+1)} \log \left(\frac{Q_d^H}{p_{sd}^H(z)} \right) dz + \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^{\infty} z^{-(\gamma+1)} \log \left(\frac{Q_d^H}{p_{sd}^B} \right) dz
\end{aligned}
$$

which evaluates to

$$
\begin{aligned}
U_d^L &= N_d^L \left[\frac{1}{2\gamma} + \log(q) \right] - N_d^L \log \left[\frac{\left(M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}} \right) z_{dd}^{HB^{\frac{1}{2}}}}{2 M_d^{H\frac{1}{2}} \left(z_{dd}^{HB^{\frac{1}{2}}} - z_{dd}^{H0^{\frac{1}{2}}} \right) + \left(M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}} \right) z_{dd}^{H0^{\frac{1}{2}}}} \right] \qquad (65) \\
&\quad + N_d^L \log \left[\frac{\left(M_d z_{dd}^{HB} \right)^{\frac{1}{2}}}{\left(M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right) z_{dd}^{H0^{\frac{1}{2}}} + \left(M_d^H z_{dd}^{HB} \right)^{\frac{1}{2}}} \right] \\
U_d^H &= N_d^H \left[\frac{1}{2\gamma} + \log(q) \right] - \sum_s b_s^\gamma J_s z_{sd}^{H0^{-\gamma}} \log \left[1 - \left(\frac{z_{dd}^{H0} w_s f_{sd}}{q M_d^H w_d} \right)^{\frac{1}{2}} \right] \qquad (66) \\
&\quad + N_d^L \log \left[\frac{\left(M_d z_{dd}^{HB} \right)^{\frac{1}{2}}}{\left(M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right) z_{dd}^{H0^{\frac{1}{2}}} + \left(M_d^H z_{dd}^{HB} \right)^{\frac{1}{2}}} \right]
\end{aligned}
$$

As $N_d^H > N_d^L$ and $z_{dd}^{H0} < z_{dd}^{HB}$ it is easy to show that $U_d^H > U_d^L$.

The log-linearized version of all variables is

$$
\begin{aligned}
\hat{N}_{sd}^{H} &= \hat{J}_s - \gamma \hat{z}_{sd}^{H0} \\
\hat{N}_{sd}^{L} &= \hat{J}_s - \gamma \hat{z}_{sd}^{HB} \\
\hat{Q}_{d}^{k} &= \frac{w_d^k}{w_d^k + qP_d^k}\hat{w}_d^k + \frac{qP_d^k}{w_d^k + qP_d^k}\hat{P}_d^k - N_d^k \\
\hat{w}_{d}^{k} &= \hat{\theta_d^k} + \hat{w}_d^k \\
\hat{J}_s &= \frac{M_s^H\theta_s^H}{M_s^H\theta_s^H + M_s^L\theta_s^L}\left(\hat{M}_s^H + \hat{\theta}_s^H\right) + \frac{M_s^L\theta_s^L}{M_s^H\theta_s^H + M_s^L\theta_s^L}\left(\hat{M}_s^L + \hat{\theta}_s^L\right) - \frac{(\gamma+1)\, f_c}{(\gamma+1)\, f_c + \sum_d f_{sd} z_{sd}^{H0^{-\gamma}}}\hat{f}_c \\
&\quad - \frac{b_s^\gamma \sum_d f_{sd} z_{sd}^{H0^{-\gamma}}}{(\gamma+1)\, f_c + b_s^\gamma \sum_d f_{sd} z_{sd}^{H0^{-\gamma}}}\hat{f}_{sd} + \frac{\gamma b_s^\gamma}{(\gamma+1)\, f_c + b_s^\gamma \sum_d f_{sd} z_{sd}^{H0^{-\gamma}}}\sum_d f_{sd} z_{sd}^{H0^{-\gamma}}\hat{z}_{sd}^{H0} \\
\hat{z}_{sd}^{H0} &= \hat{\tau}_{sd} + \hat{q} + \frac{(w_s f_{sd})^{\frac{1}{2}}}{\left(M_d^H Q_d^H\right)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}}\left[\hat{f}_{sd} - \hat{M}_d^H\right] + \frac{\left(M_d^H Q_d^H\right)^{\frac{1}{2}}}{\left(M_d^H Q_d^H\right)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}}\left[\hat{w}_s - \hat{Q}_d^H\right] \\
\hat{z}_{sd}^{HB} &= \hat{\tau}_{sd} + \hat{q} + \hat{w}_s + \frac{\left(M_d^H Q_d^H\right)^{\frac{1}{2}}}{S_d^{\frac{1}{2}}}\hat{Q}_d^H - \frac{S_d^{\frac{1}{2}} + \left(M_d^H Q_d^H\right)^{\frac{1}{2}}}{S_d^{\frac{1}{2}}}\hat{Q}_d^L - \frac{\left(M_d M_d^H\right)^{\frac{1}{2}}}{M_d - M_d^H}\frac{\left(M_d Q_d^H\right)^{\frac{1}{2}} - S_d^{\frac{1}{2}}}{S_d^{\frac{1}{2}}}\hat{M}_d \\
&\quad + \left[\frac{M_d^H}{M_d - M_d^H}\left(\frac{\left(\frac{M_d}{M_d^H}\right)^{\frac{1}{2}} S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}}{S_d^{\frac{1}{2}}}\right) - \frac{\left(M_d^H Q_d^H\right)^{\frac{1}{2}}}{S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}}\right]\hat{M}_d^H
\end{aligned}
$$

