

# Pride and Diversity in Social Economies<sup>1</sup>

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## **Abstract**

We study a two-period economy in which agents' preferences take into account relative economic position. The study builds on a decision theoretic analysis of the social emotions that underlie these concerns, i.e., envy and pride, which respond to social losses and gains, respectively. The analysis allows individual differences in their relative importance and summarizes these differences in the geometric properties of the externality function that represents relative outcome concerns.

Our main result is that envy leads to conformism in consumption behavior and pride to diversity. We thus establish a link between emotions that are object of study in psychology and neuroscience, and important features of economic variables, in the first place the equilibrium distribution of consumption and income. This research provides a tool to relate experimental and empirical studies of individual preferences for relative position and important features of macro data.

# 1 Introduction

Empirical and experimental literature has established the importance in individual choices of relative outcome concerns, especially in consumption and income. External habits, keeping up with the Joneses and other regarding preferences are common names used for this phenomenon. Early classic contributions are Veblen (1899) and Dusenberry (1949); the latter in particular is an early attempt to provide explanations of aggregate economic behavior on the basis of individual preferences over relative position. Recent empirical works show direct effects on well being of individuals, as measured by happiness indicators, of relative income (see, for example, Luttmer, 2005, and Dynan and Ravina, 2007). Other works explain economic behavior as motivated in part by concerns for relative position. For example, Charles, Hurst and Roussanov (2009) shows that visible consumption in luxury goods is well explained by status seeking (and that the level of consumption is declining in the income of the reference group). These studies use data sets and evidence of different nature, some experimental and some empirical. Though at the moment there is no wide agreement on the properties and strength of these features, these studies indicate that the effects are significant.

Our paper studies at a theoretical level how different shapes of individual preferences of agents with these social concerns affect the nature of the equilibrium and in particular the degree of inequality in the economy. Our analysis is based on the decision theoretic analysis of social preferences pursued in Maccheroni, Marinacci, and Rustichini (2012). They provide conditions on observed choices that deliver objective functions in which, to a standard utility term that depends on agents' own outcomes, is added a positional index that quantifies agents' relative outcome concerns that arise by comparing their own outcomes with those of their peers. The positional index introduces a social dimension in agents' objective functions and establishes a direct link (in addition to prices) among their choices as agents' well being now also depends on their peers' choices and outcomes. Since the foundation of the representation is choice based, the hypotheses underlying the representation are testable. Even more important for the purpose of the current paper, these features of individual preferences are reduced to few simple factors, which admit simple parametric representations that can be used in the theoretical analysis of equilibrium behavior and thus provide a foundation for calibration analysis.

Specifically, Maccheroni et al. (2012) identify two basic social emotions that, through agents' attitudes toward them, determine the shape of the positional index, that is, envy and pride. By envy we mean the negative emotion that agents experience when their outcomes fall below those of their peers, and by pride the positive emotion that they experience when they have better outcomes than their peers. Envy and pride can be viewed as the emotions that arise when agents experience, respectively, a social loss and a social gain. Like attitudes toward standard private losses and gains, also these social attitudes may well vary across individuals. As detailed in the next section, the shape of the positional index reflects these different attitudes, thus making it possible to carry out comparative statics analysis in agents' attitudes toward envy and pride.

**Two-periods economy** Our investigation of the economic consequences of these social preferences is based on an economy of agents who live two periods and produce, consume, and save for their own future consumption a single consumption good. With traditional (here called for convenience “asocial”) preferences these agents would be isolated and the economy would be in equilibrium once they solve their individual intertemporal problems. With social preferences, however, this is no longer the case: even though their choices are still independent – that is, their material outcomes are not affected by peers’ choices – now agents’ well being also depends on their peers’ outcomes. In this economy agents are thus linked only via their relative outcome concerns. This makes this setup especially well suited for our purposes since it allows to study in “purity” the equilibrium consequences of these relative concerns, with no room left for other possible interdependencies among agents’ actions that may affect the analysis. The implications of status on intertemporal choices are explored in Xia (2010) and Ray and Robson (2012). In particular, Xia shows that the inequality in initial wealth distribution affects savings and consumption, and the search for status affects these choices. In our model, agents are ex-ante identical and, in spite of this, heterogeneity in behavior is induced at equilibrium.

Our main finding is that in these social economies envy leads to conformism in equilibrium, pride to diversity. Specifically, suppose that agents are identical and that their labor supply is inelastic. In this case the choice problem they face is to select consumption in each of the two periods they live. For, they can save in the first period and store what saved for consumption in the next period. When deciding how much to consume in the first period, agents face a trade-off: if they increase consumption today they will increase their relative ranking today, but, *ceteris paribus*, also decrease their standing in the next period. They thus compare a positive effect today with a possible negative effect in the next period. This intertemporal trade-off (also noted, for example, in Binder and Pesaran, 2001, and Arrow and Dasgupta, 2009) points to a crucial feature of the preferences: the attitudes they exhibit toward social gains and losses, that is, the relative strength of the effect on individuals’ welfare of being either in a dominant or in a dominated position in their reference group.

How these social attitudes affect choices, and the solution of the intertemporal trade-offs, depends on whether or not agents are more envious than proud. The equilibrium set will be completely different in the two cases: it will be conformist when envy prevails (all agents consume the same) and diversified when, instead, pride prevails (identical agents choose a different consumption). Envious agents care more about the situation in which their consumption is below the average value. The positional index has in this case a concave kink at the origin that turns out to force equilibria to be symmetric: all agents choose the same consumption. A kink at zero, that is, at the reference point in the space of social gains and losses, is consistent with the view originating in Prospect Theory that a change in sign induces a change in marginal evaluations.

