

## ON THE DENOMINATION OF GOVERNMENT DEBT A Critique of the Portfolio Balance Approach\*

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We show that some classes of sterilized interventions have no effect on equilibrium prices and quantities. The proof does not require complete markets, Ricardian equivalence, monetary neutrality, or the law of one price. Moreover, regressions of exchange rates or interest differentials on variables measuring debt's currency composition contain no information about the effectiveness of such interventions. Other interventions require changes in monetary and fiscal policy; their effects depend, generally, on the influence of these changes on the economy and not on the intervention alone. In short, sterilized intervention is not, as the portfolio balance approach indicates, an extra policy instrument.

### 1. Introduction

We examine the effectiveness of sterilized intervention in foreign exchange markets in a theoretical monetary economy and use the results to shed light on the portfolio balance approach to international macroeconomics. That approach was one of the most actively pursued lines of research during the 1970s' revival of exchange rate theory and includes papers by Branson, Halttunen, and Masson (1977), Branson and Henderson (1985), Dornbusch (1983), Frankel (1982, 1985), Frenkel and Mussa (1985), Kouri (1976), Kouri and deMacedo (1978), Krugman (1981), and Obstfeld (1983). It continues to influence academics and policymakers today, most importantly by its theoretical support of the argument that sterilized intervention is a third instrument, independent of monetary and fiscal policy, that governments can use to influence exchange rates and other economic variables.

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We think a closer look at the theory's microfoundations exposes some flaws. We start by considering what we term *strong-form interventions*: changes in the currency composition of government debt, holding constant the time paths of monetary and fiscal policy. In sections 2 and 3 we show, for an economy with complete markets and cash-in-advance constraints on goods, that there is an equivalence class of strong-form interventions that are irrelevant; the reasoning is similar to Peled's (1985) for inflation-indexed debt. For any intervention in this class, changes in the composition of the debt do not affect equilibrium prices or quantities. In section 4 we extend this proposition to a similar economy with incomplete markets. We argue in section 5 that the irrelevance of this class of interventions depends only on budget constraints and a weak arbitrage condition on bond prices and should therefore hold for a wide range of economic environments.

Within the equivalence class, sterilized interventions – unlike most examples of the portfolio balance approach – are irrelevant. Outside this class they typically require changes in monetary or fiscal policy or both to satisfy government budget constraints. A change in the composition of the debt therefore requires changes in some combination of the time paths of taxes, government spending, or money supplies. The effect of such an intervention depends on both the changes in policy that accompany it and the structure of the economy. Until we describe exactly what policies change and how they influence economic decisions, we see no way of determining their impact. None of these considerations arise in portfolio balance models, which are models of agents' portfolio decisions alone; Branson and Henderson (1985) and Weber (1986) provide surveys. While these models have been successful in describing how agents' asset demands respond to uncertainty about exchange rates, they fail as a theory of intervention. They ignore the feedback of government and private sector portfolio decisions on budget constraints and hence on other policies and decisions. We argue in section 6 that this feedback is important and that ignoring it is a serious omission.

In section 7 we use the irrelevance results to reconsider the portfolio balance approach to exchange rate determination and to examine the evidence used within the approach to estimate the effectiveness of sterilized intervention. Typically in this approach, exchange rates or interest differentials are regressed on some variable measuring the currency composition of outstanding debt, the ratio of, say, Canadian dollar to U.S. dollar debt. The coefficient on this variable is said to measure the magnitude of the influence of intervention. In our economy, however, it does no such thing. Within the equivalence class, the value of the debt variable can be manipulated without affecting exchange rates or interest rates. It is possible, generally, to produce either a positive or a negative coefficient, even though the interventions are completely irrelevant to the equilibrium.

We conclude by speculating about channels through which intervention might affect exchange rates and other variables in practice. We observe that, in practice, the denomination of the debt may be relevant for strategic reasons, because the choice of debt instruments affects governments' strategy spaces or provides information about future policy. The United States, for example, cannot as easily inflate away its yen obligations as it can its dollar debt, so a strategy space that includes only dollar debt may yield quite different outcomes from one that includes both dollar and yen debt. This mechanism is quite different, however, from the standard portfolio balance channel.

## 2. A monetary economy

We consider a dynamic, stochastic monetary economy that combines elements of Aschauer and Greenwood (1983), Backus and Kehoe (1987), and Helpman (1981). It differs from our earlier paper in having an endogenous labor supply decision, which gives us a relatively simple way of introducing distortionary taxation and nonneutral monetary policy. Each period, the economy has, in addition to labor, a single good that is consumed by both governments and private agents, whom we refer to as consumers.

As in our earlier paper, the environment and notation extend Lucas (1984) to multiagent economies. Each period  $t$ , for  $t = 0, 1, 2, \dots$ , the economy experiences a random event  $s_t$ , which is observed by all agents. The  $t$ -period history, denoted  $s^t \equiv (s_1, s_2, \dots, s_t)$  and referred to as the state, is an element of the finite set  $S^t$ . The probability of any particular history, conditional on  $s_0$ , is denoted  $h(s^t)$ .

