Sustainable Policy Outcome with Coordinated Reforms, by Anthony Chung

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Februrary 14, 2005

Motivation

- All the papers on policy games consider the case of "one shot deviation" by government(sustainable equilibria)

- This paper considers possibility of multi-period deviation by government with its successors (renegotiation stable equilibria)

- The conjecture is that the renegotiation stable equilibrium set is smaller than set of sustainable equilibrium (which are generally large)

Motivation

- Usually: government has some power to coordinate households' policy expectations at time 0.

- But then government can affect policy expectations in any period, with similar policy announcement

- In this case bad sustainable equilibria are likely to be "renegotiated"

- Hence punishment with worst equilibrium is not credible and the whole set of equilibria shrinks

Simple 2-person game example

- Consider repeated game with following stage game

		Firm 2		
		L	M	H
	L	$10, 10$	$5, 14$	$0, 6$
Firm 1	M	$14, 5$	$7, 7$	$-5, -2$
	H	$6, 0$	$-2, -5$	$-15, -15$

- Let discount rate be $\delta = 1/2$

Simple 2-person game example

- Abreu(1988) showed that best equilibrium of (L,L) can be supported with the following punishments:

  $$\underline{Q}^1 = \{(M,H),(L,M),(L,M),...\} = \{(-5,-2),(5,14),(5,14),...\}$$

  $$\underline{Q}^2 = \{(H,M),(M,L),(M,L),...\} = \{(-2,-5),(14,5),(14,5),...\}$$

- These are worst equilibria for respective players, with payoffs (0,6) and (6,0) respectively.

Simple 2-person game example

- With renegotiation , $\{\underline{Q}^1, \underline{Q}^2\}$ cannot serve as a punishment

- So with renegotiation, (L,L) equilibrium cannot be supported (with $\delta = 1/2$)

Chung's setup

- Exactly replicates Phelan&Stacchetti (2001)

- In addition after every history government can contemplate "a reform" to a new sustainable equilibrium

- Households believe a promise to change the policy, if it is "credible"

- New policy is "credible" if government is strictly better off (compared to current equilibrium) at every history node that requires a change of strategy.

Environment

- There is a continuum of households with preferences

$$\sum_{t=0}^{\infty} \beta^t [u(c_t, l_t) + g(G_t)]$$

- There is a production technology $f(K, L)$

- Capital fully depreciates

- There is an i.i.d sunspot ,x_t that is realized at the beginning of each period. x_t, is uniformly distributed on $[0, 1]$. x^t denotes history.

Environment

- Government taxes capital and labor ($\tau_{k,t}$ and $\tau_{l,t}$) to finance G_t

- Government balances the budget every period

- Government has no commitment

Timing

1. x_t is realized and observed by everyone

2. Government sets taxes

3. Households choose c_t, l_t and k_{t+1}

Competitive Equilibrium

Definition 1 *Competitive equilibrium is*

- *Allocations* $q = \{(c_t, l_t, k_{t+1})(x^t)\}_{t=0}^{\infty}$,
- *Price system* $p = \{(w_t, r_t)(x^t)\}_{t=0}^{\infty}$,
- *tax rates* $\tau = \{\tau_t(x^t)\}_{t=0}^{\infty}$,
- *public expenditure* $G = \{G_t(x^t)\}_{t=0}^{\infty}$, *such that*

Competitive equilibrium

1. *Given* p*,* τ *and* G*, allocation maximizes household objective subject to budget constraint*

$$c_t(x^t) + k_{t+1}(x^t) = (1 - \tau_{k,t}) r_t(x^t) k_t(x^{t-1}) + (1 - \tau_{l,t}) w_t(x^t) l(x^t)$$

2. *Prices are competitive*

$$r_t(x^t) = f_k(k_t(x^{t-1}), l(x^t)), r_t(x^t) = f_l(k_t(x^{t-1}), l(x^t))$$

3. *Government's budget is balanced*

$$G_t(x^t) = \tau_{k,t}(x^t) r_t(x^t) k_t(x^{t-1}) + \tau_{l,t}(x^t) w_t(x^t) l_t(x^t)$$

Recursive Structure

- Let

  $m_{t+1} = E_{x_{t+1}}\left[(1-\tau_{k,t+1}(x_{t+1}))f_k(k_{t+1}(x^t, l_{t+1}(x^{t+1}))u_c(c_{t+1}(x^{t+1}), l_{t+1}(x^{t+1}))|x^t\right]$

- Sequence of households with preferences

$$u(c_t(x^t), l_t(x^t)) + \beta m_{t+1}(x^t) k_{t+1}(x^t)$$

- Given, a policy and initial capital stock, $q = \{(c_t, l_t, k_{t+1})(x^t)\}_{t=0}^{\infty}$ is comptetitive equilibrium allocation iff

$$(c_t, l_t, k_{t+1})(x^t) \in CE((k_t)(x^{t-1}), \tau_t(x^t), m_{t+1}(x^t)) \quad \forall t, x^t$$

The Economy as a Dynamic Game

- Let $\Gamma(k_0)$ be the dynamic game and $\Gamma(k_0, x_0)$ be the subgame after realization of x_0

- Date t action of the government is $\tau_t = (\tau_{k,t}, \tau_{l,t}) \in [0, \bar{\tau}]^2$

- Income of the household is $y_t = (1 - \tau_{k,t}) r_t k_t + (1 - \tau_{l,t}) w_t l_t$