In contrast, proud agents have a positional index with a convex kink at the origin, and this feature changes completely the structure of the equilibrium set. This local convexity turns out to be enough to make all equilibria asymmetric: although agents are identical, they will choose different consumptions. Some will choose to have a dominant position in

the current period, at the expense of a dominated one in the future, and others will choose the opposite.

Since all agents are identical, the asymmetry in behavior caused by pride is noteworthy. This asymmetry only arises out of social concerns, not because of any need of the agents to equilibrate their actions in terms of overall available resources (in fact, in this economy there is no trade). The transmission channel that our analysis examines is saving behavior. Empirically, our analysis thus suggests that, *ceteris paribus*, in economies where envy prevails agents' saving behavior should be more homogeneous, with a lower degree of inequality in outcomes in the economy. The opposite should be true if, instead, pride prevails.<sup>1</sup>

Summing up, envy and pride – modeled here as the correspondents of social losses and gains, with a similar psychological nature – turn out to have very different implications for the underlying equilibria.<sup>2</sup> This is a novel insight of our analysis that is made behaviorally well founded by the analysis of Maccheroni et al. (2012), with its preference based characterization of agents' objective functions, in particular of the shape of their positional indexes. Besides its intrinsic interest, from the methodological standpoint this behavioral foundation is important because it opens the possibility of an estimation of suitable parametric specifications of these objective functions via micro and experimental data. Once these estimates are provided in separate studies, the equilibrium effects could be determined and perhaps calibrated, just as it is done typically with other features of preferences like risk aversion.

**Related literature and outline** There is a large literature that investigates the economic consequences of relative outcome concerns. Maccheroni et al. (2012) provides a detailed bibliography. Recent relevant surveys include Sobel (2005), which also deals with relative income concerns in addition to reciprocity, Clark, Frijters, and Shields (2008), Fershtman (2008), and Heffetz and Frank (2008).

Our analysis is especially related to two strands of literatures. The first one considers the link between the shape of positional indexes and some important features of the equilibrium (see in particular Clark and Oswald, 1994 and 1998). The shape of these indexes (in particular their concavity/convexity properties) is taken as a given: these results suggest that a choice based foundation would enlighten the conclusions. In this tradition, Dupor and Liu (2003) focus on overconsumption.

The second strand models relative concerns via comparisons of ordinal ranks in outcomes. For example, Frank (1985a) studies how these comparisons affect the demand of positional goods, that is, goods on which consumers exhibit relative concerns, and that of nonpositional

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<sup>1</sup>It would be interesting to know whether these different saving behaviors would affect growth, at least at a theoretical level. In our two-period analysis we cannot, however, address this question, which is left for future research.

<sup>2</sup>As Maccheroni et al. (2012), we consider emotions as a first approximate explanation of behavior that is sufficient for choice theoretic purposes, in an utilitarian tradition that traces its origin back at least to Bentham's classic pain and pleasure calculus. Evolutionary perspectives on relative concerns have been recently considered by Rayo and Becker (2007) and Samuelson (2004).

ones.<sup>3</sup> More recently, Hopkins and Kornienko (2004) use these comparisons in a game theoretic approach to status and to its consequences on income distribution. Our paper provides, *inter alia*, a link between these strands of literature and bases its analysis on a choice based foundation that, as mentioned before, reduces the concern for relative positions to few simple factors.

The implications of the relative weight of envy and pride on the equilibrium are considered in Friedman and Ostrov (2008), which is part of the literature focusing on rank. Agents in their economy have a choice of allocating one unit of homogeneous good to private consumption,  $x$ , and to conspicuous consumption,  $y = 1 - x$ , and have concerns for their relative position in the consumption of  $y$ . The relative concern is modeled by a simple piecewise linear function assigning different coefficients to the expected shortfall and the expected excess, respectively, over the consumption of others. Agents adjust their consumption gradually in the direction of increasing utility. Friedman and Ostrov focus on the dynamic of this adjustment and find that, at steady state, the equilibrium distribution has different skew depending on whether envy or pride dominates, with a right skewed density when pride dominates. If we focus, as we do here, on the equilibria, the model they use is very simple; with the addition of effort, it is considered in Example 2.3 below where we show that, as a special case of our model, there is a multiplicity of equilibria when envy dominates, while there is a single asymmetric equilibrium when pride dominates.

Our model clarifies that the essential distinction between envy and pride dominated economies is the shape of the kink at the reference point of the average population, and does not depend on any specific functional form. We show that the intuition behind the existence of two types of economies is general, and does not depend on simple functional forms and special features of the economy (such as the double use of a single good as private and conspicuous consumption). Finally, the result generalizes to an economy with uncertainty and with two periods, where effort in production and saving is possible. This allows us to study the key intertemporal trade-off discussed before.

The paper is organized as follows. Section 2 introduces the two-periods economy that we study, while Section 3 shows how overconsumption and workaholism may arise in it. Section 4 contains the paper's main result that shows how envy leads to conformism in consumption behavior and pride to diversity. Section 5 concludes, while the Appendix (Section 6) collects proofs and reviews the decision theoretic conceptual structure.

## 2 Social equilibria in a production economy

We consider economies with a continuum of individually negligible agents. There are two main reasons for this modeling choice: it simplifies derivation and exposition, and it allows to focus on the interdependencies due to the social dimension of preferences, as will be shown momentarily.

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<sup>3</sup>The distinction between positional and nonpositional goods is due to Hirsch (1976) (see also Frank, 1985b).