The world economy consists of  $I$  countries, each of which is represented by a government and a consumer. The consumer of country  $i$  chooses consumption  $c$  and labor supply  $n$  for each state. Let  $c_t^{ij}(s^t)$  be consumer  $i$ 's purchases of the consumption good from country  $j$  in state  $s^t$  and  $c_t^i(s^t) \equiv \sum_j c_t^{ij}(s^t)$  be total consumption by consumer  $i$ . Labor, denoted  $n_t^i(s^t)$ , is supplied only in the home country. Each consumer is endowed with one unit of labor in each state. The government of country  $i$  consumes  $g_t^i(s^t) \equiv \sum_j g_t^{ij}(s^t)$  and levies a proportional tax  $\tau_t^i(s^t)$  on home output  $y_t^i(s^t)$ . It also collects an implicit inflation tax on holdings of its money.

In addition to money, the economy has a complete set of two-period contingent claims, or bonds, denominated in the currency of each country. The number of currency  $j$  denominated bonds held by consumer  $i$  is  $X_t^{ij}(s^t)$ ; each such bond is purchased in period  $t-1$  and pays one unit of currency  $j$  in period  $t$  if state  $s^t$  occurs and nothing otherwise. Its price at date  $t-1$ , in units of currency  $j$ , is  $V_t^{ij}(s^t)$ . Similarly,  $B_t^{ij}(s^t)$  is the number of such bonds issued by government  $i$ . The value of one unit of currency  $j$  is implicit in the

currency price  $p_t^j(s^t)$  of the good in state  $s^t$ . Exchange rates are denoted  $e_t^j(s^t)$ , the price in units of currency 1 of one unit of currency  $j$ .

Consumer  $i$ 's preferences are characterized by the expected utility function,

$$U_i = \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} h(s^t) u^i [c_t^i(s^t), 1 - n_t^i(s^t)] \quad \text{for } 0 < \beta < 1, \quad (2.1)$$

where each  $u^i$  is concave and increasing. Production occurs separately in each country using technologies

$$y_t^i(s^t) = f^i [n_t^i(s^t), s^t], \quad (2.2)$$

where each  $f^i$  is increasing and concave in its first argument. Domestic residents are endowed with the rights to any profits.

The utility function is maximized subject to sequences of budget constraints and cash-in-advance constraints on purchases of goods. Their form is motivated by the following story. In each period  $t$ , consumers trade money, bonds, and goods. They trade currencies and bonds in the securities market at the start of the period, after observing the current event  $s_t$ . Each household then divides into a shopper and a worker. The shoppers travel to all the countries and buy goods from their workers with local currency. The workers stay in their own country and trade the output from their labor to shoppers for local currency. Markets then close, and the shoppers return home with goods and unspent cash. The households enter the next period holding this cash, after-tax wages paid to the workers, and claims accumulated from maturing bonds.

This leads to the following constraints for the consumer of country  $i$  in state  $s^t$ . In the goods market of each country  $j$ , the consumer purchases  $c_t^{ij}(s^t)$  units of the good with  $MD_t^{ij}(s^t)$  units of local currency. The purchases must satisfy the cash-in-advance constraints

$$p_t^j(s^t) c_t^{ij}(s^t) \leq MD_t^{ij}(s^t) \quad \text{for } j = 1, 2, \dots, I. \quad (2.3)$$

As stated, consumer  $i$  enters the period, at dates  $t = 1, 2, \dots$ , with claims

$$\begin{aligned} A_t^i(s^t) \equiv & \sum_j e_t^j(s^t) X_t^{ij}(s^t) \\ & + e_t^i(s^t) p_{t-1}^i(s^{t-1}) [1 - \tau_{t-1}^i(s^{t-1})] y_{t-1}^i(s^{t-1}) \\ & + \sum_j e_t^j(s^t) [MD_{t-1}^{ij}(s^{t-1}) - p_{t-1}^j(s^{t-1}) c_{t-1}^{ij}(s^{t-1})], \quad (2.4a) \end{aligned}$$

measured in units of currency 1. We assume that at date  $t = 0$  the last two

terms are zero. In the securities market, consumer  $i$  acquires  $MD_i^{jj}(s^t)$  units of currency  $j$  and  $X_{t+1}^{ij}(s^t, s_{t+1})$  bonds subject to the budget constraint

$$\sum_j e_i^j(s^t) MD_i^{jj}(s^t) + \sum_{j, s_{t+1}} e_i^j(s^t) V_{t+1}^j(s^{t+1}) X_{t+1}^{ij}(s^{t+1}) = A_i^j(s^t). \tag{2.4b}$$

These two relations together will be referred to as the consumer's budget constraint, eq. (2.4).