- Date t action of household is $a_t = (l_t, \theta_t) \in [0, 1]^2$

- $c_t = (1 - \theta_t) y_t$ and $k_{t+1} = \theta_t y_t$

The Economy as a Dynamic Game

- A public history is a triplet $h^t = (x^t, \tau^t, a^t) \in H^t$

- Strategy of the government

$$\{\sigma_G(t)\}_{t=0}^{\infty} \in \Sigma_G \text{ with } \tau_t = \sigma_G(t)(h^{t-1}, x_t)$$

- Strategy for the household

$$\{\sigma_C(t)\}_{t=0}^{\infty} \in \Sigma_C \text{ with } a_t = \sigma_C(t)(h^{t-1}, x_t, \tau_t)$$

The Economy as a Dynamic Game

- Every $\sigma = (\sigma_G, \sigma_C) \in \Sigma$ induces a continuation strategy profile $\sigma|_{h^{t-1}} \in \Sigma$ after each $h^{t-1} \in H^{t-1}$

$$
\begin{aligned}
\sigma_G|_{h^{t-1}}(s)(\widehat{h}^{s-1}, \widehat{x}_s) &= \sigma_G(t+s)(h^{t-1}, \widehat{h}^{s-1}, \widehat{x}_s) \\
\sigma_G|_{h^{t-1}}(s)(\widehat{h}^{s-1}, \widehat{x}_s, \widehat{\tau}_s) &= \sigma_G(t+s)(h^{t-1}, \widehat{h}^{s-1}, \widehat{x}_s, \widehat{\tau}_s) \\
\sigma_G|_{h^{t-1}}(s)(\widehat{h}^{s-1}, \widehat{x}_s, \widehat{\tau}_s, \widehat{a}_s) &= \sigma_G(t+s)(h^{t-1}, \widehat{h}^{s-1}, \widehat{x}_s, \widehat{\tau}_s, \widehat{a}_s))
\end{aligned}
$$

for all $s \geq 0$, $\widehat{h}^{s-1} \in H^{s-1}, \widehat{x}_s \in [0,1], \widehat{\tau}_s \in [0,\bar{\tau}]^2$, and $\widehat{a}_s \in [0,1]$

The Economy as a Dynamic Game

- Let $\Phi(k_0, \sigma) = (\Phi_C(k_0, \sigma), \Phi_G(k_0, \sigma))$be the value correspondence

$$\begin{aligned}
\Phi_C(k_0, \sigma) &= E\left[(1 - \tau_{k,0}(x_0)) f_k(k_0, l_0(x_0)) u_c(c_0(x_0), l_0(x_0))\right] \\
\Phi_G(k_0, \sigma) &= (1 - \beta)\left[\sum_{t=0}^{\infty} \beta^t [u(c_t, l_t) + g(G_t)]\right]
\end{aligned}$$

Sustainable Equilibrium

Definition 2 $\sigma \in \Sigma$ *is a Sustainable Equilibrium for* $\Gamma(k_0)$ *iff* $\forall$ $t \geq 0$, *history* $h^{t-1} \in H^{t-1}$ *with capital* k_t. *and* $x_t \in [0,1]$

(i) $\forall \hat{\tau}_t \in [\underline{\tau}, \overline{\tau}]^2$

$$\Phi_G(k_t, \sigma|_{(h^{t-1}, x_t)}) \geq \Phi_G(k_t, \sigma|_{(h^{t-1}, x_t, \hat{\tau}_t)})$$

(ii) *The outcome path and allocation generated by* $\sigma|_{(h^{t-1}, x_t)}$ *is a CE after all* $\hat{\tau}_t \in [\underline{\tau}, \overline{\tau}]^2$ *for economy with initial capital* k_t

Sustainable Equilibrium

- At each date and after each history, government does not have incentive for any **idiosyncratic** deviation.

- The above is subject to the restriction that every continuation of Sustainable Equilibrium is itself a Sustainable Equilibrium.

Sequential Dominance

Definition 3 *A strategy profile $\widehat{\sigma}$ sequentially dominates another strategy profile σ ($\widehat{\sigma} \succ \sigma$), iff*

whenever

$$\widehat{\sigma}_G(t)(h^{t-1}, x_t) \neq \sigma_G(t)(h^{t-1}, x_t)$$

we have

$$\Phi_G(k_t, \widehat{\sigma}|_{(h^{t-1}, x_t)}) > \Phi_G(k_t, \sigma|_{(h^{t-1}, x_t)})$$

for all $t \geq 0$, history $h^{t-1} \in H^{t-1}$ with corresponding capital k_t, and $x_t \in [0, 1]$,

Renegotiation-Stable Equilibrium

Definition 4 *$\sigma \in \Sigma$ is a Renegotiation-Stable Equilibrium for $\Gamma(k_0)$ iff*

(i) σ is a Sustainable Equilibrium for $\Gamma(k_0)$

(ii) $\forall h^{t-1}$ and $\sigma|_{h^{t-1}}$, $\nexists$ an alternative Sustainable Equilibrium $\hat{\sigma}$ such that

$$\hat{\sigma} \neq \sigma|_{h^{t-1}} \text{ and } \hat{\sigma} \succ \sigma|_{h^{t-1}}$$

Renegotiation-Stable Equilibrium

- A RSE is a Sustainable Equilibrium that can not be negotiated away by any systematic enforceable deviation

- The RSE policy if adopted and expected by rational expectation, will be followed by all future optimizing governments

Recursive Characterization of Equilibrium Correspondence

- The goal is to characterize the following object

  $V^{RSE}(k_0) = \{\Phi(k_0, \sigma) | \sigma$ is renegotiation-stable equilibrium of $\Gamma(k_0)\}$