There is a single consumption good, which can be either consumed or saved. We consider a *production economy*, in which a technology is available that allows agents to save for the future any amount of the good that they do not consume in the first period. In this economy there is no room for trade: each agent produces, consumes, and saves for his own future consumption. There are no markets and prices and, with conventional asocial objective functions, this economy is in equilibrium (see Definition 1) when agents just solve their individual intertemporal problems (presented in eq. (1)). As a result, it is an equilibrium notion limited in scope, with no need of considering any form of mutual compatibility of agents' choices. If, however, agents have our social objective functions, this is no longer the case. In fact, when agents' own consumption choices are affected by their peers' choices, a link among all such choices naturally emerges. Even without any trading, in this case there is a sensible notion of mutual compatibility of the agents' choices and, therefore, a more interesting equilibrium notion becomes appropriate (given in Definition 2).

In the economies we consider, therefore, interaction among agents is only due to the social dimension of consumption. This allows us to study the equilibrium effects of this social dimension in “purity,” without other factors intruding into the analysis. This is why we consider these economies. Later, in Section 5, we will briefly discuss a market economy.

## 2.1 The economy

We now turn to the formal details. The set  $I$  of agents is a complete nonatomic probability space  $(I, \Lambda, \lambda)$ . This is the set of agents that are relevant for social comparisons that our reference agent makes. In applications, it is likely to be substantially smaller than the entire universe of people in the planet, or even in the economy. For example, it might be restricted to the individuals in the same environment (neighborhood, ethnic group, or company where the individual is employed).

In particular, we denote by  $M^n$  the collection of all  $\Lambda$ -measurable functions  $\phi : I \rightarrow \mathbb{R}^n$  and by  $L^n$  the subset of  $M^n$  consisting of bounded functions. The technology gives a real (gross) return  $R > 0$ . Agents live two periods and in each of them they work and consume; in period one they can also store. In the first period each agent  $i$  selects a consumption/effort pair  $(c_{i,0}, e_{i,0}) \in \mathbb{R}_+^2$ , evaluated by a utility function  $u_i : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ . Effort is transformed in consumption good according to an individual production function  $F_{i,0} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ .

There is technological uncertainty, resolved in the second period, described by a stochastic production function  $F_{i,s} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  that depends on a finite space  $S$  of states of Nature, endowed with a probability  $P$ . With the usual abuse of notation we set  $S = \{1, 2, \dots, S\}$  and  $S_0 = \{0, 1, 2, \dots, S\}$ , and we write  $p_s$  instead of  $P(s)$ . The production functions  $\{F_{i,s}\}_{s \in S_0}$  use a physical capital, whose amount is exogenously fixed in each period and state (capital accumulation is thus not studied here).

In the second period each agent  $i$  works and consumes. He thus selects in each state  $s$  a consumption/effort pair  $(c_{i,s}, e_{i,s}) \in \mathbb{R}_+^2$ , again evaluated by the same utility function  $u_i$  of the first period. Finally, effort is a limited resource: for each  $i$  there is a vector  $h_i \in \mathbb{R}_+^{S+1}$  such that  $e_{i,s}$  cannot exceed  $h_{i,s}$  for all  $s \in S_0$ . The agent has a discount factor  $\beta \in (0, 1)$ .

Summing up, the intertemporal problem of agent  $i$  in the economy is:

$$\max_{(c_i, e_i) \in B_i} U_i(c_i, e_i), \quad (1)$$

where

$$U_i(c_i, e_i) = u_i(c_{i,0}, e_{i,0}) + \beta \sum_{s \in S} p_s u_i(c_{i,s}, e_{i,s}) \quad \forall (c_i, e_i) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1},$$

and  $B_i$  is the subset of  $\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$  consisting of all  $(c_i, e_i)$  such that:

- (i)  $(c_i, e_i) \in \mathbb{R}_+^{S+1} \times \prod_{s=0}^S [0, h_{i,s}]$ .
- (ii)  $c_{i,0} \leq F_{i,0}(e_{i,0})$ .
- (iii)  $c_{i,s} = F_{i,s}(e_{i,s}) + R(F_{i,0}(e_{i,0}) - c_{i,0})$  for all  $s \in S$ .

The set  $B_i$  is never empty since in every period and state each agent can consume all he produces. Next we make a first assumption on the economy.

H.1 For each agent  $i \in I$ :

- (i)  $u_i : \mathbb{R}_+^2 \rightarrow \mathbb{R}$  is continuous.
- (ii)  $F_{i,s} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is increasing and continuous for all  $s \in S_0$ .

This assumption guarantees that the (nonempty) set  $B_i$  is compact, and that the objective function  $U_i$  is continuous. By the Weierstrass Theorem, problem (1) thus admits a solution. A consumption/effort profile  $(c, e) \in M^{S+1} \times M^{S+1}$  is *feasible* if  $(c_i, e_i) \in B_i$  for all  $i \in I$ .

**Definition 1** *A feasible consumption/effort profile  $(c^*, e^*)$  is an asocial equilibrium for the economy if*

$$U_i(c_i^*, e_i^*) \geq U_i(c_i, e_i) \quad \forall (c_i, e_i) \in B_i,$$

for  $\lambda$ -almost all  $i \in I$ .

As mentioned before, this equilibrium notion just requires that agents individually solve their problems (1), with no interaction.

## 2.2 Equilibrium with social preferences

We turn now to social preferences, conceptually the model we consider relies on the axiomatic analysis of Social Decision Theory developed in Maccheroni et al. (2012), here adapted to the case of a continuum of agents. The basic structure is presented in Section 6.1 of the Appendix.

