ELSEVIER Journal of Monetary Economics 37 (1996) 203–223

JOURNAL OF Monetary ECONOMICS

Optimality of the Friedman rule in economies with distorting taxes

V.V. Chari[a], Lawrence J. Christiano[b], Patrick J. Kehoe*[,c]

[a] *Department of Economics, University of Minnesota, Minneapolis, MN 55455, USA*
[b] *Department of Economics, Northwestern University, Evanston, IL 60208, USA*
[c] *Department of Economics, University of Pennsylvania, Philadelphia, PA 19104, USA*

(Received August 1993; final version received March 1996)

Abstract

We find conditions for the Friedman rule to be optimal in three standard monetary models. Our main contribution is to shed light on two issues in the literature. First, the conventional view maintains that when money is a final good, its services should be taxed. Moreover, if money demand is interest-inelastic, its services should be taxed heavily. We show that this view is incorrect. Second, there is an ongoing controversy about whether the optimality of the Friedman rule is connected to the intermediate goods result from public finance. We resolve this controversy by showing a deep connection between these results.

Key words: Optimal monetary policy; Inflation tax; Ramsey policy

JEL classification: E52; E61; E63

1. Introduction

In a classic contribution, Friedman (1969) argues that optimal monetary policy requires setting nominal interest rates to zero. Phelps (1973) argues that in

*Corresponding author.

Chari is affiliated with University of Minnesota and Federal Reserve Bank of Minneapolis; Christiano with Northwestern University, National Bureau of Economic Research, Federal Reserve Bank of Minneapolis, and Federal Reserve Bank of Chicago; Kehoe with University of Pennsylvania, Federal Reserve Bank of Minneapolis, and National Bureau of Economic Research. We are grateful for the comments of an insightful referee. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, the Federal Reserve Bank of Chicago, or the Federal Reserve System.

SSDI 0304-3932(96)01252-4

economies in which governments must raise revenues with distorting taxes, it is optimal to tax all goods, including the liquidity services derived from holding money. Hence, Phelps argues that there is no theoretical presumption that the Friedman rule is optimal when there are distorting taxes. Indeed, he goes on to argue that 'if, as is often maintained, the demand for money is highly interest-inelastic, then liquidity is an attractive candidate for heavy taxation at least from the standpoint of monetary and fiscal efficiency' (Phelps, 1973, p. 82). In this paper, we analyze three standard monetary economies with distorting taxes: a cash–credit model, a money-in-the-utility-function model, and a shopping time model. The conditions for the optimality of the Friedman rule in the shopping time model are extensively analyzed in the literature (see Kimbrough, 1986a, 1986b; Faig, 1988; Woodford, 1990; Guidotti and Vegh, 1993; Correia and Teles, 1994). The main contribution of this paper is to develop conditions for the optimality of the Friedman rule in the cash–credit and the money-in-the-utility-function economies. The common features of the requirements for optimality are simple homotheticity and separability conditions similar to those in the public finance literature on optimal uniform commodity taxation.

There appears to be a widespread consensus in the literature that in economies with distorting taxes in which money is a final good, the Friedman rule is typically not optimal. Furthermore, if the demand for money is interest-inelastic, liquidity services should be taxed heavily. In support of the consensus view, Kimbrough (1986b) argues that: 'Phelps (1973) finds that money should be taxed because he introduces it directly through the utility function. As a result, money is similar to other consumption goods.' We show that in the money-in-the-utility-function model, the Friedman rule turns out to be optimal if the consumer's preferences are homothetic in money and the consumption good and weakly separable in leisure. These conditions are consistent with elasticities for the money demand function, which range from zero to infinity. These results show that the consensus view is incorrect.

There is also extensive discussion in the literature about the optimality of the Friedman rule when money is an intermediate good. Kimbrough (1986a) shows that the Friedman rule is optimal in a shopping time model when the transactions technology is constant returns-to-scale. He argues that this result holds because money is an intermediate good in this economy, and the standard public finance result (see Diamond and Mirrlees, 1971a, 1971b) is that intermediate goods should not be taxed. More generally, Kimbrough (1986b, p. 137) claims that the Friedman rule will be optimal in 'any economy in which, in equilibrium, scarce resources are used up in the transactions process and agents can economize on these transactions costs by holding money' because money is an intermediate good in all such cases. Woodford (1990) demonstrates that this claim is incorrect because there are technologies in the shopping time model for which the Friedman rule is not optimal. Woodford uses this demonstration to argue that there is no connection between optimality of the Friedman rule and the intermediate-goods result.

We show that the Friedman rule is optimal in all three economies under similar homotheticity and separability conditions. Thus, for example, even though money is a final good in the money-in-the-utility-function economy and an intermediate good in the shopping time economy, we find that the conditions for the optimality of the Friedman rule in the two economies look quite similar. At one level, this similarity suggests that there is no connection between the optimality of the Friedman rule and the intermediate-goods result (we subscribe to this view in Chari, Christiano, and Kehoe, 1993). Indeed, if there is a connection with the public finance literature at all, it appears to be with the results on the optimality of uniform commodity taxation. This connection seems natural, because our homotheticity and separability conditions are similar to those used to establish the optimality of uniform taxation (see Atkinson and Stiglitz, 1972).

At a deeper level, however, there is a close connection between the optimality of the Friedman rule and the intermediate-goods result when our conditions hold, and there is no connection when our conditions do not hold. For all three monetary economies, when our homotheticity and separability conditions hold, the optimality of the Friedman rule follows from the intermediate-goods result. To prove this, we show that under such conditions, all three monetary economies can be reinterpreted as real intermediate-goods economies, and the optimality of the Friedman rule in the monetary economies follows directly from the intermediate-goods result in the reinterpreted real economies.

In contrast, when our conditions do not hold, there is no such connection. To prove this, we show that when our conditions do not hold, there are a couple of possibilities. First, there are monetary economies in which the Friedman rule holds which cannot be reinterpreted as real intermediate-goods economies. Second, there are monetary economies which can be reinterpreted as real intermediate-goods economies but in which the Friedman result does not hold.

2. A cash–credit economy

Consider a simple production economy populated by a large number of identical, infinitely-lived consumers. In each period $t = 0, 1, \ldots$, the economy experiences one of finitely many events s_t. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of period 0, of any particular history s^t is $\mu(s^t)$. The initial realization s_0 is given. This suggests a natural commodity space in which goods are differentiated by histories.