- We do it using the methodology of APS(1990) and Atkeson(1991) that is developed by Phelan and Stacchetti(2001) to apply to the model that we consider here

Recursive Characterization of Equilibrium Correspondence

- By definition we have

$$V^{RSE} \subseteq V^{SE} \subseteq V^{CE}$$

- The first task is to refine V^{CE} using the recursive procedure of Phelan and Stacchetti(2001) to get V^{SE}

- Next we use an iterative procedure to refine V^{SE}

- Every step involves solving a *DP* with incentive constraint to find *a lower bound* of RSE payoffs for government

Recursive Characterization of Equilibrium Correspondence

Definition 5 *A compact and convex valued* $W : [\underline{k}, \overline{k}] \to \mathbb{R}^2_+$ *is a value correspondence. Given* $W(.)$ *Let*

- $\overline{W}(k, m) = \max_v \{v | (m, v) \in W(k)\}$

- $\underline{W}(k, m) = \min_v \{v | (m, v) \in W(k)\}$

- $\overline{W}(k) = \max_m \overline{W}(k, m)$

- $\underline{W}(k) = \min_m \underline{W}(k, m)$

Recursive Characterization of Equilibrium Correspondence

Definition 6 *Let $\mathcal{B}(.)$ be a map on value correspondences. A value correspondence W is self-generating w.r.t $\mathcal{B}(.)$ iff*

$$W \subseteq \mathcal{B}(W)$$

Recursive Characterization of Equilibrium Correspondence

Definition 7 *A vector* $\xi = (\tau, l, c, k_+, m_+, v_+)$ *is consistent w.r.t* W *at* $k \in [\underline{k}, \overline{k}]$ *if*

$$(l, c, k_+) \in CE(k, \tau, m_+), \quad k_+ \in [\underline{k}, \overline{k}] \text{ and } (m_+, v_+) \in W(k_+)$$

also a value correspondence for ξ *is the pair* $(\Psi_C(k, \xi), \Psi_G(k, \xi))$

$$\begin{aligned} \Psi_G(k, \xi) &= (1 - \beta)[u(c, l) + g(\tau_k f_k(k, l)k + \tau_l f_l(k, l)l)] + \beta v_+ \\ \Psi_C(k, \xi) &= (1 - \tau_k) f_k(k, l) u_c(c, l) \end{aligned}$$

Recursive Characterization of Equilibrium Correspondence

Definition 8 *Given W, for all $k \in [\underline{k}, \overline{k}]$, define B_{CE}*

$$B_{CE}(W)(k) = co(\{\Psi(k, \xi)|\xi \text{ is consistent w.r.t } W \text{ at } k\})$$

Recursive Characterization of Equilibrium Correspondence

- The idea is to find CE allocations with continuation values in W

- Note, there is no idea of sequential rationality here, yet

- It is easy to show that B_{CE} is monotone and preserves compactness

Recursive Characterization of Equilibrium Correspondence

Proposition 1 *Let W we a value correspondence*

(i) $W \subseteq B_{CE}(W) \Rightarrow B_{CE}(W) \subseteq V^{CE}$

(ii) V^{CE} *is the largest correspondence* W *such that* $W = B_{CE}(W)$

(iii) Let $B_{CE}(W_0) \subseteq W_0$ *and* $V^{CE} \subseteq W_0$*. Then*

$$V^{CE} = \lim_{n \to \infty} W_n$$

with

$$W_{n+1} = B_{CE}(W_n) \quad \forall n \geq 0$$

Recursive Characterization of Equilibrium Correspondence

Definition 9 *For all* $k \in [\underline{k}, \overline{k}]$*, and* $\tau' \in [\underline{\tau}, \overline{\tau}]^2$*, define the government's worst punishment*

$$\pi_W(k, \tau') = \min_{(c', l', k'_+, m'_+, v'_+)} \Psi(k, \xi') \text{ subject to}$$

$$\xi' = (\tau', c', l', k'_+, m'_+, v'_+) \text{ is consistent w.r.t } W \text{ at } k$$

Recursive Characterization of Equilibrium Correspondence

Definition 10 ξ *is admissible w.r.t* W *at* k *iff*

(i) it is consistent w.r.t W *at* k

(ii) $\Psi_G(k, \xi) \geq \overline{\pi}_W(k)$

where

$$\overline{\pi}_W(k) = \max_{\tau' \in [\underline{\tau}, \overline{\tau}]^2} \pi_W(k, \tau')$$

is the payoff of the best deviation for government (at k*)*

Recursive Characterization of Equilibrium Correspondence

- Define

$$B_{SE}(W)(k) = co(\{\Psi(k,\xi)|\xi \text{ is admissile w.r.t W at } k\})$$

- B_{SE} is monotone and preserves compactness

Recursive Characterization of Equilibrium Correspondence

Proposition 2 *Let* W *we a value correspondence*

(i) $W \subseteq B_{SE}(W) \Rightarrow B_{SE}(W) \subseteq V^{SE}$

(ii) V^{SE} *is the largest correspondence* W *such that* $W = B_{SE}(W)$

(iii) Let $B_{SE}(W_0) \subseteq W_0$ *and* $V^{SE} \subseteq W_0$*. Then*

$$V^{SE} = \lim_{n\to\infty} W_n$$

with

$$W_{n+1} = B_{SE}(W_n) \quad \forall n \geq 0$$

Value Correspondence for RSE

- The road map is the following: first we introduce the *cautious renegotiations*

- Renegotiations is cautious if at each date, household expect that government re-optimize again next period

- We argue that it is sufficient to consider only the cautious renegotiations

- We argue that payoff associated with the best cautious renegotiation can serve as lower bound of payoffs associates with RSE

Value Correspondence for RSE

- Fix arbitrary $\sigma \in \Sigma$ in $\Gamma(k)$ and given history of x^t define

$$(\mathcal{T}_t, \mathcal{C}_t, \mathcal{L}_t, \mathcal{M}_t, \mathcal{G}_t, \mathcal{K}_{t+1})(\sigma)(x^t)$$

  to be date t allocation induced by σ with $\mathcal{K}_0(\hat{\sigma}) = k$.