$$
\begin{aligned}
\hat{P}_d^L &= \hat{N}_d^L - \hat{q} - \frac{S_d M_d^{H^{\frac{1}{2}}}\left[M_d^{\frac{1}{2}} - M_d^{H^{\frac{1}{2}}}\right] + \left(M_d^H Q_d^H\right)^{\frac{1}{2}} M_d \left[\left(M_d^H Q_d^H\right)^{\frac{1}{2}} + S_d^{\frac{1}{2}}\right]}{2\left(M_d - M_d^H\right)}\hat{M}_d \\
&\quad -\hat{Q}_d^H \frac{\left(M_d^H Q_d^H\right)^{\frac{1}{2}}\left[S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right]}{2S_d} + \hat{Q}_d^L \frac{2S_d + \left(S_d M_d^H Q_d^H\right)^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)}{2S_d} \\
&\quad -\hat{M}_d^H \frac{M_d^{H^{\frac{1}{2}}}\left[S_d^{\frac{1}{2}} - \left(M_d Q_d^H\right)^{\frac{1}{2}}\right]}{2\left(M_d - M_d^H\right)\left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right)}\left[\left(S_d + 2\left(M_d^H Q_d^H S_d\right)^{\frac{1}{2}}\right)\left(M_d^{\frac{1}{2}} - M_d^{H^{\frac{1}{2}}}\right) - M_d^{\frac{1}{2}} M_d^H Q_d^H\right] \\
\hat{P}_d^H &= \frac{\left(M_d^{H^{\frac{1}{2}}} + M_d^{\frac{1}{2}}\right) P_d^L - M_d^{\frac{1}{2}} P_d^H}{M_d^{H^{\frac{1}{2}}} P_d^H}\hat{q} + \frac{M_d^{H^{\frac{1}{2}}} + M_d^{\frac{1}{2}}}{M_d^{H^{\frac{1}{2}}}}\frac{P_d^L}{P_d^H}\hat{P}_d^L + \left[\frac{q}{2}\left(P_d^L - P_d^H\right) + \gamma\left(w_d^H - w_d^L\right)\right]\frac{\hat{M}_d^H}{qP_d^H} \\
&\quad + \frac{M_d^{\frac{1}{2}}}{qM_d^{H^{\frac{1}{2}}} P_d^H}\left[\frac{q}{2}P_d^L - \gamma w_d^L\right]\hat{M}_d + \frac{2\gamma w_d^H}{qP_d^H}\hat{w}_d^H - \frac{2\gamma w_d^L}{qP_d^H M_d^{H^{\frac{1}{2}}}}\left(M_d^{H^{\frac{1}{2}}} - M_d^{\frac{1}{2}}\right)\hat{w}_d^L \\
&\quad - \frac{\gamma Q_d^{H^{\frac{1}{2}}}\hat{Q}_d^H}{qM_d^{H^{\frac{1}{2}}} P_d^H}\sum_s b_s^\gamma J_s z_{sd}^{H0^{-\gamma}}\left(w_s f_{sd}\right)^{\frac{1}{2}} - \frac{2\gamma Q_d^{H^{\frac{1}{2}}}}{qM_d^H P_d^H}\sum_s b_s^\gamma J_s z_{sd}^{H0^{-\gamma}}\left(w_s f_{sd}\right)^{\frac{1}{2}}\left[\hat{J}_s - \gamma \hat{z}_{sd}^{H0} + \frac{1}{2}\hat{w}_s + \frac{1}{2}\hat{f}_{sd}\right]
\end{aligned}
$$