In each period t, there are three goods: labor and two consumption goods (a cash good and a credit good). A constant returns-to-scale technology is available to transform labor $\ell(s^t)$ into output. The output can be used for private consumption of either the cash good $c_1(s^t)$ or the credit good $c_2(s^t)$ or for government consumption $g(s^t)$. Throughout, we will take government consumption to be exogenously specified.

The resource constraint is

$$c_1(s^t) + c_2(s^t) + g(s^t) = \ell(s^t). \tag{2.1}$$

The preferences of each consumer are given by

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) U(c_1(s^t), c_2(s^t), \ell(s^t)), \tag{2.2}$$

where the utility function U is strictly concave and satisfies the Inada conditions.

In period t, consumers trade money, assets, and goods in particular ways. At the start of period t, after observing the current state s_t, consumers trade money and assets in a centralized securities market. The assets are one-period, non-state-contingent, nominal claims. Let $M(s^t)$ and $B(s^t)$ denote the money and the nominal bonds held at the end of the securities market trading. Let $R(s^t)$ denote the gross nominal return on these bonds payable in period $t+1$ in all states $s^{t+1} = (s^t, s_{t+1})$. After this trading, each consumer splits into a worker and a shopper. The shopper must use the money to purchase cash goods. To purchase credit goods, the shopper issues nominal claims, which are settled in the securities market in the next period. The worker is paid in cash at the end of each period.

This environment leads to the following constraint for the securities market:

$$\begin{aligned} M(s^t) + B(s^t) = {} & R(s^{t-1})B(s^{t-1}) + M(s^{t-1}) - p(s^{t-1})c_1(s^{t-1}) \\ & - p\,(s^{t-1})c_2(s^{t-1}) + p(s^{t-1})(1 - \tau(s^{t-1}))\ell(s^{t-1}), \end{aligned} \tag{2.3}$$

where p is the price of the consumption good and τ is the tax rate on labor income. The left side of (2.3) is the nominal value of assets held at the end of securities market trading. The first term on the right side is the value of nominal debt bought in the preceding period. The next two terms are the shopper's unspent cash. The fourth term is the payments for credit goods, and the last term is the after-tax receipts from labor services. We will assume that the holdings of real debt $B(s^t)/p(s^t)$ are bounded above and below by some arbitrarily large constants. Purchases of cash goods must satisfy the following cash-in-advance constraint:

$$p(s^t)c_1(s^t) \leq M(s^t). \tag{2.4}$$

We let $x(s^t) = (c_1(s^t), c_2(s^t), \ell(s^t), M(s^t), B(s^t))$ denote an allocation for consumers at s^t, and we let $x = (x(s^t))$ denote an allocation for all s^t. The initial stock of money M_{-1} and the initial stock of nominal debt B_{-1} are given.

Money is introduced into and withdrawn from the economy through open market operations in the securities market. The constraint facing the government in this market is

$$M(s^t) - M(s^{t-1}) + B(s^t) = R(s^{t-1})B(s^{t-1}) + p(s^{t-1})g(s^{t-1}) \\ - p(s^{t-1})\tau(s^{t-1})\ell(s^{t-1}). \tag{2.5}$$

The terms on the left side of this equation are the assets sold by the government. The first term on the right is the payments on debt incurred in the preceding period, the second term is the payment for government consumption, and the third term is tax receipts. Notice that government consumption is bought on credit. We let $\pi(s^t) = (\tau(s^t), p(s^t), R(s^t))$ denote a policy for the government at s^t, and we let $\pi = (\pi(s^t))$ denote an allocation for all s^t.

Consider now the policy problem faced by the government. Suppose an institution or a commitment technology exists through which the government can bind itself to a particular sequence of policies once and for all at period 0. We model this technology by having the government choose a policy $\pi = (\pi(s^t))$ at the beginning of time and then having consumers choose their allocations. Since the government needs to predict how consumer allocations and prices will respond to its policies, consumer allocations and prices are described by rules that associate allocations with government policies. Formally, allocation rules are sequences of functions $x(\pi) = (x(s^t \mid \pi))$ that map policies π into allocations x.

A *Ramsey equilibrium* is a policy and an allocation rule $x(\cdot)$ with the following conditions: (i) the policy π maximizes

$$\sum_{t,s^t} \beta^t \mu(s^t) U\left(c_1(s^t \mid \pi), c_2(s^t \mid \pi), \ell(s^t \mid \pi)\right)$$

subject to (2.5), with allocations given by $x(\pi)$, and (ii) for every π', the allocation $x(\pi')$ maximizes (2.2) subject to the bounds on debt purchases and to (2.3) and (2.4) evaluated at the policy π'.

In this equilibrium, the consumer maximizes (2.2) subject to (2.3), (2.4), and the bounds on debt. Money earns a gross nominal return of one. If bonds earn a gross nominal return of less than one, then the consumer can make infinite profits by buying money and selling bonds. Thus in any equilibrium, $R(s^t) \geq 1$. The consumer's first-order conditions imply that $U_1(s^t)/U_2(s^t) = R(s^t)$; thus in any equilibrium, the following constraint must hold:

$$U_1(s^t) \geq U_2(s^t). \tag{2.6}$$

This feature of the competitive equilibrium constrains the set of Ramsey allocations.

The allocations in the Ramsey equilibrium solve a simple programming problem called the *Ramsey allocation problem.* As is well known, if the initial stock of nominal assets held by consumers is positive, then welfare is maximized by increasing the initial price level to infinity. If the initial stock is negative, then welfare is maximized by setting the initial price level so low that the government raises all the revenue it needs without levying any distorting taxes. To make the problem interesting, we set the initial sum of nominal assets of consumers $M_{-1}+B_{-1}$ to zero. In terms of notation, it will be convenient here and throughout the paper to let $U_i(s^t)$, $i = 1,2,3$, denote the marginal utilities at state s^t. Using standard techniques (see Lucas and Stokey, 1983; Chari, Christiano, and Kehoe, 1991) we can establish:

Proposition 1 (Ramsey Allocations). *The consumption and labor allocations in the Ramsey equilibrium solve the Ramsey allocation problem*

$$\max \sum_t \sum_{s^t} \beta^t \mu(s^t) U\left(c_1(s^t), c_2(s^t), \ell(s^t)\right)$$

subject to (2.1), (2.6), and

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[U_1(s^t)c_1(s^t) + U_2(s^t)c_2(s^t) + U_3(s^t)\ell(s^t)\right] = 0. \tag{2.7}$$

The proof of the proposition has two parts: Any competitive equilibrium allocation must satisfy (2.1), (2.6), and (2.7); conversely, any allocation satisfying (2.1), (2.6), and (2.7) can be decentralized as a competitive equilibrium. Thus the resource constraint (2.1), the no-return-dominance constraint (2.6), and the implementability constraint (2.7) completely characterize the competitive equilibrium allocations. We use this characterization in Section 4.