- Let $\alpha^t(\sigma)(x^t)$ be time t history induced by σ

Value Correspondence for RSE

Definition 11 *Given $\sigma, \widehat{\sigma} \in \Sigma$ for $\Gamma(k)$, a strategy profile $\widehat{\sigma}$ sequentially dominates another strategy σ at time 0 ($\widehat{\sigma} \succ^{0} \sigma$) iff*

$$\widehat{\sigma} \succ \sigma \text{ and } \widehat{\sigma}_G(0) \neq \sigma_G(0)$$

- *Note that $\widehat{\sigma} \succ^{0} \sigma \Rightarrow \widehat{\sigma} \succ \sigma$*

- *if $\widehat{\sigma} \succ \sigma$ then $exists$ a history h^t such that $\widehat{\sigma}|_{h^t} \succ^{0} \sigma|_{h^t}$*

Value Correspondence for RSE

Definition 12 *A strategy profile $\sigma \in \Sigma^{SE}$ is cautious given V^{SE} iff*

$$\mathcal{M}_t(\sigma)(x^t) \in argmax_m \overline{V^{SE}}(\mathcal{K}_t(\sigma)(x^t), m), \forall x^t, t \geq 0$$

Value Correspondence for RSE

- Government, given set V, irrespective of promises picks a best response in V

- Households pick investment expecting government to re-optimize

- Main Idea - if value of sustainable equilibrium is less than value of cautious strategy - government will be better off to propose a reform.

- Cautious strategies are enough to characterize the whole value set if for every (v, m) in $V(k)$ that can be renegotiated there is sustainable equilibrium, renegotiable with cautious strategy.