Governments face similar constraints. The cash-in-advance constraints are, for each country  $i$ ,

$$p_i^j(s^t) g_i^{jj}(s^t) = MG_i^{jj}(s^t) \quad \text{for } j = 1, 2, \dots, I, \tag{2.5}$$

where  $MG_i^{jj}(s^t)$  is the amount of currency  $j$  held by government  $i$  for transactions purposes in state  $s^t$ . The budget constraints are

$$\begin{aligned} & \sum_{j, s_{t+1}} e_i^j(s^t) V_{t+1}^j(s^{t+1}) B_{t+1}^{ij}(s^{t+1}) \\ &= \sum_j e_i^j(s^t) B_t^{ij}(s^t) + \sum_j e_i^j(s^t) MG_i^{jj}(s^t) \\ & \quad - e_i^j(s^t) [M_t^i(s^t) - M_{t-1}^i(s^{t-1})] \\ & \quad - e_i^j(s^t) p_{t-1}^i(s^{t-1}) \tau_{t-1}^{ij}(s^{t-1}) y_{t-1}^j(s^{t-1}), \end{aligned} \tag{2.6}$$

where the last term is zero at date 0. Both (2.5) and (2.6) hold for all countries  $i$  and states  $s^t$ .

The constraints are completed by boundary conditions. We assume, first, that initial assets and liabilities balance across agents:  $\sum_i X_0^{ij} = \sum_i B_0^{ij}$ , for all  $i$  and  $j$ . The terminal conditions are more subtle. Consider any infinite history  $s^\infty$ , an element of the countable set  $S^\infty$ . Then, for any sequence of subhistories  $s^t$  leading to  $s^\infty$ ,

$$\lim_{t \rightarrow \infty} \inf e_i^j(s^t) Q_t^j(s^t) X_t^{ij}(s^t) = 0 \tag{2.7}$$

and

$$\lim_{t \rightarrow \infty} \inf e_i^j(s^t) Q_t^j(s^t) B_t^{ij}(s^t) = 0, \tag{2.8}$$

for all  $i$  and  $j$ , where  $Q_t^j(s^t) = \prod_{\tau=1}^t V_\tau^j(s^\tau)$  is the date 0, or present-value,

price of one unit of currency  $j$  in state  $s^t$ . The idea is that the date 0 value of any asset holdings must go to zero along any path – roughly, the value of assets and liabilities must grow more slowly, in the limit, than the rate of interest.

We define an equilibrium for this economy after describing some standard notation. For any variable  $z_t(s^t)$ , let  $z$  denote the set of elements  $z_t(s^t)$ , one for each  $t$  and  $s^t$ . For a superscripted variable like  $y_t^i(s^t)$  or  $c_t^{ij}(s^t)$ , let  $(z^t)$  and  $(z^{ij})$  denote the sets  $\{z^1, z^2, \dots, z^I\}$  and  $\{z^{11}, z^{12}, \dots, z^{II}\}$ , respectively. Finally, let  $(\pi^i) = [(g^{ij}), (\tau^i), (M^i), (MG^{ij}), (B^{ij})]$  denote the complete set of government policies and  $\pi^i$  the policies of government  $i$ .

*Definition.* An equilibrium in this economy is a set of allocations  $[(c^{ij}), (n^i)]$ , money and asset positions  $[(MD^{ij}), (X^{ij})]$ , prices  $[(p^i), (V^i), (e^i)]$ , and government policies  $(\pi^i)$  satisfying these conditions:

- *Market clearing.* For each date  $t$ , history  $s^t$ , and country  $j$ , markets for goods, assets, and money clear:

$$\sum_i [c_t^{ij}(s^t) + g_t^{ij}(s^t)] = y_t^j(s^t) = f^j[n_t^j(s^t), s^t], \quad (2.9)$$

$$\sum_i X_t^{ij}(s^t) = \sum_i B_t^{ij}(s^t), \quad (2.10)$$

$$\sum_i [MD_t^{ij}(s^t) + MG_t^{ij}(s^t)] = M_t^j(s^t). \quad (2.11)$$

- *Consumer maximization.* For each consumer  $i$ , the quantities  $c^{ij}$ ,  $n^i$ ,  $MD^{ij}$ , and  $X^{ij}$ , for  $j = 1, 2, \dots, I$ , maximize utility (2.1) subject to the production function (2.2), the cash-in-advance constraints (2.3), the budget constraints (2.4), the initial conditions  $X_0^{ij}$ , and the terminal conditions (2.7).

- *Government budget balance.* For each government  $i$ , policies obey the cash-in-advance constraint (2.5), the budget constraint (2.6), the initial condition  $B_0^{ij}$ , and the terminal condition (2.8).

### 3. An irrelevance result

We prove that in our economy there is a class of sterilized interventions that have no effect on the equilibrium. We start by distinguishing between two types of intervention. One, which we label *strong-form intervention*, is a change in the currency composition of the debt, with no change in any other policy variables. To be precise, consider an initial policy  $(\pi^i) = [(g^{ij}), (\tau^i), (M^i), (MG^{ij}), (B^{ij})]$ . We say a policy is a strong-form intervention with respect to  $(\pi^i)$  if it agrees with  $(\pi^i)$  in every respect but  $(B^{ij})$ . The other

type of intervention, which we call *weak form*, allows changes in other policies as well. Rogoff (1984), for example, considers changes in  $(B^{ij})$ , holding constant the time paths of money supplies but allowing the possibility of changes to fiscal policy.