Consider utility functions of the form

$$U(c_1, c_2, \ell) = V(w(c_1, c_2), \ell), \tag{2.8}$$

where w is homothetic. We then have:

Proposition 2 (Optimality of the Friedman Rule). *For utility functions of the form* (2.8), *the Ramsey equilibrium has* $R(s^t) = 1$ *for all* s^t.

Proof. Consider for a moment the Ramsey allocation problem with constraint (2.6) dropped. Let λ denote the Lagrange multiplier on (2.7) and $\beta^t \mu(s^t)\gamma(s^t)$ denote the Lagrange multiplier on (2.1). The first-order conditions for $c_i(s^t)$, $i = 1,2$, in this problem are

$$(1+\lambda)U_i(s^t) + \lambda \left[\sum_{j=1}^{2} U_{ji}(s^t)c_j(s^t) + U_{3i}(s^t)\ell(s^t)\right] = \gamma(s^t). \tag{2.9}$$

Note that a utility function which satisfies (2.8) also satisfies

$$\sum_{j=1}^{2} U_{j1}(s^t)c_j(s^t)/U_1(s^t) = \sum_{j=1}^{2} U_{j2}(s^t)c_j(s^t)/U_2(s^t). \tag{2.10}$$

To see this, recall that homotheticity implies that for any constant $\alpha > 0$,

$$\frac{U_1(\alpha c_1(s^t), \alpha c_2(s^t), \ell)}{U_2(\alpha c_1(s^t), \alpha c_2(s^t), \ell)} = \frac{U_1(c_1(s^t), c_2(s^t), \ell(s^t))}{U_2(c_1(s^t), c_2(s^t), \ell(s^t))}. \tag{2.11}$$

Differentiating (2.11) with respect to α and evaluating it at $\alpha = 1$ gives (2.10). Next, dividing (2.9) by U_i and noting that $U_{3i}/U_i = V_{12}/V_1$ for $i = 1, 2$, we have that

$$(1+\lambda) + \lambda \left[\sum_{j=1}^{2} \frac{U_{ji}(s^t) c_j(s^t)}{U_i(s^t)} + \frac{V_{12}(s^t)}{V_1(s^t)} \ell(s^t) \right] = \frac{\gamma(s^t)}{U_i(s^t)}. \tag{2.12}$$

Using (2.10), we have that the left side of (2.12) has the same value for $i = 1$ and for $i = 2$. It follows that $U_1(s^t)/U_2(s^t) = 1$. Since the solution to the less constrained problem satisfies (2.6), it is also a solution to the Ramsey allocation problem. From the consumer's first-order condition, $U_1(s^t)/U_2(s^t) = R(s^t)$, and thus $R(s^t) = 1$. ■

It is interesting to relate our results to Phelps' arguments for taxing liquidity services. Our results suggest that the connection between the interest elasticity of money demand and the desirability of taxing liquidity services is, at best, tenuous. To see this, suppose that the utility function is of the form

$$U(c_1, c_2, \ell) = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma} + V(\ell). \tag{2.13}$$

Then the consumer's first-order condition $U_1/U_2 = R$ becomes

$$\frac{m^{-\sigma}}{(c-m)^{-\sigma}} = R, \tag{2.14}$$

where m is real money balances and $c = c_1 + c_2$. The implied elasticity of money demand η is given by

$$\eta = \frac{1}{\sigma} R^{1/\sigma - 1} / (1 + R^{1/\sigma - 1}). \tag{2.15}$$

Evaluating this elasticity at $R = 1$ gives $\eta = 1/2\sigma$, and thus the elasticity of money demand can range from 0 to ∞. Nevertheless, all preferences in this class satisfy our homotheticity and separability conditions, and hence the Friedman rule is optimal.

There are two points to note about the generality of this result. First, notice that restricting w to be homogeneous of degree 1 does not reduce the generality of the result, since we can write $w(\cdot) = g(f(\cdot))$, where g is monotone and f is homogeneous of degree 1, and simply reinterpret V accordingly. Second, the proof can be easily extended to economies with more general production technologies, including those with capital accumulation. To see how, consider modifying the resource constraint (2.1) to

$$f\left(c_1(s^t), c_2(s^t), g(s^t), \ell(s^t), k(s^t), k(s^{t-1})\right) = 0, \tag{2.16}$$

where f is constant returns-to-scale and modifying the consumer's and the government's budget constraints appropriately. Let capital income be net of depreciation taxed at rate $\theta(s^t)$, and let capital be a credit good, although the same result goes through if capital is a cash good. For this economy, combining the consumer's and the firm's first-order conditions gives

$$\frac{U_1(s^t)}{U_2(s^t)} = R(s^t)\frac{f_1(s^t)}{f_2(s^t)}. \tag{2.17}$$

Thus the optimality of the Friedman rule requires that $U_1(s^t)/U_2(s^t) = f_1(s^t)/f_2(s^t)$. The constraint requiring that $R(s^t) \geq 1$ now implies that

$$\frac{U_1(s^t)}{U_2(s^t)} \geq \frac{f_1(s^t)}{f_2(s^t)}, \tag{2.18}$$

and the implementability condition (1.7) now reads

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[U_1(s^t)c_1(s^t) + U_2(s^t)c_2(s^t) + U_3(s^t)\ell(s^t)\right]$$
$$= U_c(s_0)\left[(1 - \theta(s_0))(f_6(s_0) - \delta)\right] k_{-1}, \tag{2.19}$$

where k_{-1} is the initial capital stock. Since the tax on initial capital $\theta(s_0)$ acts like a lump-sum tax, it is optimal to set it as high as possible. To make the problem interesting, we follow the standard procedure of fixing it exogenously. The Ramsey allocation problem is to choose allocations to maximize utility subject to (2.16), (2.18), and (2.19). It is easy to show that for preferences of the form (2.8), the analog of (2.12) has the right side multiplied by $f_i(s^t)$, $i = 1,2$. This analog implies that $U_1(s^t)/U_2(s^t) = f_1(s^t)/f_2(s^t)$, and thus the Friedman rule holds.

The intuition for the proposition is as follows. In this economy, the tax on labor income implicitly taxes consumption of the cash good and the credit good at the same rate. A standard result in public finance is that if the utility function is separable in leisure and the subutility function over consumption goods is homothetic, then the optimal policy is to tax all consumption goods at the same rate (Atkinson and Stiglitz, 1972). If $R(s^t) > 1$, the cash good is effectively taxed at a higher rate than the credit good, since cash goods must be paid for immediately but credit goods are paid for with a one-period lag. Thus, with such preferences, efficiency requires that $R(s^t) = 1$ and, therefore, that monetary policy follow the Friedman rule.