Given a common social value function  $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ , the social objective function  $V_i$  of each agent now depends on the entire profile of consumption and effort, as follows:

$$V_i(c, e) = U_i(c_i, e_i) + \gamma_i \left( v(c_{i,0}) - \int_I v(c_0) d\lambda \right) + \beta \sum_{s \in S} p_s \gamma_i \left( v(c_{i,s}) - \int_I v(c_s) d\lambda \right) \quad (2)$$

for all  $(c, e) \in L_+^{S+1} \times L_+^{S+1}$  (see the discussion of this representation in Section 6.1).

The function  $v$  represents the social value that the decision maker attaches to consumption, his own and of others. This index is different from the individual specific utility  $u_i$ , and describes the valuation given to the consumption good from the social point of view. For example in a model of conspicuous consumption the value  $v$  describes the signaling value of the good.

The increasing function  $\gamma_i : \mathbb{R} \rightarrow \mathbb{R}$  describes the effect of social comparison on the welfare of the individual. In Section 6.1 we show how this function can be identified from choice, and how it is a special case of a more general structure axiomatized in Maccheroni et al. (2012). Note that in the formulation adopted in (2) we are assuming that only consumption has a social dimension, while effort (and so leisure) is only valued privately. That is, consumption is more positional than effort/leisure, in the terminology of Hirsch (1976) and Frank (1985). This was a classic assumption in Veblen's analysis, justified by the lower degree of observability of effort relative to consumption. For this reason, effort is not an argument of the function  $\gamma$ .

Note that the separability condition identified in Dufwenberg et al. (2011) (Definition 2 p. 616) is not satisfied. Such condition requires that the preference over choices controlled by an agent be independent of the allocations and profiles of budget sets of others. In their general equilibrium model, the condition is necessary and sufficient to guarantee that the demand of an agent is independent of the allocation and characteristics of the other agents. Similarly, the separability condition in Dubey and Shubik (1985), who use a strategic setup similar to the one adopted here, is not satisfied.

Other papers in the literature have examined how social preferences affect market equilibria. For example, Gebhardt (2004) models financial markets as a coordination game (because the underlying utility is reference dependent) with multiple equilibria. When agents coordinate again, the valuation of assets may adjust, inducing output fluctuations, which may be large. Also related is Schmidt (2011), who stresses that social preferences are indeed irrelevant under the conditions identified by Dufwenberg et al. (2011). These conditions however fail to hold when uncertainty is important (as it is in financial markets) or when contracts are incomplete (as in labor markets).

The equilibrium notion relevant for our social preferences is a Nash equilibrium for a continuum of agents.

**Definition 2** *A feasible consumption/effort profile  $(c^*, e^*) \in L^{S+1} \times L^{S+1}$  is a social equilibrium for the economy if*

$$V_i(c^*, e^*) \geq V_i(c_i, c_{-i}^*, e_i, e_{-i}^*) \quad \forall (c_i, e_i) \in B_i, \quad (3)$$

for  $\lambda$ -almost all  $i \in I$ . We say that an equilibrium is symmetric if it is a constant  $\lambda$ -almost everywhere, and asymmetric otherwise.

This equilibrium notion requires a mutual compatibility of agents' choices and is thus qualitatively very different from that of Definition 1, a difference entirely due to the social dimension of our preferences.

A key theoretical issue is the existence of social equilibria, which is necessary to have a solution concept that can be the object of our comparative statics analysis. To prove it, we need the following mild assumption. Point (i) says that the effort and production capacities are limited, points (ii) to (iv) are standard assumptions.

H.2 The following conditions are satisfied:

- (i)  $\sup_{i \in I} (F_{i,s}(h_{i,s}) + h_{i,s}) < \infty$  for all  $s \in S_0$ .
- (ii)  $\gamma_i : \mathbb{R} \rightarrow \mathbb{R}$  is increasing and continuous, with  $\gamma_i(0) = 0$ , for all  $i \in I$ .
- (iii) the real valued functions  $u_{(\cdot)}(x, y)$ ,  $h_{(\cdot),s}$ ,  $F_{(\cdot),s}(z)$ , and  $\gamma_{(\cdot)}(t)$  are  $\Lambda$ -measurable on  $I$  for each fixed  $(x, y, z, t) \in \mathbb{R}_+^3 \times \mathbb{R}$  and  $s \in S_0$ .
- (iv)  $v : \mathbb{R}_+ \rightarrow \mathbb{R}$  is increasing and continuous.

We can now prove a general existence result for economies. The proof relies on existence results of Schmeidler (1973) and Balder (1995).

**Theorem 1** *In a economy satisfying assumptions H.1 and H.2 there exists a social equilibrium.*

### 2.3 Example

To introduce and motivate the analysis that follows we consider a very simple example, close to the setup in Friedman and Ostrov (2008). We add to their structure the choice of effort, as in our general setup. To keep the analysis simple we consider a single period economy and do not introduce uncertainty (there is only one state).