A.3 Calibration

Begin by observing that $w_d M_d^H \theta_d^H = IncShare_d Y_d$ and $w_d M_d^L \theta_d^L = (1 - IncShare_d)Y_d$. This in turn implies that $M_d^L \theta_d^L = \frac{1-Incshare_d}{IncShare_d} M_d^H \theta_d^H$. Next, notice that θ_d^H can be expressed as

$$
\theta_d^H = \frac{IncShare_d Y_d}{M_d^H w_d}
$$

.

The value imported from t into d is simply the total revenue received by firms from t selling to d. Define $A_d = S_d^{\frac{1}{2}} + \left(M_d^H Q_d^H\right)^{\frac{1}{2}} + \frac{2\gamma M_d^{H^{\frac{1}{2}}}}{M_d^{\frac{1}{2}} - M_d^{H^{\frac{1}{2}}}}\left(\left(M_d Q_d^H\right)^{\frac{1}{2}} - S_d^{\frac{1}{2}}\right)$, then the import share of country s in d is given by

$$
\begin{aligned}
\lambda_{td} &= \frac{R_{td}}{\sum_s R_{sd}} \qquad (67) \\
&= \frac{b_t^\gamma J_t z_{td}^{H0^{-\gamma}}\left[M_d^H Q_d^H + A_d\left(\frac{z_{td}^{HB}}{z_{td}^{H0}}\right)^{-\gamma}\left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right) + 2\gamma\left[M_d^H Q_d^H w_t f_{td}\right]^{\frac{1}{2}}\right]}{\sum_s J_s b_s^\gamma z_{sd}^{H0^{-\gamma}}\left[M_d^H Q_d^H + A_d\left(\frac{z_{sd}^{HB}}{z_{sd}^{H0}}\right)^{-\gamma}\left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right) + 2\gamma\left[M_d^H Q_d^H w_s f_{sd}\right]^{\frac{1}{2}}\right]} \qquad (68)
\end{aligned}
$$

where $\frac{z_{sd}^{HB}}{z_{sd}^{H0}} = \left[\frac{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}}{M_d^{H\frac{1}{2}}}\right]^2 \left[\frac{\left(M_d^H Q_d^H\right)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}}{S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}}\right]^2$. Since $\sum_s R_{sd} = Y_d$ total value of imports can be written as $R_{sd} = \lambda_{sd} Y_d$. The value of all imports into d equals the value of all exports from d, $\sum_s R_{sd} = \sum_s R_{ds}$

$$Y_t = \sum_d R_{td} = \sum_d Y_d \lambda_{td} \tag{69}$$

$$= \sum_d Y_d \frac{b_t^\gamma J_t z_{td}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{td}^{HB}}{z_{td}^{H0}}\right)^{-\gamma} \left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right) + 2\gamma \left[M_d^H Q_d^H w_t f_{td}\right]^{\frac{1}{2}}\right]}{\sum_s J_s b_s^\gamma z_{sd}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{sd}^{HB}}{z_{sd}^{H0}}\right)^{-\gamma} \left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right) + 2\gamma \left[M_d^H Q_d^H w_s f_{sd}\right]^{\frac{1}{2}}\right]} \tag{70}$$