To make this intuition precise, consider a real barter economy with the same preferences (2.2) and resource constraint (2.1) as the monetary economy and with commodity taxes on the two consumption goods. Consider a date 0 representation of the budget constraints. The consumer's budget constraint is

$$\sum_t \sum_{s^t} q(s^t)\left[(1+\tau_1(s^t))c_1(s^t) + (1+\tau_2(s^t))c_2(s^t)\right] = \sum q(s^t)\ell(s^t), \quad (2.20)$$

and the government's budget constraint is

$$\sum_t \sum_{s^t} q(s^t)g(s^t) = \sum_t \sum_{s^t} q(s^t)\left[\tau_1(s^t)c_1(s^t) + \tau_2(s^t)c_2(s^t)\right], \quad (2.21)$$

where $q(s^t)$ is the price of goods at date t and state s^t. A Ramsey equilibrium for this economy is defined in the obvious fashion. The Ramsey allocation problem for this barter economy is similar to that in the monetary economy except that there is no constraint (2.6).

The consumer's first-order conditions imply that

$$\frac{U_1(s^t)}{U_2(s^t)} = \frac{1+\tau_1(s^t)}{1+\tau_2(s^t)}. \quad (2.22)$$

Thus Ramsey taxes satisfy $\tau_1(s^t) = \tau_2(s^t)$ if and only if, in the Ramsey allocation problem of maximizing (2.2) subject to (2.1) and (2.7), the solution has $U_1(s^t)/U_2(s^t) = 1$. We can then use the argument in Proposition 2 to show:

Proposition 3 (Optimality of Uniform Commodity Taxation). *For utility functions of the form (2.8), the Ramsey equilibrium has $\tau_1(s^t) = \tau_2(s^t)$ for all s^t.*

Thus with homotheticity and separability in the period utility function, the optimal taxes on the two consumption goods are equal at each state. Notice that this proposition does not imply that commodity taxes are equal across states (that is, $\tau_i(s^t)$ may not equal $\tau_j(s^r)$ for $t \neq r$, $i,j = 1,2$).

We have shown that if the conditions for uniform commodity taxation are satisfied in the barter economy, then in the associated monetary economy the Friedman rule is optimal. Of course, since the allocations in the monetary economy must satisfy (2.6) while those in the barter economy need not, there are situations in which uniform commodity taxation is not optimal in the barter economy but in which the Friedman rule is optimal in the monetary economy. To see this, consider preferences of the form

$$U(c_1, c_2, \ell) = \frac{c_1^{1-\sigma_1}}{1-\sigma_1} + \frac{c_2^{1-\sigma_2}}{1-\sigma_2} + V(\ell). \quad (2.23)$$

The first-order conditions to the Ramsey problem in the barter economy imply that

$$\frac{U_1(s^t)}{U_2(s^t)} = \frac{c_1(s^t)^{-\sigma_1}}{c_2(s^t)^{-\sigma_2}} = \frac{1+\lambda(1-\sigma_2)}{1+\lambda(1-\sigma_1)}. \quad (2.24)$$

Clearly $U_1(s^t) \geq U_2(s^t)$ if and only if $\sigma_1 \geq \sigma_2$. For cases in which $\sigma_1 = \sigma_2$, these preferences satisfy condition (2.6), and both uniform commodity taxation and the Friedman rule are optimal. If $\sigma_1 > \sigma_2$, neither uniform commodity taxation nor the Friedman rule is optimal. It is optimal to tax good 1 at a higher rate than good 2. In the barter economy, this higher taxation is accomplished by setting $\tau_1(s^t) > \tau_2(s^t)$, while in the monetary economy it is accomplished by setting $R(s^t) > 1$. More interesting is that, when $\sigma_1 < \sigma_2$, uniform commodity taxation is not optimal but the Friedman rule is. To see this note that when $\sigma_1 < \sigma_2$, the solution in the monetary economy ignoring the constraint $U_1(s^t) \geq U_2(s^t)$ violates this constraint. Thus, this constraint binds, and in the monetary economy, $U_1(s^t) = U_2(s^t)$. Thus, in the barter economy, it is optimal to tax good 1 at a lower rate than good 2, and this is accomplished by setting $\tau_1(s^t) < \tau_2(s^t)$. In the monetary economy, it is not feasible to tax good 1 at a lower rate than good 2, since $R(s^t) \geq 1$, and the best feasible solution is to set $R(s^t) = 1$.

In this section, we have focused on the Lucas and Stokey (1983) cash–credit version of the cash-in-advance model. It turns out that in the simpler cash-in-advance model without credit goods, the inflation rate and the labor tax rate are indeterminate. The first-order conditions for a deterministic version of that model are the cash-in-advance constraint, the budget constraint, and

$$-\frac{U_{1t}}{U_{2t}} = R_{t+1}(1 - \tau_t), \tag{2.25}$$

$$\frac{1}{\beta}\frac{U_{1t}}{U_{1t+1}} = \frac{R_{t+1}p_t}{p_{t+1}}, \tag{2.26}$$

where the period utility function is $U(c_t, \ell_t)$ and R_{t+1} is the nominal interest rate from period t to period $t+1$. Here, only the products $R_{t+1}(1-\tau_t)$ and $R_{t+1}p_t/p_{t+1}$ are pinned down by the allocations. Thus, the nominal interest rate, the tax rate, and the inflation rate are not individually determined. There are a whole variety of ways to decentralize the Ramsey allocation. In particular, trivially, both the Friedman rule and arbitrarily high rates of inflation are optimal.