Our analysis in this section concerns strong-form interventions. We start with an arbitrage condition:

*Lemma 1.* *In any equilibrium, bond prices and exchange rates obey the relation*

$$V_{t+1}^j(s^{t+1}) = V_{t+1}^1(s^{t+1}) e_{t+1}^j(s^{t+1}) / e_t^j(s^t), \quad (3.1)$$

for all states  $s^t$  and currencies  $j$ .

*Proof.* Any prices not satisfying (3.1) allow unlimited arbitrage profits. They are therefore inconsistent with consumer maximization; hence, with equilibrium. ■

We can now state and prove the irrelevance proposition. Since output and inflation taxes are distortionary in this economy, the proposition clearly does not depend on Ricardian equivalence or monetary neutrality. Also, since all elements of government policy but bond supplies are held constant, the interventions are of the strong form.

*Proposition 1.* *Consider an equilibrium consisting of consumer decisions  $[(c^{ij}), (n^i), (MD^{ij}), (X^{ij})]$ , prices  $[(p^i), (V^i), (e^i)]$ , and government policies  $[(g^{ij}), (\tau^i), (M^i), (MD^{ij}), (B^{ij})]$ . Then any change in bond policies to  $(\hat{B}^{ij})$  satisfying, for each state,*

$$\sum_j e_t^j(s^t) \hat{B}_t^{ij}(s^t) = \sum_j e_t^j(s^t) B_t^{ij}(s^t), \quad (3.2)$$

is an equilibrium for some choice of  $(\hat{X}^{ij})$  satisfying

$$\sum_j e_t^j(s^t) \hat{X}_t^{ij}(s^t) = \sum_j e_t^j(s^t) X_t^{ij}(s^t) \quad (3.3)$$

at the original values for all other variables.

*Proof.* We simply verify that the new bond and asset positions satisfy the conditions for an equilibrium at the original prices. Since only asset quantities have changed in the conjectured equilibrium, we need only check conditions involving these quantities. Take the government budget constraints first: The second term in (2.6) is obviously unchanged by the new bond supplies. From

Lemma 1 we can rewrite the first term as

$$\sum_{s_{t-1}} V_{t+1}^1(s^{t+1}) \sum_j e_{t+1}^j(s^{t+1}) B_{t+1}^{ij}(s^{t+1}),$$

which is unchanged for the same reason. The new policy therefore satisfies government  $j$ 's budget constraints at the original prices.

We have some flexibility in choosing  $(\hat{X}^{ij})$ . One possibility is

$$\hat{X}_t^{ij}(s^t) = X_t^{ij}(s^t) + \hat{B}_t^{ij}(s^t) - B_t^{ij}(s^t),$$

which clearly satisfies condition (3.3) of the proposition and, by the same reasoning as above, consumers' budget constraints. Summing over  $i$  we verify that the new bond supplies and asset positions satisfy the market-clearing condition for bonds and that the new values constitute an equilibrium at the original prices. ■

The conclusion is that we have an equivalence class, defined by (3.2), within which sterilized interventions are irrelevant. In this sense they have no effect on the equilibrium. The logic is that only the state-contingent payoffs matter to anyone's decision, and this is not affected by these interventions. Note, too, that the proposition sidesteps the issue of existence, saying only that, if an equilibrium exists, then Proposition 1 applies to it. Lucas and Stokey (1987) provide an existence proof for a related economy, but as far as we know existence has not been demonstrated for our environment. The same issue arises more forcefully in the incomplete markets version of the next section, for which existence of equilibrium cannot generally be guaranteed even in nonmonetary economies.

#### 4. Irrelevance with incomplete markets

In the last section we specified a complete set of nominal, state-contingent claims markets. Here we restrict ourselves to a smaller set of markets. Using the same logic, we extend the proposition to this environment. The restrictions on markets, however, limit the equivalence class of bond policies that support any equilibrium. We provide necessary and sufficient conditions for this class to be nonempty for any given equilibrium.

Assume, then, that consumers and governments can buy and sell only a limited set of two-period assets with exogenously specified, state-contingent yields. For each currency  $j$ , let there be  $K_j$  types of bonds, each with a different vector of state-contingent payoffs. Each bond of type  $k$ , for  $k = 1, 2, \dots, K_j$ , is bought in state  $s^{t-1}$  and yields  $D_{kt}^j(s^{t-1}, s_t)$  units of currency  $j$  at date  $t$  for each event  $s_t$ . Let  $B_{kt}^{ij}(s^{t-1})$  denote the number of this type of

bond issued by government  $i$  and  $X_{kt}^{ij}(s^{t-1})$  the number held by consumer  $i$ . The price, in units of currency  $j$ , is  $W_{kt}^j(s^{t-1})$ . The total number of bonds is  $K = \sum_j K_j$ .