The utility function of the agent depends on a private consumption good  $x$  and a conspicuous consumption good  $y$ . Agents produce both goods in a private economy, and pay a quadratic cost of effort of the sum of the two goods,  $\frac{c}{2}(x+y)^2$ . The utility function of the agent depends on the pair  $(x, y)$  and on the average conspicuous consumption  $y^*$  in the economy, as follows:

$$U(x, y, y^*) = x + \pi(y - y^*)1_{\{y \geq y^*\}} + \eta(y - y^*)1_{\{y \leq y^*\}} - \frac{c}{2}(x + y)^2 \quad (4)$$

where  $\eta$  is the envy parameter and  $\pi$  the pride parameter, both positive. The cases where  $\pi$  and  $\eta$  are both larger than 1 or both smaller than 1 (which is the marginal utility of the private consumption good  $x$ ) are less interesting: in the first the  $x$  good is produced in zero quantity, in the second the  $y$  good is. We thus consider two cases:

**Envy dominates pride.** Let  $\eta > 1 > \pi$ . The economy has a continuum of symmetric equilibria: any profile  $(x, y)$ , with  $y = y^*$  and

$$x + y = \frac{1}{c},$$

corresponds to an equilibrium in which all agents produce and consume  $(x, y)$ .

This is a concave economy (in particular the utility function is concave in both arguments), with a twist: there is a kink at  $y^*$  and this kink is endogenous. Intuitively,  $y^*$  sets a reference consumption level in the economy, and the kink varies the cost of deviating from it above and below. As long as this reference value is not too large, that is, the marginal cost of producing is not larger than the marginal utility from  $x$ , and sets the benchmark, there is an equilibrium where all agents choose such reference level as the level of their own conspicuous consumption.

Looking at details, the marginal utility net of cost with respect to  $y$  is decreasing, and falls from the value  $\eta - c(x + y^*)$  to  $\pi - c(x + y^*)$  at the kink. Consider now all the pairs  $(x, y^*)$  for which the marginal cost  $c(x + y^*)$  is equal to the marginal utility from  $x$ . For such pairs the first order condition for the private consumption  $x$  is satisfied, and the net marginal utility of  $y$  falls from positive to negative as  $y$  crosses the value  $y^*$ . The agents who take the value  $y^*$  as given will also choose this level of conspicuous consumption as optimal. Thus, all such pairs are equilibria.

**Pride dominates envy.** Let  $\eta < 1 < \pi$ . The economy has a unique asymmetric equilibrium. The population is split into two groups, of size  $1 - p$  and  $p$ , consuming  $(x_1, y_1)$  and  $(x_2, y_2)$ , respectively. The equilibrium values are:

$$x_1 = \frac{1}{c}, y_1 = 0 \quad \text{and} \quad x_2 = 0, y_2 = \frac{\pi}{c}, \tag{5}$$

with  $y^* = py_2$ .

The net marginal utility with respect to the private consumption is decreasing; that of  $y$  is not, and jumps up from a low to a high value at the kink, when consumption of  $y$  begins to deliver pride. Thus, the first order condition can be satisfied for two different values of the conspicuous consumption, and thus two consumption profiles. One has high conspicuous consumption and low private consumption, and the other an opposite profile. If the utility from these two different values of  $y$  and the corresponding optimal choice of  $x$  are equal, then the agents will be indifferent between these two types of behavior.

There is only one such pair. Looking at details, consider agents who choose a conspicuous consumption less than the public average  $y^*$ , call that value  $y_1$ . If the optimal private consumption is strictly positive then the net marginal utility from  $x$  has to be zero, that is  $c(x_1 + y_1) = 1 > \eta$ , and so  $y_1 = 0$ . On the other side, consider agents who choose a level larger than  $y^*$ , call it  $y_2$ ; since this value is positive, the net marginal utility must be zero, so  $c(x_2 + y_2) = \pi > 1$ , and therefore  $x_2 = 0$ . We conclude that the only pair of equilibrium values is the one described in (5). Note that the values are unique, and determined by the

parameters  $\pi, \eta, c$ . The condition that the utility from the two choices is equal determines the value of  $y^*$  or, equivalently,  $p$  from  $y^* = p \frac{\pi}{c}$  and the equation

$$p = \frac{\pi^2 - 1}{2\pi(\pi - \eta)}.$$

### 3 Overconsumption and workaholism

In the example just discussed the nature of equilibrium is deeply affected by the relative weight that envy and pride have in the utility of agents. When envy dominates, conformism is the equilibrium outcome, and when pride dominates, diversity is the outcome. The comparison between the two relative effects is ultimately a comparison between the effect of gains and losses, and therefore shares important features with the central comparison in Prospect Theory (PT). In PT, the comparison is in the private domain; in our environment it is done in the social domain because what is compared is, for instance, the wealth of the individual and that of the comparison group. When the comparison is not favorable we can think that it is associated with a social loss, and viceversa in the case of a favorable comparison.

The parallel with PT might suggest by analogy that losses loom larger than gains in the social domain as well. Whether they do is, however, an empirical question. There are plausible reasons why the parallel might not hold perfectly: for example, in social environments many final outcomes (for example in the competition for mating) are allocated on the basis of winner-take-all mechanisms. In these cases the prize attached to gains is substantially higher than in the private domain, and the cost attached to loss also more substantial. The larger advantage associated with gains might be reflected in a larger weight of gains in the utility function of individuals. The question has been recently examined in the experimental literature. For example, in Bault et al. (2008) and Bault et al. (2011), the experimental setups are close in spirit to the environment described in our theory, and it is found that social gains loom larger than social losses.

The first phenomenon we consider here is how overconsumption and workaholism can arise in a social equilibrium. This is an often mentioned behavioral consequence of concerns for relative consumption and here Proposition 1 shows how it emerges in our general analysis. Empirical evidence on this phenomenon can be found, for example, in Bowles and Park (2005) and the references therein. Anecdotal evidence is reported in Rivlin (2007), who describes Silicon Valley workaholic executives as “working class millionaires”.