The number of firms trading with a country is given by N_{sd}^H. The average sales per firm is given by

$$\frac{R_{sd}}{N_{sd}^H} = \frac{M_d^H Q_d^H + 2\gamma \left[M_d^H Q_d^H w_t f_{td}\right]^{\frac{1}{2}} + A_d \left(\frac{z_{td}^{HB}}{z_{td}^{H0}}\right)^{-\gamma} \left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right)}{2\left(\gamma + \frac{1}{2}\right)} \tag{71}$$

Finally the number of firms from s that serve destination s relative to destination j is given by

$$\frac{N_{sd}^H}{N_{sj}^H} = \left(\frac{z_{sd}^{H0}}{z_{sj}^{H0}}\right)^{-\gamma} \tag{72}$$

$$= \left(\left[\frac{M_d^H}{M_j^H}\right] \left[\frac{\left(M_j^H Q_j^H\right)^{\frac{1}{2}} - (w_s f_{sj})^{\frac{1}{2}}}{\left(M_d^H Q_d^H\right)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}}\right]^2\right)^{-\gamma} \tag{73}$$

A.3.1 Calibration if $f_{sd} = 0$

The value imported from s into d is simply the total revenue received by firms in s selling to d. Define $A_d = S_d^{\frac{1}{2}} + \left(M_d^H Q_d^H\right)^{\frac{1}{2}} + \frac{2\gamma M_d^{H\frac{1}{2}}}{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}} \left(\left(M_d Q_d^H\right)^{\frac{1}{2}} - S_d^{\frac{1}{2}}\right)$, then the import share of country t in d is given by

$$\lambda_{td} = \frac{R_{td}}{\sum_s R_{sd}} \tag{74}$$

$$= \frac{b_t^\gamma J_t z_{td}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{td}^{HB}}{z_{td}^{H0}}\right)^{-\gamma} \left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right)\right]}{\sum_s b_s^\gamma J_s z_{sd}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{sd}^{HB}}{z_{sd}^{H0}}\right)^{-\gamma} \left(S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}\right)\right]} \tag{75}$$

Using $\frac{z_{sd}^{HB}}{z_{sd}^{H0}} = \left[\frac{\left(M_d Q_d^H\right)^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)^{\frac{1}{2}}}{S_d^{\frac{1}{2}} - \left(M_d^H Q_d^H\right)}\right]^2$ and $z_{sd}^{H0} = \frac{q}{Q_d^H} \tau_{sd} w_s$ and $J_s = \frac{Y_s}{(\gamma+1) w_s f_c}$

$$\lambda_{td} = \frac{b_t^\gamma J_t z_{td}^{H0-\gamma}}{\sum_s b_s^\gamma J_s z_{sd}^{H0-\gamma}} \tag{76}$$

$$= \frac{\frac{Y_t}{w_t}\left(\frac{b_t}{\tau_{td} w_t}\right)^\gamma}{\sum_s \frac{Y_s}{w_s}\left(\frac{b_s}{\tau_{sd} w_s}\right)^\gamma} \tag{77}$$

Next, since $\sum_s R_{sd} = Y_d$ total value of imports can be written as $R_{sd} = \lambda_{sd} Y_d$. Finally, the value of all imports into d equals the value of all exports from d, $\sum_s R_{sd} = \sum_s R_{ds}$, so

$$Y_d = \sum_t R_{dt} = \sum_t Y_t \lambda_{dt}$$

$$= \sum_t Y_t \frac{\frac{Y_d}{w_d}\left(\frac{b_d}{\tau_{dt} w_d}\right)^\gamma}{\sum_s \frac{Y_s}{w_s}\left(\frac{b_s}{\tau_{st} w_s}\right)^\gamma}$$

$$\frac{w_d^{\gamma+1}}{b_d^\gamma} = \sum_t \frac{Y_t}{\sum_s \frac{Y_s}{w_s}\left(\frac{b_s \tau_{dt}}{w_s \tau_{st}}\right)^\gamma} \tag{78}$$

Equation (78) implicitly solves wages in d as a function of wages in all other countries. Setting a wage to be the numeraire solves for all wages.