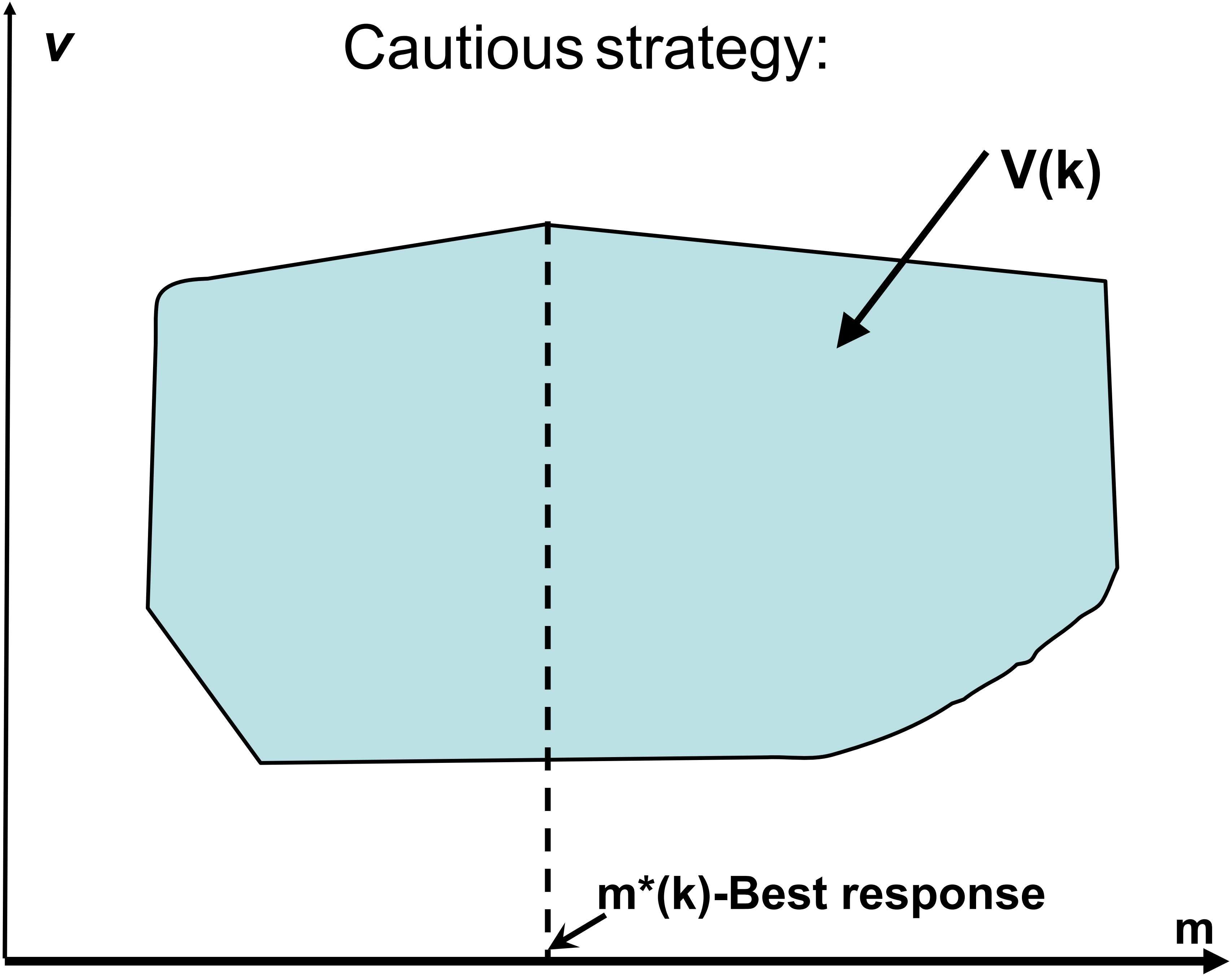
Value Correspondence for RSE

- **Claim:** Given any $(m, v) \in V^{SE}(k), k \in [\underline{k}, \overline{k}]$ there exits $\sigma \in \Sigma^{SE}$ with $\Phi(k, \sigma) = (m, v)$ such that if σ can be renegotiated by another $\hat{\sigma} \in \Sigma^{SE}$, then continuation of $\hat{\sigma}$ is cautious w.r.t V^{SE}

- Let $\overline{m}(k) = \{argmax_m \overline{V^{SE}}(k, m)\}$

- **Claim:** $\overline{m}(k)$ is a function

Cautious strategy:



Idea of the proof (of sufficiency of cautious strategies)

- Construct the sustainable equilibrium such that every renegotiation is renegotiation is necessarily cautious.

- Idea: any renegotiation $\widehat{\sigma} \succ^0 \sigma, \;\; \widehat{\sigma} \neq \sigma$ prescribes an out-of-equilibrium path.

- So manipulating out-of-equilibrium beliefs of σ we can make sure that any renegotiation is cautious.

- All we have to make sure that if $\widehat{\sigma}|_{\alpha^t(\widehat{\sigma})}$ is not cautious - for a history $\widehat{\sigma}|_{\alpha^t(\widehat{\sigma})}$ payoff of σ is higher

Value Correspondence for RSE

- Define the best optimal cautious strategy with path q by

$$\overline{S}(k_0) = max_q(1-\beta)\sum_{t=0}^{\infty}\beta^t[u(c_t, l_t) + g(G_t)]$$