3. A money in the utility function economy

Consider the following monetary economy. Labor is transformed into consumption goods according to

$$c(s^t) + g(s^t) = \ell(s^t). \tag{3.1}$$

The preferences of the representative consumer are given by

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) U(M(s^t)/p(s^t), c(s^t), \ell(s^t)). \tag{3.2}$$

In period t, the consumer's budget constraint is

$$p(s^t)c(s^t) + M(s^t) + B(s^t) = M(s^{t-1}) + R(s^{t-1})B(s^{t-1}) + p(s^t)(1 - \tau(s^t))\ell(s^t). \tag{3.3}$$

The holdings of real debt $B(s^t)/p(s^t)$ are bounded below by some arbitrarily large constant, and the holdings of money are bounded below by zero. Let M_{-1} and $R_{-1}B_{-1}$ denote the initial asset holdings of the consumer. The budget constraint of the government is given by

$$B(s^t) = R(s^{t-1})B(s^{t-1}) + p(s^t)g(s^t) - [M(s^t) - M(s^{t-1})] - p(s^t)(1 - \tau(s^t))g(s^t). \tag{3.4}$$

A Ramsey equilibrium for this economy is defined in the obvious fashion. We set the initial stock of assets to zero for reasons similar to those given in the preceding section. Let $m(s^t) = M(s^t)/p(s^t)$ denote the real balances in the Ramsey equilibrium. Using logic similar to that in Proposition 1, we can show that the consumption and labor allocations and the real money balances in the Ramsey equilibrium solve the Ramsey allocation problem

$$\max \sum_t \sum_{s^t} \beta^t \mu(s^t) U(m(s^t), c(s^t), \ell(s^t)) \tag{3.5}$$

subject to (3.1) and

$$\sum \beta^t \left[m(s^t)U_1(s^t) + c(s^t)U_2(s^t) + \ell(s^t)U_3(s^t)\right] = 0. \tag{3.6}$$

The resource constraint (3.1) and the implementability constraint (3.6) completely characterize the set of competitive equilibrium allocations.

We are interested in finding conditions under which the Friedman rule is optimal. Now the consumer's first-order conditions imply that

$$\frac{U_1(s^t)}{U_2(s^t)} = 1 - \frac{1}{R(s^t)}. \tag{3.7}$$

Thus for the Friedman rule to hold, namely, $R(s^t) = 1$, it must be true that

$$\frac{U_1(s^t)}{U_2(s^t)} = 0. \tag{3.8}$$

Since the marginal utility of consumption goods will be finite, (3.8) will hold only if $U_1(s^t) = 0$, namely, if the marginal utility of real money balances is zero. Intuitively, under the Friedman rule, it is optimal to satiate the economy with real money balances.

We are interested in economies for which preferences are not satiated with any finite level of money balances and for which the marginal utility of real money balances converges to zero as the level of real money balances converges to infinity. That is, for each c and ℓ, $U_1(m,c,\ell) \to 0$ as $m \to \infty$. Intuitively, in such economies, the Friedman rule holds exactly only if the value of real money balances is infinite, and for such economies, there would be no solution to the Ramsey allocation problem. To get around this technicality, we consider an economy in which the level of real money balances is exogenously bounded by a constant. We will say that the Friedman rule is optimal if, as this bound on real money balances increases, the associated nominal interest rates in the Ramsey equilibrium converge to one. With this in mind, consider modifying the Ramsey allocation problem to include the constraint

$$m(s^t) \leq \bar{m}. \tag{3.9}$$

Consider preferences of the form

$$U(m,c,\ell) = V(w(m,c),\ell), \tag{3.10}$$

where w is homothetic. We then have:

Proposition 4 (Optimality of the Friedman Rule). *If the utility function is of the form (3.10), then the Friedman rule is optimal.*

Proof. The Ramsey allocation problem is to maximize (3.2) subject to (3.1), (3.6), and (3.9). Consider a less constrained version of this problem in which constraint (3.9) is dropped. Let $\beta^t\mu(s^t)\gamma(s^t)$ and λ denote the Lagrange multipliers on constraints (3.1) and (3.6). The first-order condition for real money balances and consumption are

$$(1+\lambda)U_1(s^t) + \lambda\left[m(s^t)U_{11}(s^t) + c(s^t)U_{21}(s^t) + \ell(s^t)U_{31}(s^t)\right] = 0 \tag{3.11}$$

$$(1+\lambda)U_2(s^t) + \lambda\left[m(s^t)U_{12}(s^t) + c(s^t)U_{22}(s^t) + \ell(s^t)U_{32}(s^t)\right] = \gamma(s^t). \tag{3.12}$$

Since the utility function satisfies (3.10), it follows that

$$\frac{m(s^t)U_{11}(s^t) + c(s^t)U_{21}(s^t)}{U_1(s^t)} = \frac{m(s^t)U_{12} + c(s^t)U_{22}(s^t)}{U_2(s^t)}. \tag{3.13}$$

Using the form of (3.10), we can rewrite (3.11) and (3.12) as

$$(1+\lambda) + \lambda\left[\frac{m(s^t)U_{11}(s^t) + c(s^t)U_{21}(s^t)}{U_1(s^t)} + \ell(s^t)\frac{V_{21}(s^t)}{V_1(s^t)}\right] = 0 \tag{3.14}$$

$$(1+\lambda) + \lambda\left[\frac{m(s^t)U_{12}(s^t) + c(s^t)U_{22}(s^t)}{U_2(s^t)} + \ell(s^t)\frac{V_{21}(s^t)}{V_1(s^t)}\right] = \frac{\gamma(s^t)}{U_1(s^t)}. \tag{3.15}$$

Using (3.13), we can show that

$$\frac{U_1(s^t)}{U_2(s^t)} = 0 \tag{3.16}$$

in the less constrained problem. Hence the associated $m(s^t)$ is arbitrarily large, and thus for any finite bound $\bar{m}$, the constraint (3.9) binds in the original problem. The result then follows from (3.7). ■

Again, restricting w to be homogeneous does not reduce the generality of the result.

Clearly, there are preferences which do not satisfy (3.10) for which the Friedman rule is optimal. Consider

$$U(m, c, \ell) = \frac{m^{1-\sigma_1}}{1-\sigma_1} + \frac{c^{1-\sigma_1}}{1-\sigma_2} + V(\ell). \tag{3.17}$$

The first-order condition in the Ramsey problem for money balances $m(s^t)$ is

$$[1 + \lambda(1 - \sigma_1)]m(s^t)^{-\sigma_1} = 0. \tag{3.18}$$

Unless the endogenous Lagrange multiplier λ just happens to equal $(\sigma_1 - 1)^{-1}$, (3.17) implies that the Friedman rule is optimal. Note that for cases in which $\sigma_1 \neq \sigma_2$, (3.16) does not satisfy (3.10).