We focus on a single period version of the economy. As pointed out in the Introduction, different trade-offs arise in more general intertemporal settings: for example, consuming more today leads to lower saving and, possibly, to lower future consumption. The tendency to overconsumption and workaholism that we identify in the single period setting might be then offset by other forces.

To ease notation, we drop the subscripts 0; that is,  $c_i$  and  $e_i$  stand for  $c_{i,0}$  and  $e_{i,0}$ ,

respectively. The asocial problem of each agent  $i \in I$  is then given by

$$\max_{(c_i, e_i) \in B_i} u_i(c_i, e_i) \quad (6)$$

where  $B_i = \{(c_i, e_i) \in \mathbb{R}_+^2 : 0 \leq e_i \leq h_i, c_i = F_i(e_i)\}$ .

Here a feasible consumption/effort profile  $(c^*, e^*) \in L \times L$  is an asocial equilibrium if  $(c_i^*, e_i^*)$  is a solution of problem (6) for  $\lambda$ -almost all  $i \in I$ .

H.3 For each agent  $i \in I$ :

- (i)  $u_i$  is twice continuously differentiable on  $\mathbb{R}_{++}^2$ ,  $\partial u_i / \partial x > 0$ , and the Hessian matrix  $\nabla^2 u_i$  is negative definite.
- (ii)  $F_i$  is twice differentiable on  $\mathbb{R}_{++}$ ,  $F_i' > 0$  and  $F_i'' < 0$ .

**Lemma 1** *If H.1, H.2, and H.3 hold, then there exists a ( $\lambda$ -a.e.) unique asocial equilibrium.*

The social objective functions  $V_i$  take the form

$$V_i(c, e) = u_i(c_i, e_i) + \gamma_i \left( v(c_i) - \int_I v(c) d\lambda \right),$$

and a feasible pair  $(c^*, e^*) \in L \times L$  is a social equilibrium if (3) holds.

To state the result we need a condition and some notation. The special form that  $B_i$  has in this case guarantees that a consumption/effort profile  $(c, e) \in M \times M$  is feasible if and only if  $e_i \in [0, h_i]$  and  $c_i = F_i(e_i)$  for all  $i \in I$ . Under H.2-(i), feasible profiles are thus determined by effort profiles that belong to the supnorm closed and convex set  $E = \{e \in L : 0 \leq e \leq h\}$ . The value of the social objective function can be thus written as

$$W_i(e) = u_i(F_i(e_i), e_i) + \gamma_i \left( v(F_i(e_i)) - \int_I v(F_i(e_l)) d\lambda(l) \right) \quad \forall e \in E, i \in I. \quad (7)$$

An equilibrium  $(c^*, e^*)$  is *internal* if  $e^* \in \text{int}E$  and *strongly Pareto inefficient* if it is strongly Pareto dominated, that is, there is  $\varepsilon > 0$  and a feasible  $(c, e) \in M \times M$  such that

$$V_i(c, e) \geq V_i(c^*, e^*) + \varepsilon$$

for  $\lambda$ -almost all  $i \in I$ .

We can now state the needed assumption.

H.4 The following conditions are satisfied:

- (i)  $v$  is differentiable on  $\mathbb{R}_{++}$  and  $v' > 0$ .
- (ii)  $\gamma_i$  is differentiable on  $\mathbb{R}$  for all  $i \in I$ , and  $\inf_{(i,t) \in I \times \mathbb{R}} \gamma_i'(t) > 0$ .

$$(iii) \quad \sup_{|x|, |y|, |t| \leq n, i \in I} |u_i(x, y) + \gamma_i(t)| < \infty \text{ and } \sup_{|x| \leq n, i \in I} |v'(x) + F'_i(x)| < \infty$$

for all  $n \in \mathbb{N}$ .<sup>4</sup>

$$(iv) \quad W : E \rightarrow L \text{ is strictly differentiable on } \text{int}E.<sup>5</sup>$$

In the empirical literature on overconsumption the statement that “consumption is excessive” is usually derived from international comparisons. Here we give a precise definition comparing the equilibrium level of consumption to the level one has in the asocial economy taken as benchmark. We do the same for overwork, or “workaholism”. In the empirical literature (see Harpaz and Snir, 2003), workaholism is defined sometimes in absolute terms (as the “individual’s steady and considerable allocation of time to work-related activities and thoughts, which does not derive from external necessities”). An alternative definition, and corresponding measurement, closer to our analysis is as “time invested in work, while controlling the financial needs for this investment”. Such control implicitly compares the actual (social) effort and that explained by (asocial) material needs.

All this motivates the following definition, which is well posed by Lemma 1.

**Definition 3** *Let  $(\hat{c}, \hat{e})$  be the asocial equilibrium of the economy. A social equilibrium  $(c^*, e^*)$  exhibits over-consumption if,  $\lambda$ -almost everywhere,  $c_i^* > \hat{c}_i$ ; it exhibits workaholism if,  $\lambda$ -almost everywhere,  $e_i^* > \hat{e}_i$ .*

We can now state the announced result.

**Proposition 1** *If assumptions H.1-H.4 hold, then internal social equilibria are strongly Pareto inefficient and exhibit overconsumption and workaholism.*

Overconsumption and workaholism thus characterize equilibria in the single period version of the economy. We studied here in detail the Pareto inefficiency of the equilibria to stress the negative features of these equilibria.

This result confirms a general intuition about social preferences (see, e.g., Dupor and Liu, 2003, for a related point).