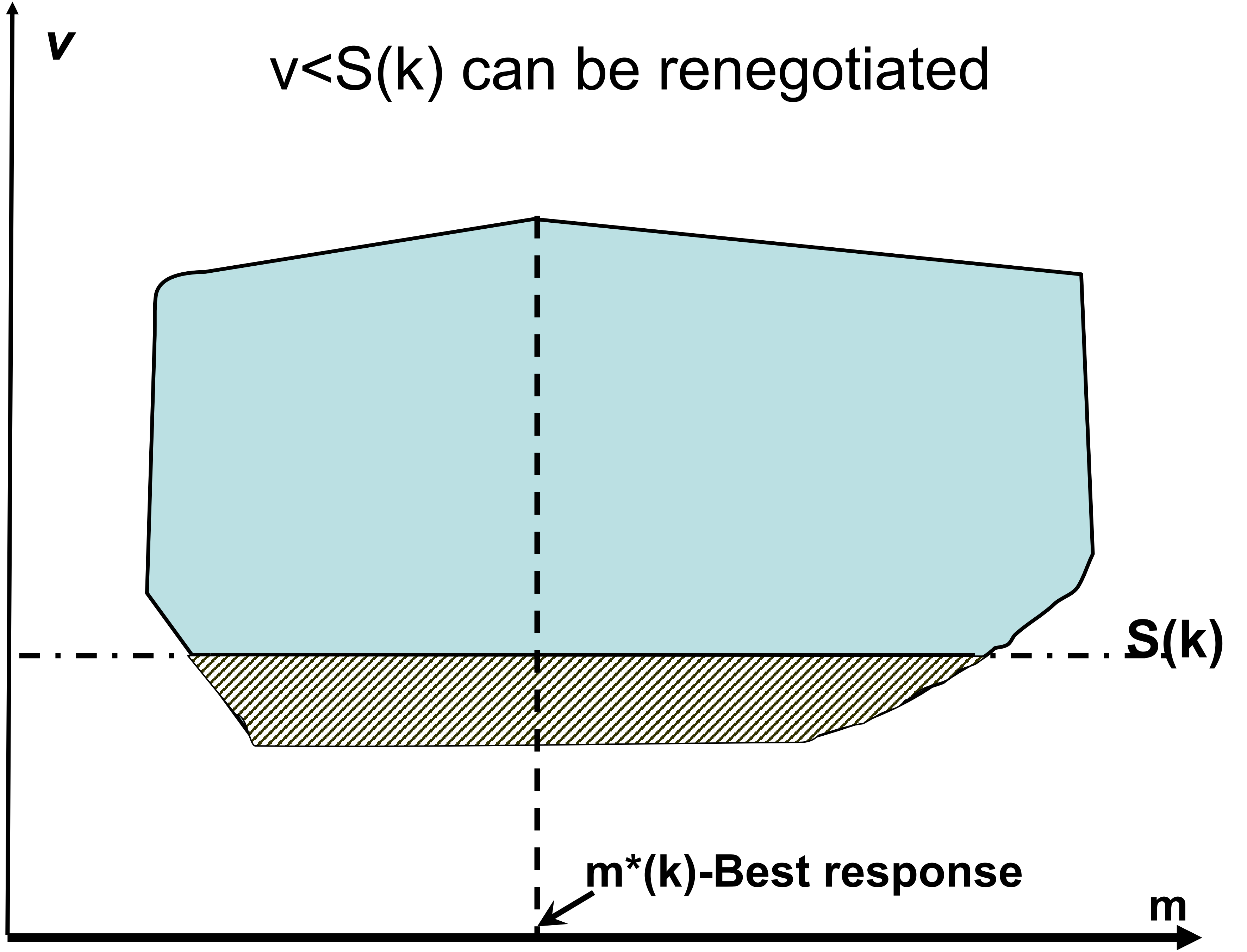
subject to

$$(1-\tau_{k,t})f_{k,t}u_{c,t} = \overline{m}(k_t)$$
$$\{u_{l,t} + (1-\tau_{l,t}f_{l,t}\}(1-l_t)) = 0$$
$$u_{c,t} = \beta\overline{m}(k_{t+1})$$
$$c_t + k_{t+1} = (1-\tau_{k,t})f_{k,t}k_t + (1-\tau_{l,t}f_{l,t}l_t$$
$$c_t + k_{t+1} + G_t = f_t$$
$$V_t(q) \geq \underline{V}^{SE}(k_t)$$

Value Correspondence for RSE

Proposition 3 *Given* $k \in [\underline{k}, \overline{k}]$ *and* $(m, v) \in V^{SE}(k)$*, if* $v < \overline{S}(k)$*, then for every* $\sigma \in \Sigma^{SE}$ *with* $\Phi_G(k, \sigma) = v$*, there exists* $\hat{\sigma} \neq \sigma$ *(in* Σ^{SE}*) such that* $\hat{\sigma} \succ \sigma$

- Also it is easy to show that if $(m, v) \in V^{SE}(k)$ with $v \geq \overline{S}(k)$ then $\exists\ \sigma$ such that $\Phi(k, \sigma) = (m, v)$ and σ is not sequentially dominated by any V^{SE}-cautious strategy v<S(k) can be renegotiated



Value Correspondence for RSE

Definition 13 V *is a candidate equilibrium correspondence if*

1. $V \subseteq B_{SE}(V)$

2. $\forall \sigma \in \Sigma^{SE}$, $k \in [\underline{k}, \overline{k}]$,

$$\Phi(k, \sigma) \in V(k)$$

implies $\exists \hat{\sigma} \in \Sigma^{SE}$ *such that* $\hat{\sigma} \neq \sigma|_{h^t}$ *and* $\hat{\sigma} \succ \sigma|_{h^t}$ *for some* h^t

3. *Given* $k \in [\underline{k}, \overline{k}]$ *and* $(m, v) \in V^{SE}(k)$, $\exists \sigma \in \Sigma^V$ *with* $\Phi(k, \sigma) = (m, v)$ *such that can not be sequentially dominated by any* $\hat{\sigma} \in \Sigma^{SE}$ *with* $\Phi(k_t, \hat{\sigma}) \notin V(k_t)$ *for al* h^{t-1} *with corresponding* k_t

Value Correspondence for RSE

- for $W \subseteq V$ Let

$$\eta_W(k) = \max\{\overline{\pi}_W(k), \overline{S}_V(k)\}$$

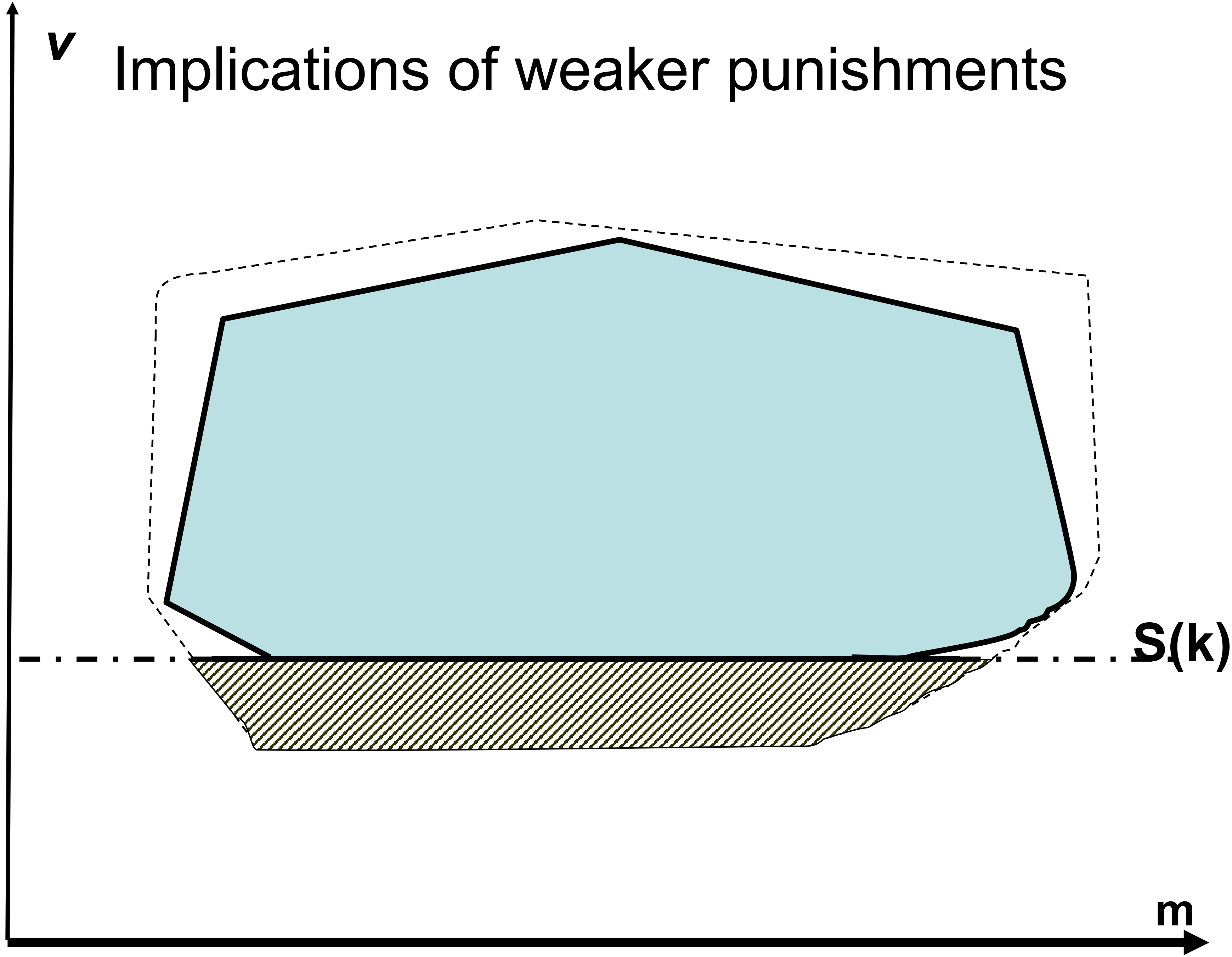
- We say ξ is V-cautious w.r.t W if it is consistent w.r.t W and