In related work, Woodford (1990) considers the optimality of the Friedman rule in this model. He characterizes the policy that maximizes steady-state utility rather than the one that maximizes the discounted value of utility. His Ramsey problem is

$$\max U(m, c, \ell) \tag{3.19}$$

subject to

$$c + g \leq \ell, \tag{3.20}$$

$$U_1 m + U_2 c + U_3 \ell = (1 - \beta) U_1 m. \tag{3.21}$$

Woodford shows that if consumption and real balances are gross substitutes, the Friedman rule is not optimal. Of course, there are functions which satisfy our homotheticity and separability assumptions which are gross substitutes, for example,

$$U(m, c, \ell) = \frac{m^{1-\sigma}}{1-\sigma} + \frac{c^{1-\sigma}}{1-\sigma} + V(\ell). \tag{3.22}$$

The reason for the difference in the results arises from the difference in the implementability constraints. The first-order conditions to our problem are similar to those in Woodford's problem, except that his include derivatives of the right side of (3.21). Notice that in Woodford's problem, if $\beta = 1$ and preferences satisfy our homotheticity and separability conditions, then the Friedman rule is optimal.

4. A shopping time monetary economy

Consider a monetary economy along the lines of Kimbrough (1986a, 1986b). Labor is transformed into consumption goods according to

$$c(s^t) + g(s^t) \leq \ell(s^t). \tag{4.1}$$

The preferences of the representative consumer are given by

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) U\left(c(s^t), \ell(s^t) + \phi(c(s^t), M(s^t)/p(s^t))\right), \tag{4.2}$$

where U is concave, $U_1 > 0$, $U_2 < 0$, $\phi_1 > 0$, and $\phi_2 < 0$. The function $\phi(c_1, M/p)$ describes the amount of time it takes to obtain c units of the consumption good when the consumer has M/p units of real money balances. We assume $\phi_1 > 0$ so that, with the same amount of money, it takes more time to obtain more consumption goods. We also assume $\phi_2 < 0$ so that, with more money, it takes less time to obtain the same amount of consumption goods. The budget constraints of the consumer and the government are the same as in (3.3) and (3.4).

The Ramsey equilibrium is defined in the obvious fashion. Letting $m(s^t) = M(s^t)/p(s^t)$ and setting the initial nominal assets to zero, we can show that the consumption, labor allocations, and real money balances in the Ramsey equilibrium solve the problem

$$\max \sum_t \sum_{s^t} \beta^t \mu(s^t) U\left(c(s^t), \ell(s^t) + \phi(c(s^t), m(s^t))\right)$$

subject to (4.1) and

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[c(s^t)(U_1(s^t) + \phi_1(s^t) U_2(s^t)) \right.$$
$$\left. + \ell(s^t) U_2(s^t) + m(s^t) \phi_2(s^t) U_2(s^t)\right] = 0. \tag{4.3}$$

The resource constraint (4.1) and the implementability constraint (4.3) completely characterize the set of competitive allocations.

From the consumer's first-order conditions, it follows that $R(s^t) = 1$ if $\phi_2 = 0$. We then have:

Proposition 5 (Optimality of the Friedman Rule). *If ϕ is homogeneous of degree k and $k \geq 1$, then the Friedman rule is optimal.*

Proof. The first-order conditions to the Ramsey problem with respect to $m(s^t)$ and $\ell(s^t)$ are given by

$$U_2\phi_2 + \lambda[cU_{12}\phi_2 + U_{22}\phi_2(\phi_1 c + \phi_2 m + \ell) + U_2\phi_2 + U_2(\phi_{12}c + \phi_{22}m)] = 0 \tag{4.4}$$

and

$$U_2 + \lambda[cU_{12} + U_{22}(\phi_1 c + \phi_2 m + \ell) + U_2] + \mu = 0, \tag{4.5}$$

where μ is the multiplier on the resource constraint.

Suppose first that $\phi_2 \neq 0$ so that the optimal policy does not follow the Friedman rule. Then, from (4.4) and (4.5), we have that

$$-\frac{\lambda U_2(\phi_{12}c + \phi_{22}m)}{\phi_2} + \mu = 0. \tag{4.6}$$

Now, under the condition that $\phi(c,m)$ is homogeneous of degree k and $k \geq 1$, we have that $\phi_2(\gamma c, \gamma m) = \gamma^{k-1}\phi_2(c,m)$. Differentiating with respect to γ and evaluating at $\gamma = 1$, we have that $c\phi_{12} + m\phi_{22} = (k-1)\phi_2$, and thus

$$\frac{c\phi_{12} + m\phi_{22}}{\phi_2} \geq 0. \tag{4.7}$$

Since $\lambda \geq 0$, $U_2 < 0$, and $\mu \geq 0$, (4.6) and (4.7) contradict each other. ■

Note that this proof does not go through if $\phi(c,m)$ is homogeneous of degree less than unity. Using a different approach, however, Correia and Teles (1994) prove that the Friedman rule is optimal when $\phi(c,m)$ is homogeneous of any degree.

5. Reinterpreting monetary economies as real economies

In this section, we examine the relationship between the optimality of the Friedman rule and the intermediate-goods result. The relationship is the following: First, if the homotheticity and separability conditions hold, then in all three monetary models, the optimality of the Friedman rule follows from the intermediate-goods result. Second, if these conditions do not hold, then in all three economies, there is no connection between the optimality of the Friedman rule and the intermediate-goods result.

To establish these results, we proceed as follows. We begin by setting up the notation for a simple real intermediate economy and review the intermediate-goods result for that economy. We then show that when our homotheticity and separability conditions hold, all three monetary economies can be reinterpreted as real economies with intermediate goods. For each of the three monetary economies, we establish that the optimality of the Friedman rule in the monetary economy follows from the intermediate-goods result in the reinterpreted real economy. This proves the first result.

Next we consider monetary economies which do not satisfy our conditions. We establish our second result with a couple of examples. We start with an example in which the monetary economy can be reinterpreted as a real intermediate-goods economy but in which the Friedman rule does not hold in the monetary economy. We then give an example of a monetary economy in which the Friedman rule holds, but this economy cannot be reinterpreted as a real intermediate-goods economy.

We begin by reviewing the intermediate-goods result for a class of economies, which will be useful in our reinterpretations. There are three final goods – private consumption x, government consumption g, and labor l – and an intermediate good z. The utility function is $U(x,\ell)$. The technology set for producing the final consumption good using labor ℓ_1 and the intermediate good is described by

$$f(x,z,\ell_1) \leq 0, \tag{5.1}$$

where f is a constant returns-to-scale production function. There is a technology set for producing the intermediate good and the government consumption using labor ℓ_2 described by

$$h(z,g,\ell_2) \leq 0, \tag{5.2}$$

where h is a constant returns-to-scale production function. The consumer's problem is to maximize

$$U(x,\ell_1+\ell_2)$$

subject to

$$p(1+\tau)x \leq w(\ell_1+\ell_2), \tag{5.3}$$

where p and w are the prices of the consumption good and labor and τ is the tax on the consumption good. The firm that produces private consumption goods maximizes profits

$$px - w\ell_1 - q(1+\eta)z \tag{5.4}$$

subject to (5.1), where q is the price of intermediate goods and η is the tax on intermediate goods. The firm that produces intermediate goods and government consumption goods maximizes profits

$$qz + pg - w\ell_2 \tag{5.5}$$

subject to (5.2).