The definition of equilibrium here is similar to that in section 2. The market-clearing conditions for goods and money are unchanged. In place of the market-clearing condition for assets (2.10), though, we have, for bonds of each currency  $j$  and type  $k$ ,

$$\sum_i X_{kt}^{ij}(s^{t-1}) = \sum_i B_{kt}^{ij}(s^{t-1}), \tag{4.1}$$

for every state. The budget constraints change as follows. For consumer  $i$  the second term in (2.4b) is replaced by

$$\sum_{j,k} e_t^j(s^t) W_{kt}^j(s^t) X_{k,t+1}^{ij}(s^t),$$

and the first term on the right side of (2.4a) by

$$\sum_{j,k} e_t^j(s^t) D_{kt}^j(s^t) X_{kt}^{ij}(s^{t-1}).$$

These terms represent purchases and payoffs, respectively, of assets. Similarly, for government  $i$  in constraint (2.6) the left side is replaced by

$$\sum_{j,k} e_t^j(s^t) W_{kt}^j(s^t) B_{k,t+1}^{ij}(s^t),$$

and the first term on the right by

$$\sum_{j,k} e_t^j(s^t) D_{kt}^j(s^t) B_{kt}^{ij}(s^{t-1}).$$

With these changes, the definition of equilibrium in section 2 applies with incomplete markets as well.

The analysis begins with the arbitrage condition. For any state  $s^t$  let  $Z_{t+1}(s^t)$  denote a portfolio of bonds. Thus  $Z_{t+1}(s^t)$  is a  $K$ -dimensional vector with typical element  $Z_{k,t+1}^j(s^t)$ . Then:

*Lemma 2. In any equilibrium, bond prices satisfy the following condition. Any portfolios  $Z_{t+1}(s^t)$  and  $\hat{Z}_{t+1}(s^t)$  that satisfy, for all  $s_{t+1}$ ,*

$$\begin{aligned} & \sum_j e_{t+1}^j(s^{t+1}) \sum_k D_{k,t+1}^j(s^{t+1}) Z_{k,t+1}^j(s^t) \\ &= \sum_j e_{t+1}^j(s^{t+1}) \sum_k D_{k,t+1}^j(s^{t+1}) \hat{Z}_{k,t+1}^j(s^t), \end{aligned} \tag{4.2}$$

also satisfy

$$\begin{aligned} & \sum_j e_t^j(s^t) \sum_k W_{k,t+1}^j(s^t) Z_{k,t+1}^j(s^t) \\ &= \sum_j e_t^j(s^t) \sum_k W_{k,t+1}^j(s^t) \hat{Z}_{k,t+1}^j(s^t). \end{aligned} \quad (4.3)$$

The proof mirrors that of Lemma 1. Then we have:

*Proposition 2. Consider an equilibrium consisting of consumer decisions  $[(c^{ij}), (n^i), (MD^{ij}), (X_k^{ij})]$ , prices  $[(p^i), (W_k^i), (e^i)]$ , and government policies  $[(g^{ij}), (\tau^i), (B_k^{ij}), (M^i), (MD^{ij})]$ . Then any change in bond policies to  $(\hat{B}_k^{ij})$  such that, for each state,*

$$\sum_{j,k} e_t^j(s^t) D_{kt}^j(s^t) \hat{B}_{kt}^{ij}(s^{t-1}) = \sum_{j,k} e_t^j(s^t) D_{kt}^j(s^t) B_{kt}^{ij}(s^{t-1}) \quad (4.4)$$

is an equilibrium for some choice of  $(\hat{X}^{ij})$  satisfying

$$\sum_{j,k} e_t^j(s^t) D_{kt}^j(s^t) \hat{X}_{kt}^{ij}(s^{t-1}) = \sum_{j,k} e_t^j(s^t) D_{kt}^j(s^t) X_{kt}^{ij}(s^{t-1}) \quad (4.5)$$

at the original values for all other variables.

The proof is similar to that of Proposition 1: we use the arbitrage conditions to verify that the conjectured equilibrium satisfies the equilibrium condition (4.1) and the budget constraints of governments and consumers. The conditions defining the equivalence class, however, are quite different. In the earlier proposition it was obvious that the class was nonempty: for any date and state there are always multiple bond policies that yield the same vector of payoffs and allow nontrivial interventions satisfying (3.2). Here that need not be true. For a particular country  $i$ , consider bond policies  $B^i$  and  $\hat{B}^i$ , and let  $\Delta B^i$  be the difference between the two:

$$\Delta B_{kt}^{ij}(s^{t-1}) \equiv \hat{B}_{kt}^{ij}(s^{t-1}) - B_{kt}^{ij}(s^{t-1}).$$

Think of  $\Delta B^i$  as the set of interventions required to pursue policy  $\hat{B}^i$  starting from policy  $B^i$ . As an illustration, consider a one-time bilateral intervention by country  $i$  between currencies 1 and  $j$  in state  $s^{t-1}$ , in which it exchanges a linear combination of the  $K_1$  currency 1 bonds for another combination of

currency  $j$  bonds. Then the policy satisfies condition (4.4) only if

$$\sum_{k=1}^{K_1} e_t^1(s^t) D_{kt}^1(s^t) \hat{B}_{kt}^1(s^{t-1}) = \sum_{k=1}^{K_j} e_t^j(s^t) D_{kt}^j(s^t) B_{kt}^{jj}(s^{t-1}), \quad (4.6)$$

for all feasible events  $s_t$ .