$$\Psi_G(k, \xi) \geq \eta_W(k)$$

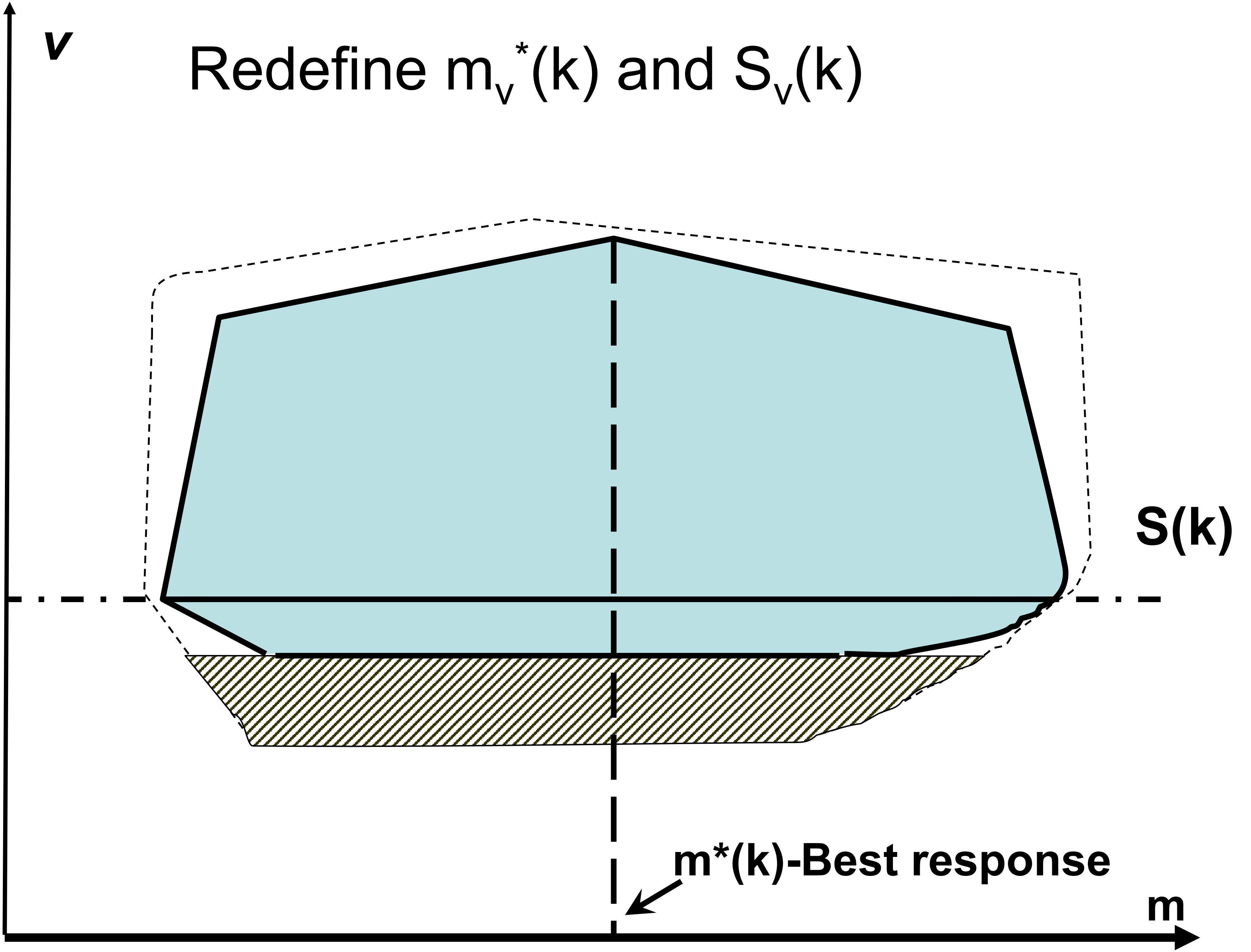
- Define

$$B_V(W)(k) = co(\{\Psi(k, \xi) | \xi \text{ is } V\text{-cautious w.r.t } W \text{ at } k\})$$

Implications of weaker punishments



Redefine $m_v^*(k)$ and $S_v(k)$



Value Correspondence for RSE

Proposition 5

$$\hat{B}^{\infty} = \lim_{n \to \infty} \hat{B}^{n}(V^{SE}) \neq \emptyset$$

Proposition 6

$$V^{RSE} = \hat{B}^{\infty}(V^{SE})$$

Application of RSE concept to simple dynamic game

- Consider Chari-Kehoe "Sustainable plans"
  - no physical connection between periods
  - one shot game Nash equilibrium has no investment
- Dynamic game between government and competitive households
- Worst sustainable equilibrium - government ignores all promises and plays a one-shot best response.