## 4 Equilibrium anticonformism

We now study how conformism and anticonformism can characterize the consumption choices of agents in social equilibria, depending, as anticipated in the Introduction, on whether either envy or pride prevails among agents. Since the rise of anticonformism is our main interest, in order to better focus on this issue we consider a version of the economy in which agents are *identical*, so that the social dimension of their preferences is the only possible cause of heterogeneous consumption choices. We also assume that labor is supplied inelastically, say  $e_{i,s} = \bar{e}_s > 0$  for all  $i \in I$  and  $s \in S_0$ . To ease notation, we set  $F_0(\bar{e}_0) = \bar{x}_0 > 0$  and

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<sup>4</sup>This implies that  $W(E)$  consists of bounded functions.

<sup>5</sup>See pages 30-32 of Clarke (1983) for the properties of strict differentiability.

$F_s(\bar{e}_s) = \bar{x}_s > 0$ , and we drop effort as argument of the utility function  $u$ . This is the version of the two-periods economy that we discussed in the Introduction.

In this case, the asocial intertemporal problem of each identical agent  $i$  is

$$\max_{c \in [0, \bar{x}_0]} U(c), \quad (8)$$

where  $U(c) = u(c) + \beta \sum_{s \in S} p_s u(\bar{x}_s + R(\bar{x}_0 - c))$  for all  $c \in [0, \bar{x}_0]$ . A (first period) consumption profile  $c \in M$  is feasible if it belongs to the set  $C = \{c \in M : 0 \leq c \leq \bar{x}_0\}$ , and is an asocial equilibrium if  $c_i$  solves problem (8) for  $\lambda$ -almost all  $i \in I$ . Clearly, all asocial equilibria are symmetric (see Definition 2) provided  $U$  is unimodal.

H.5 The following conditions are satisfied:

- (i)  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$  is continuous and strictly concave on  $\mathbb{R}_+$ , differentiable on  $(0, +\infty)$  with  $u' > 0$ .
- (ii)  $U'_+(0) \geq 0 \geq U'_-(\bar{x}_0)$ .

The standard condition (i) implies that  $U : \mathbb{R}_+ \rightarrow \mathbb{R}$  is continuous and strictly concave on  $\mathbb{R}_+$ , and differentiable on  $(0, +\infty)$ . In turn, this implies that condition (ii) holds if and only if the asocial problem (8) has either a unique interior solution or a unique boundary solution with  $U'_+(0) = 0$  or  $U'_-(\bar{x}_0) = 0$ .<sup>6</sup>

Given  $\gamma : \mathbb{R} \rightarrow \mathbb{R}$  and  $v : \mathbb{R}_+ \rightarrow \mathbb{R}$  and  $c \in C$ , agent  $i$ 's social objective function becomes

$$\begin{aligned} V_i(c) = & u(c_i) + \beta \sum_{s \in S} p_s u(\bar{x}_s + R(\bar{x}_0 - c_i)) + \gamma(v(c_i) - \int_I v(c_i) d\lambda(\iota)) \\ & + \beta \sum_{s \in S} p_s \left[ \gamma \left( v(\bar{x}_s + R(\bar{x}_0 - c_i)) - \int_I v(\bar{x}_s + R(\bar{x}_0 - c_i)) d\lambda(\iota) \right) \right]. \end{aligned} \quad (9)$$

Here a  $c^* \in C$  is a social equilibrium if, for  $\lambda$ -almost all  $i \in I$ ,

$$V_i(c^*) \geq V_i(c_i, c_{-i}^*) \quad \forall c_i \in [0, \bar{x}_0].$$

We will use the following assumption on  $\gamma$  and  $v$ .

H.6 The following conditions are satisfied:

- (i)  $v : \mathbb{R}_+ \rightarrow \mathbb{R}$  is continuous and concave on  $\mathbb{R}_+$ , differentiable on  $(0, +\infty)$  with  $v' > 0$ .
- (ii)  $v'(\bar{x}_0) \leq \sum_s p_s v'(\bar{x}_s)$ .
- (iii)  $\gamma : \mathbb{R} \rightarrow \mathbb{R}$  is continuous and increasing, with  $\gamma(0) = 0$ .

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<sup>6</sup>The solution is internal when the inequalities in (ii) are strict.

The condition (ii) can be interpreted as stating that the social value function  $v$  is not too concave or that the second period endowments  $\bar{x}_s$  are not too large. An example in which the condition is automatically satisfied is given by a linear function  $v$ .

Assumptions H.5 and H.6 imply assumptions H.1 and H.2 in the version of the two-periods economy that we are considering in this section. By Theorem 1, social equilibria thus exist under Assumptions H.5 and H.6. Next we characterize them according to the nature of the kink. In particular, we show that envy leads to conformism, that is, to symmetric social equilibria, while pride leads to diversity, that is, to asymmetric social equilibria.<sup>7</sup>

**Theorem 2** *If assumptions H.5 and H.6 hold, then:*

- (i) *All social equilibria are asymmetric provided  $D_+\gamma(0) > D^-\gamma(0)$ .*
- (ii) *All social equilibria are symmetric provided  $\gamma$  is concave.*

Point (i), together with the existence Theorem 1, jointly form the main result of the paper and motivate its title: equilibria exist and they are all asymmetric. Note that equilibria are in pure strategies, that is, each agent chooses a profile of consumption and effort, not a probability over consumption and effort. As argued in Section 6.3, thanks to the analysis of Maccheroni et al. (2012) the convex kink  $D_+\gamma(0) > D^-\gamma(0)$  at zero has a clear behavioral interpretation as a more proud than envious social attitude. According to point (i) and Theorem 1, when the identical agents of this economy feature such attitude, social equilibria exist and are asymmetric. As mentioned in the Introduction, this means that in equilibrium agents have to solve differently the trade-off that they face between consuming more either today or tomorrow – which ensures, *ceteris paribus*, a dominant position either today or tomorrow.