Relation (4.6) is a spanning condition, defining subspaces of redundant assets. To see this, let  $A_t^j(s^{t-1})$  denote the matrix of state-contingent payoffs of the  $K_j$  currency  $j$  bonds: its  $k$ th column is the payoff vector  $e_t^j(s^{t-1}, s_t) D_{kt}^j(s^{t-1}, s_t)$ , with one element for each  $s_t$ . Thus  $A_t^j(s^{t-1})$  has dimensions  $\dim(s_t) \times K_j$ . Then there exist nontrivial interventions between currencies 1 and  $j$  satisfying (4.6) if and only if

$$\text{span}[A_t^1(s^{t-1})] \cap \text{span}[A_t^j(s^{t-1})] \neq \{0\}, \quad (4.7)$$

where the span of a matrix is the space generated by linear combinations of its columns. In words, there exists a linear combination of the  $K_1$  currency 1 bonds that reproduces the payoffs of a nontrivial linear combination of currency  $j$  bonds. As long as this condition is satisfied, there exist nontrivial interventions between currencies 1 and  $j$  that are irrelevant in the sense of the proposition.

The idea is easily extended to more general interventions in currency 1. With similar reasoning we see that nontrivial multilateral interventions satisfying (4.4) are possible between currency 1 and other currencies if and only if

$$\text{span}[A_t^1(s^{t-1})] \cap \text{span}[A_t^2(s^{t-1}), \dots, A_t^j(s^{t-1})] \neq \{0\}. \quad (4.8)$$

As long as this condition is satisfied, the equivalence class is nonempty.

### 5. Extensions

We have proved irrelevance propositions for two theoretical economies, allowing both incomplete markets and nonneutral monetary and fiscal policies, but analogous results hold for a wide range of economic environments. The proofs make it clear that they depend on only two features of the model: an arbitrage condition on bond prices and budget constraints. Most of the specifics of the model, therefore, have no bearing on the propositions.

Take the assumptions that each country has a single representative agent and a single traded good. It should be clear that we need only change the notation to extend the irrelevance proposition to economies with nontraded goods, multiple traded goods, capital, and multiple heterogeneous agents within countries. None of these would change the logic of either the arbitrage

condition or budget constraints. Note in particular that the propositions will hold even when purchasing power parity fails.

A more subtle distinction concerns the lifetimes of private agents. A popular alternative to the representative agent paradigm is the overlapping generations framework, used in international macroeconomics by Buiter (1981) and Persson (1984), and the closely related model of Frenkel and Razin (1986). In each of these economies an arbitrage condition applies and government policies obey a sequence of budget constraints. Since our formulation is based on sequence constraints, the extension of the irrelevance proposition to economies with overlapping generations is immediate. The use of sequence constraints distinguishes our results from the Ricardian equivalence theorem, which depends on being able to reduce the sequence of budget constraints to a single present-value constraint. Ours do not require this and therefore hold in models with incomplete markets and overlapping generations.

With regard to money, we opted for the cash-in-advance approach, but similar results hold for other models. In models of money based on overlapping generations or money-in-the-utility function, the way in which money interacts with the real economy may be quite different. Still, arbitrage conditions and budget constraints hold. The irrelevance proposition should hold here, too.

## **6. Weak-form interventions**

So far we have considered only sterilizations of the strong form, which rule out changes in monetary and fiscal policy when the currency composition of the debt changes. We have shown that, if such interventions satisfy certain conditions, they are irrelevant in the sense that different bond policies support the same equilibrium prices and allocations. We now consider interventions that do not satisfy conditions (3.2) or (4.4). Typically the bond policies induced by this kind of intervention will violate governments' budget constraints unless other policy variables, in addition to bond supplies, are changed at the same time. We are faced, in other words, with weak-form interventions.

The question is, how does the equilibrium differ for economies with different bond supplies? The first step is to specify precisely the policy experiment involved, including any changes to monetary and fiscal variables. The answer, then, must surely depend on what these changes are and how they impinge on the economy.

Even in our simple model, the government has a large number of feasible policy responses. In every state it might change spending, adjust the tax rate, print money, or execute some combination of the three to satisfy its budget constraint. Each of these instruments typically has a different effect on the equilibrium. A decline in government spending, for example, changes the net supply of goods available for private consumption, while an increase in the tax

rate distorts consumers' decisions to save, consume, and supply labor. Similarly, a decline in spending affects price levels and exchange rates differently than an increase in the money supply. Unless we specify as part of the intervention exactly what other policy changes go with it, we cannot determine the effect on equilibrium prices and quantities.