It is easy to show that the Ramsey allocation problem is given by

$$\max U(x, \ell_1 + \ell_2)$$

subject to (5.1), (5.2), and

$$xU_x + (\ell_1 + \ell_2)U_\ell = 0. \tag{5.6}$$

We then have:

Proposition 6 (Intermediate-Goods Result). *The solution to the Ramsey allocation problem satisfies production efficiency; namely, the marginal rates of transformation are equated across technologies. Equivalently, it is optimal to set the tax on intermediate goods* $\eta = 0$.

Proof. For this economy, production efficiency is equivalent to

$$\frac{f_z}{f_\ell} = -\frac{h_z}{h_\ell}. \tag{5.7}$$

Solving the Ramsey allocation problem, we obtain the following first-order conditions for z, ℓ_1, and ℓ_2, respectively:

$$vf_z = -\mu h_z, \tag{5.8}$$

$$U_\ell + \lambda[xU_{\ell x} + U_\ell + \ell U_{ll}] + vf_\ell = 0, \tag{5.9}$$

$$U_\ell + \lambda[xU_{\ell x} + U_\ell + \ell U_{ll}] + \mu h_\ell = 0, \tag{5.10}$$

where v, μ, and λ are the multipliers on (5.1), (5.2), and (5.6). Combining (5.9) and (5.10) gives $vf_\ell = \mu h_\ell$ which, combined with (5.8), establishes (5.7).

The first-order conditions for profit maximization for the firms imply that

$$\frac{f_z}{f_\ell} = \frac{q(1+\eta)}{w} = -\frac{h_z}{h_\ell}(1+\eta). \tag{5.11}$$

Thus, if (5.7) holds, (5.11) implies $\eta = 0$. ■

The intermediate-goods result holds in more general settings in which there are (possibly infinitely) many goods and many production technologies. We have assumed that the production technologies satisfy constant returns-to-scale. If there are increasing returns-to-scale, there are standard problems with the existence of a competitive equilibrium. If there are decreasing returns-to-scale, the intermediate-goods result continues to hold, provided that pure profits can be fully taxed away.

We begin by reinterpreting the cash–credit economy as a real production economy with intermediate goods. Under our homotheticity and separability assumptions, the period utility is $U(w(c_{1t}, c_{2t}), \ell_t)$ and the resource constraint is

$$c_{1t} + c_{2t} + g_t = \ell_t. \tag{5.12}$$

Since the gross nominal interest cannot be less than unity, the allocations in the monetary economy must satisfy

$$w_1(c_{1t}, c_{2t}) \geq w_2(c_{1t}, c_{2t}). \tag{5.13}$$

The reinterpreted economy is an infinite sequence of real static economies. In each period, there are two intermediate goods z_{1t} and z_{2t}, a final private consumption good x_t, labor ℓ_t, and government consumption g_t. The intermediate goods z_{1t} and z_{2t} in the real economy correspond to the final consumption goods c_{1t} and c_{2t} in the monetary economy. The period utility function is $U(x_t, \ell_t)$. The technology set for producing the final good x_t is given by

$$f^1(x_t, z_{1t}, z_{2t}, \ell_t) = w(z_{1t}, z_{2t}) - x_t \leq 0, \tag{5.14}$$

$$f^2(x_t, z_{1t}, z_{2t}, \ell_t) = w_2(z_{1t}, z_{2t}) - w_1(z_{1t}, z_{2t}) \leq 0, \tag{5.15}$$

while the technology for producing the intermediate goods and government consumption is given by

$$h(z_{1t}, z_{2t}, g_t, \ell_t) = z_{1t} + z_{2t} + g_t - \ell_t \leq 0. \tag{5.16}$$

The real economy and the monetary economy are obviously equivalent. The intermediate-goods result for the real economy is that the Ramsey allocations satisfy production efficiency. For this economy, because the marginal rate of transformation between z_1 and z_2 is one in the intermediate-goods technology, production efficiency requires that

$$\frac{w_1}{w_2} = 1. \tag{5.17}$$

Recall that in the monetary economy, the Friedman rule is optimal when (5.17) holds. Thus the intermediate-goods result in the real economy implies the optimality of the Friedman rule in the monetary economy.

One might wonder whether this implication holds more generally. Is it true that whenever the monetary economy can be reinterpreted as an intermediate-goods economy, the Friedman rule is optimal in the monetary economy? The answer is no. Suppose that the utility function $U(c_1, c_2, \ell)$ is of separable form $V(w(c_1, c_2), \ell)$, but that it does not have a representation in which w exhibits constant returns-to-scale. Suppose that w exhibits decreasing returns. For example, suppose that $w(c_1, c_2) = (c_1 + k)^\alpha c_2^{1-\alpha}$, where k is a constant. In the intermediate-goods reinterpretation, the constant k can be thought of as a scarce factor inelastically supplied by the household. The intermediate-goods result holds, provided that the returns to the scarce factor are fully taxed away. If it is not possible to tax the returns to the scarce factor, the intermediate-goods result does not hold. It is easy to show that the Friedman rule is not optimal in the monetary economy. In a sense, the Friedman rule is not optimal because in the monetary economy, there is no sensible interpretation under which it is possible to tax the parameter k.

Next, one might ask the following question. Is it true that whenever the Friedman rule is optimal in the monetary economy, there exists an analogous intermediate-goods economy? Again, the answer is no. Consider, for example, Ramsey allocation problems in which the constraint $U_1 \geq U_2$ binds, but in which the utility function is not separable. The Friedman rule is optimal, but the monetary economy cannot be reinterpreted as an intermediate-goods economy.

In Section 3, we showed that, under our homotheticity and separability assumptions, the optimality of the Friedman rule follows from the optimality of uniform commodity taxation. Here we showed that the optimality of the Friedman rule follows from the intermediate-goods result. There is no inconsistency between these findings because the uniform taxation result actually follows from the intermediate-goods result. We illustrate this claim in a simple example. Consider an economy with two consumption goods, c_1 and c_2, and labor. The utility function is $U(w(c_1, c_2), \ell)$, where w is homothetic, and the resource constraint is $c_1 + c_2 + g = \ell$. The uniform taxation result requires that the tax rate on c_1 and c_2 be the same, namely, $w_1/w_2 = 1$. Consider the following intermediate-goods reinterpretation of this economy. There are two intermediate goods z_1 and z_2 corresponding to c_1 and c_2 and there is a final good x. The intermediate goods produce output according to the constant returns-to-scale function $x = f(z_1, z_2)$. The intermediate goods are produced according to $z_1 + z_2 + g = \ell$. The utility function is given by $U(g(x), \ell)$, where g is a monotone increasing function and $g \circ f = w$. Such choices of g and f are possible since w is homothetic. Production efficiency requires that $f_1/f_2 = 1$, which implies that $w_1/w_2 = 1$. Clearly, this link between the uniform taxation result and the intermediate-goods result holds in general.