Application of RSE concept to simple dynamic game

- Hence Worst Sustainable equilibrium is the same as "Cautious strategy" equilibrium in Chung(2004) paper

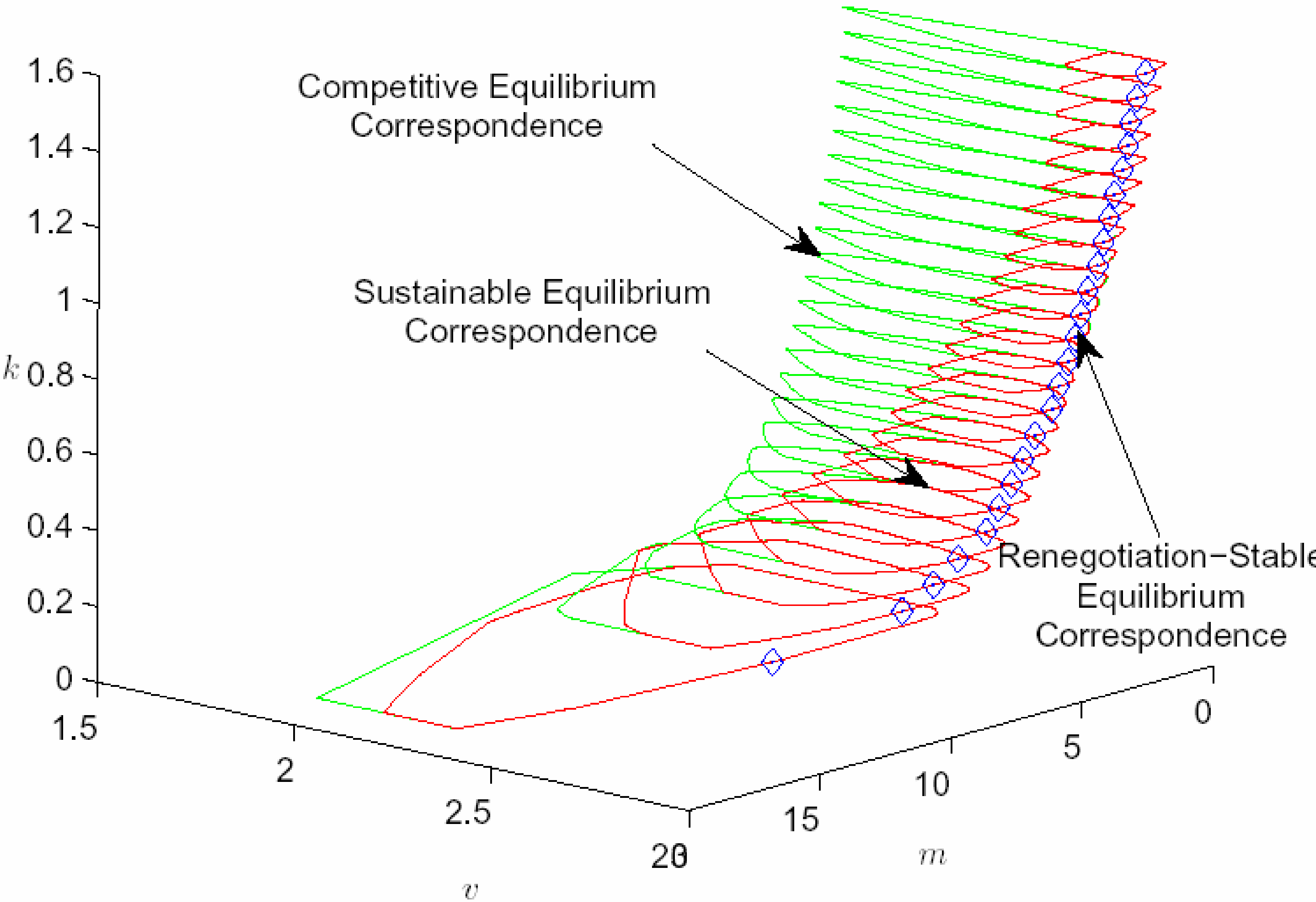
- Allowing renegotiation does not shrink equilibrium set in Chari-Kehoe

Value Correspondence for RSE

Proposition 4 *If* V *is nonempty candidate equilibrium correspondence, then*

$$V \supseteq \lim_{n\to\infty} B_V^n(V) \neq \emptyset$$

and $\hat{B}(V) = \lim_{n\to\infty} B_V(V)$ *is a candidate equilibrium correspondence*



Competitive Equilibrium
Correspondence
Sustainable Equilibrium
Correspondence
Renegotiation−Stable
Equilibrium
Correspondence
k
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
v
1.5
2
2.5
m
0
5
10
15
20

Discussion

- When best V^{SE}-cautious strategy is strictly better than the worst sustainable equilibrium in Phelan and Stacchetti(2001), than it should not be possible to support some of the best equilibria, since punishments below V^{SE}-cautious strategy are not credible.

- That is applying $\widehat{B}(V^{SE})(k)$ correspondence to V^{SE} should not only eliminate $v < \overline{S}(k)$, but also make some of the best equilibria unsustainable. (unless they can be supported by reversion to static best response)

- This is clearly not the case on picture