Once we specify the experiment, its impact will depend on the structure of the economy. Suppose, for example, that we always adjust the tax on labor to maintain budget balance and consider two different economic structures: one in which labor supply is inelastic and one in which it's elastic. In the former, interventions should have little effect on the equilibrium, since the tax does not greatly distort consumers' decisions, including the supply of labor. But in the latter structure, interventions should have a large effect. Generally speaking, the magnitude of the effect depends on how these other aspects of policy influence the economy. Until we take a stand on the economic structure, we see no way of deciding how interventions affect the economy. As an example of such a stand, consider interventions accompanied by changes in lump-sum taxes in representative agent economies. Obstfeld (1981, 1982), Sargent and Smith (1986), and Stockman (1979, 1983) prove that weak-form interventions of this sort are irrelevant, since the timing of taxes does not affect any economic decisions; that is, even some classes of weak-form interventions are irrelevant when the Ricardian equivalence theorem applies.

We summarize briefly: Interventions of the strong form, satisfying conditions (3.2) or (4.4), are irrelevant in the sense of Propositions 1 and 2. Other sterilized interventions typically require changes in other government policies to satisfy government budget constraints. Their effects depend on what these other policy changes are and on the structure of the economy. When other policy changes accompany changes in bond supplies, which change affects the equilibrium becomes a semantic issue: the impact can as easily be attributed to the changes in monetary and fiscal policy as to the intervention per se. In short, in our class of models at least, sterilized intervention cannot be considered a separate policy instrument.

## **7. The portfolio balance approach**

The analysis leaves us concerned about using portfolio balance models to study sterilized interventions. These models ignore the inevitable consequences of government budget constraints. They claim to determine the effects of intervention without specifying either the complete package of policies intended by the term intervention or the structure of the real economy. Our discussion of weak-form intervention can be seen as a thinly veiled critique of this practice.

We are also skeptical of the interpretation placed on econometric work that uses the portfolio balance approach as its maintained hypothesis. In most of

this work, the exchange rate or uncovered interest differential is regressed on a variable summarizing the currency composition of government debt, like the ratio of foreign- to domestic-denominated debt. Branson, Halttunen, and Masson (1977), Frankel (1982), and Rogoff (1984) are good examples of this work. If the coefficient on the debt variable is nonzero, the approach is judged to be a success and intervention effective. In our model, however, both regressions can yield a wide range of outcomes, even for interventions known to be irrelevant. The reason is that the equivalence class defined by Proposition 1, to take a specific example, gives us enough freedom in choosing bond supplies that for a given equilibrium price path we can choose a path for the debt variable that covaries in an arbitrary way with the exchange rate or interest differential.

The application to exchange rates is fairly direct, so we consider interest differentials, which we proceed to define. We have in mind the complete markets economy of sections 2 and 3, but adapt some of the notation from section 4. For each currency  $j$ , we consider an arbitrary bond with payoffs  $D^j$  and price  $W^j$ , where for simplicity of notation we have dropped the  $k$  subscripts. The price of this bond, which is a combination of the pure state-contingent claims examined in sections 2 and 3, is simply the sum of the prices of those claims:

$$W_t^j(s^t) = \sum_{s_{t+1}} V_{t+1}^j(s^{t+1}) D_{t+1}^j(s^{t+1}). \quad (7.1)$$

The (gross) nominal return on the currency 1 bond between periods  $t$  and  $t + 1$ , measured in units of currency 1, is

$$r_{t+1}^1(s^t, s_{t+1}) \equiv D_{t+1}^1(s^t, s_{t+1}) / W_t^1(s^t).$$

From (7.1) we see that this satisfies

$$1 = \sum_{s_{t+1}} V_{t+1}^1(s^t, s_{t+1}) r_{t+1}^1(s^t, s_{t+1}). \quad (7.2)$$

We can make this more familiar by adopting the convention, standard in financial economics, of normalizing prices by probabilities. Let

$$v_{t+1}^j(s^t, s_{t+1}) = V_{t+1}^j(s^t, s_{t+1}) / h(s_{t+1} | s^t) \quad \text{for } j = 1, 2, \dots, I,$$

where  $h(s_{t+1} | s^t) \equiv h(s_{t+1}) / h(s^t)$  is the conditional probability of event  $s_{t+1}$

given state  $s^t$ . Then (7.2) becomes

$$1 = \sum_{s_{t+1}} h(s_{t+1}|s^t) v_{t+1}^1(s^t, s_{t+1}) r_{t+1}^1(s^t, s_{t+1}) = E_t v_{t+1}^1 r_{t+1}^1$$

or

$$E_t r_{t+1}^1 = 1/E_t v_{t+1}^1 - \text{cov}_t(v_{t+1}^1, r_{t+1}^1)/E_t v_{t+1}^1, \tag{7.3}$$

where  $E_t$  and  $\text{cov}_t$  denote the expectation and covariance, respectively, conditional on  $s^t$ . Eq. (7.3) states that the expected return on the currency 1 bond equals the return on a safe bond, which pays one unit of currency 1 for all events  $s_{t+1}$ , plus a covariance term that we will refer to as a risk premium. The risk premium is a function, in general, of the current state  $s^t$ .