The construction of the intermediate-goods economy for the money-in-the-utility-function economy is straightforward. Recall that in the monetary economy, under our homotheticity and separability conditions, the period utility function is $U(w(m_t, c_t), \ell_t)$ and the resource constraint is

$$c_t + g_t = \ell_t. \tag{5.18}$$

The reinterpreted economy is again an infinite sequence of real static economies. In each period, there are two intermediate goods z_{1t} and z_{2t}, a final private consumption good x_t, labor ℓ_t, and government consumption g_t. The intermediate goods z_{1t} and z_{2t} correspond to money m_t and the consumption good c_t in the monetary economy. The technology set for producing the final good x_t is given by

$$f(x_t, z_{1t}, z_{2t}, \ell_t) = w(z_{1t}, z_{2t}) - x_t \leq 0.$$

The technology set for producing intermediate goods and consumption is given by

$$h(x_t, z_{1t}, z_{2t}, \ell_t) = z_{2t} + g_t - \ell_t \leq 0.$$

The real and monetary economies are obviously equivalent. Production efficiency in the intermediate-goods economy requires that the marginal rates of transformation between z_1 and z_2 in the two technologies be equated. Since the marginal rate of transformation between z_1 and z_2 in the intermediate-goods technology is zero ($h_2/h_3 = 0$), it follows that $w_1/w_2 = 0$. Thus, production efficiency in the intermediate-goods economy implies optimality of the Friedman rule in the monetary economy.

Finally, consider the shopping time model. Recall that in the monetary economy, the period utility function is $U(c_t, \ell_t + \phi(m_t, c_t))$ and the resource constraint is

$$c_t + g_t \leq \ell_t.$$

In the intermediate-goods economy, in each period there are two intermediate goods z_{1t} and z_{2t}, a final private consumption good x_t, labor ℓ_t, and government consumption g_t. There are three technologies. The final good x_t is produced using labor ℓ_{1t} and the second intermediate good z_{2t} according to

$$f(x_t, z_{1t}, z_{2t}, \ell_{1t}) = \min(\ell_{1t}, z_{2t}) - x_t \leq 0.$$

The technology for producing the intermediate goods is given by

$$h^1(x_t, z_{1t}, z_{2t}, \ell_{2t}) = \phi(z_{1t}, z_{2t}) - \ell_{2t} \leq 0.$$

The technology for producing government consumption is given by

$$h^2(x_t, z_{1t}, z_{2t}, \ell_{3t}) = g_t - \ell_{3t} \leq 0.$$

The period utility function is $U(x_t, \ell_{1t} + \ell_{2t} + \ell_{3t})$.

The mapping between the real and monetary economies is more subtle in this case and was devised by Correia and Teles (1994). The first intermediate good z_{1t} corresponds to money. The second intermediate good, which has no direct counterpart in the monetary economy, can be thought of as transactions services produced using labor and money which are required for the purchase of the final consumption good. Here production efficiency requires that the marginal rates of transformation between z_1 and z_2 in the technologies f and h^1 be equated, and hence $\phi_1 = 0$. Thus production efficiency in the intermediate-goods economy implies the optimality of the Friedman rule in the monetary economy.

6. Conclusion

In this paper, we analyzed the optimality of the Friedman rule in three monetary models with distorting taxes. In all three models, the Friedman rule is optimal if preferences satisfy homotheticity and separability conditions similar to those in the literature on uniform commodity taxation. We showed that there is no obvious connection between the optimality of the Friedman rule and the interest elasticity of money demand. We also showed that there is a close connection between the optimality of the Friedman rule and the intermediate-goods result when our conditions hold and that there is no connection when our conditions do not hold.

References

Atkinson, A.B. and J.E. Stiglitz, 1972, The structure of indirect taxation and economic efficiency, Journal of Public Economics 1, 97–119.

Chari, V.V., L.J. Christiano, and P.J. Kehoe, 1991, Optimal fiscal and monetary policy: Some recent results, Journal of Money, Credit, and Banking 23, 519–539.

Chari, V.V., L.J. Christiano, and P.J. Kehoe, 1993, Optimality of the Friedman rule in economies with distorting taxes, Research Department staff report 158 (Federal Reserve Bank of Minneapolis, Minneapolis, MN).

Correia, I. and P. Teles, 1994, Money as an intermediate good and the welfare cost of the inflation tax, Manuscript (Bank of Portugal, Lisbon).

Diamond, P.A. and J.A. Mirrlees, 1971a, Optimal taxation and public production I: Production efficiency, American Economic Review 61, 8–27.

Diamond, P.A. and J.A. Mirrless, 1971b, Optimal taxation and public production II: Tax rules, American Economic Review 61, 261–268.

Faig, M., 1988, Characterization of the optimal tax on money when it functions as a medium of exchange, Journal of Monetary Economics 22, 137–148.

Friedman, M., 1969, The optimum quantity of money, in: The optimum quantity of money and other essays (Aldine, Chicago IL) 1–50.

Guidotti, P.E. and C.A. Vegh, 1993, The optimal inflation tax when money reduces transactions costs: A reconsideration, Journal of Monetary Economics 31, 189–205.

Kimbrough, K.P., 1986a, The optimum quantity of money rule in the theory of public finance, Journal of Monetary Economics 18, 277–284.

Kimbrough, K.P., 1986b, Inflation, employment, and welfare in the presence of transactions costs, Journal of Money, Credit, and Banking 18, 127–140.

Lucas, R.E., Jr. and N.L. Stokey, 1983, Optimal fiscal and monetary policy in an economy without capital, Journal of Monetary Economics 12, 55–93.

Phelps, E.S., 1973, Inflation in the theory of public finance, Swedish Journal of Economics 75, 67–82.

Woodford, M., 1990, The optimum quantity of money, in: B.M. Friedman and F.H. Hahn, eds., Handbook of monetary economics, Vol. 2 (North-Holland, Amsterdam) 1067–1152.