We follow a similar procedure for an arbitrary bond denominated in currency  $j$ . The state-contingent rate of return, in units of currency 1, of a bond with payoffs  $D^j$  and price  $W^j$  is

$$r_{t+1}^j(s^t, s_{t+1}) \equiv D_{t+1}^j(s^t, s_{t+1}) e_{t+1}^j(s^t, s_{t+1}) / e_t^j(s^t) W_t^j(s^t).$$

Using (7.1) and arbitrage condition (3.1), we can express the expected return as

$$E_t r_{t+1}^j = 1/E_t v_{t+1}^1 - \text{cov}_t(v_{t+1}^1, r_{t+1}^j)/E_t v_{t+1}^1, \tag{7.4}$$

which has the same interpretation as (7.3). The bonds may have different returns, however, because their risk premiums differ.

Variables like these have been used by Danker et al. (1985), Frankel (1982), Rogoff (1984), and others to test the hypothesis that sterilized intervention has no effect. From (7.3) and (7.4) we see that the interest differential on two bonds denominated in different currencies can be expressed as

$$r_{t+1}^1 - r_{t+1}^j = -\text{cov}_t(v_{t+1}^1, r_{t+1}^1 - r_{t+1}^j) / E_t v_{t+1}^1 + \varepsilon_{t+1}, \tag{7.5}$$

where  $\varepsilon_{t+1}$  is a forecast error with a conditional mean of zero. Since  $r^j$  has been defined in units of currency 1, the interest differential in (7.5) is the (ex post) deviation from uncovered interest parity for these two bonds.

The issue is whether this deviation can be ‘explained’ by a variable measuring the currency composition of the government debt, as in Rogoff’s (1984) eq. (2). In Rogoff’s example, country 1 is Canada and country 2 the United States and the explanatory variable is the ratio of Canadian dollar to U.S. dollar debt, which is

$$\sum_i B^{i1}(s^t) / e^2(s^t) \sum_i B^{i2}(s^t)$$

in the notation of sections 2 and 3. Under the conditions of Proposition 1, multiple paths for this variable are consistent with a given path for interest rates. The regression contains no information, under these conditions, about the effectiveness of sterilized intervention. For weak-form interventions, this need not be the case: the coefficient will be a function of the type of intervention followed and the economic structure.

## **8. Conclusion**

We have seen that the conditions needed in theory for the denomination of government debt to be irrelevant are weaker than previously suspected: taxes and money can affect real variables, foreign and domestic goods can be imperfect substitutes, and agents may face an incomplete set of markets. As a direct consequence, we can construct for a large class of economies equilibria in which interventions are irrelevant, yet regressions of exchange rates or interest differentials on variables measuring the currency composition of the debt – regressions used in the literature to test the portfolio balance approach and the effectiveness of sterilized intervention – can yield virtually any outcome. This leaves open the possibility that other classes of models may lead to different interpretations of these regressions. We have little hope, however, that the coefficient will be interpretable, as claimed in the portfolio balance literature, as a simple function of the risk aversion of consumers and the stochastic properties of asset returns. Other interventions in our economy do have real effects because they require, to satisfy governments' budget constraints, changes in monetary or fiscal policy. Without a complete description of these additional policy changes, an intervention is not a well-defined experiment. Models that purport to measure the effects of an intervention without a complete description of the experiment and the environment seem to us misleading. We are doubtful, in short, of the portfolio balance model's claim that sterilized intervention is an extra policy instrument.

Nevertheless, that approach has proved useful in other respects, particularly in describing how economic agents deal with currency risk. The asset pricing part of the literature – including papers by Branson and Henderson (1985, esp. sec. 3), Frankel and Engel (1984), and Kouri and deMacedo (1978) – has extended our theoretical understanding of how agents diversify internationally and pointed out discrepancies between specific versions of the theory and observed portfolio decisions. No doubt future work along these lines will contribute more to our knowledge of risk premiums on foreign currency bonds and forward contracts.

We conclude by speculating about channels through which interventions might influence the economy in practice. As we mentioned, some asset swaps must be accompanied by changes in taxes, government spending, or money supplies. All of these changes can affect equilibrium prices and quantities in a

broad range of models. Perhaps more important, though, is the potential for governments to use foreign debt in a strategic way. Reports on the Carter administration's issue of Deutsche mark treasury bills suggest that they were used primarily as a signal that the United States would follow monetary and fiscal policies consistent with a strong U.S. dollar. Current talk about yen-denominated debt has the same flavor. Perhaps further progress on the denomination of government debt can be made by formalizing these aspects of government policy.

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