The minimal local convexity that the kink  $D_+\gamma(0) > D^-\gamma(0)$  gives to the positional index  $\gamma$  is enough to break any equilibrium symmetry. Point (ii) shows that, instead, when  $\gamma$  is concave such symmetry is fully restored.

**Outline of the proof of (i)** The proof of the important point (i) is not trivial. Its basic intuition, however, can be seen in a special case. Suppose that  $v$  is linear; actually, set  $v(x) = x$ . Moreover, for simplicity suppose that  $\gamma$  has both left and right derivatives, denoted  $\gamma'_+$  and  $\gamma'_-$  respectively. The kink condition  $D_+\gamma(0) > D^-\gamma(0)$  then reduces to  $\gamma'_+(0) > \gamma'_-(0)$ .

In this special case it is easy to check that the objective function (9) takes the stark simple form

$$V_i(c) = U(c_i) + \gamma(c_i - m) + \beta\gamma(R(m - c_i)), \quad (10)$$

where  $m = \int c_i d\lambda$ . The remarkable feature of (10) is that, thanks to the condition  $v(x) = x$ , the second period term  $\beta\gamma(R(m - c_i))$  takes a simple deterministic form, that is, uncertainty

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<sup>7</sup>The left and right Dini derivatives denoted  $D^-$  and  $D_+$  are defined in Section 6.4, and coincide with the usual left and right derivatives when they exist.

does not matter for the positional index. This drastically simplifies the form of  $V_i(c)$  relative to what happens when  $v$  is not linear.

To ease notation, denote by  $V_i^+$  and  $V_i^-$  the left and right derivatives of  $V_i$ , respectively. Some algebra shows that, for  $c_i > 0$ , it holds

$$V_i^+(c_i) = U'(c_i) + \gamma'_+(c_i - m) - \beta R \gamma'_-(R(m - c_i))$$

and

$$V_i^-(c_i) = U'(c_i) + \gamma'_-(c_i - m) - \beta R \gamma'_+(R(m - c_i)).$$

Let  $c^*$  be a symmetric consumption profile, so that  $c_i^* = m^*$  for  $\lambda$ -almost all agents  $i$ . For simplicity, suppose  $c_i^* > 0$  for each  $i \in I$ . It is easy to see that the profile  $c^*$  is an equilibrium only if the first order condition  $V_i^+(c_i^*) \leq 0 \leq V_i^-(c_i^*)$  holds for  $\lambda$ -almost all agents  $i$ . That is,

$$U'(c_i^*) + \gamma'_+(0) - \beta R \gamma'_-(0) \leq 0 \leq U'(c_i^*) + \gamma'_-(0) - \beta R \gamma'_+(0).$$

In turn this implies

$$\gamma'_+(0) - \gamma'_-(0) \leq \beta R (\gamma'_-(0) - \gamma'_+(0)). \quad (11)$$

Since  $\gamma'_+(0) > \gamma'_-(0)$ , the inequality (11) implies  $\gamma'_-(0) - \gamma'_+(0) > 0$ , a contradiction. We conclude that the first order necessary condition  $V_i^+(c_i^*) \leq 0 \leq V_i^-(c_i^*)$  cannot be satisfied under the kink  $\gamma'_+(0) > \gamma'_-(0)$ . Hence, all equilibria must be asymmetric.

#### 4.1 Kinks and asymmetric equilibria

By Theorem 2, the convex kink  $D_+\gamma(0) > D^-\gamma(0)$  is a sufficient condition for all equilibria to be asymmetric, with a sharp behavioral meaning in terms of social attitudes. We now explore whether it is necessary, and conclude that it is not: symmetric equilibria do not exist if the curvature of the  $\gamma$  function is large enough to counterbalance the curvature of the utility function, and there are no equilibria at the boundary.

To see why this is the case, for simplicity consider again the special case  $v(x) = x$ . We also assume that  $\gamma$  is continuously differentiable, so that it has no kinks, and that the function  $U$  is continuously differentiable and strictly concave. We claim that there are economies with these properties that have only asymmetric equilibria. Suppose that

$$U'(\bar{x}_0) + \gamma'(0)(1 - \beta R) < 0 < U'(0) + \gamma'(0)(1 - \beta R). \quad (12)$$

For example, an Inada condition on  $U$  at 0 and  $\beta R > 1$  suffice. Then, there exists a unique scalar  $m^* \in (0, \bar{x}_0)$  that satisfies the first order condition  $U'(m^*) + \gamma'(0)(1 - \beta R) = 0$ . Consider the symmetric profile  $c^*$  such that  $c_i^* = m^*$  for  $\lambda$ -almost all agents  $i$ . By condition (12), if a symmetric equilibrium exists it must be  $m^*$ . This profile is an equilibrium only if the second order condition

$$U''(m^*) + \gamma''(0)(1 + \beta R^2) \leq 0. \quad (13)$$

is satisfied. In this case  $c^*$  may (the condition is only local) be a symmetric equilibrium. If it is not, then there is no symmetric equilibrium. We conclude that all equilibria must be asymmetric if condition (12) is satisfied and (13) is violated.

The intuition is clear: if we exclude boundary equilibria, then equilibria are asymmetric when the local gain from diversification – given by the  $\gamma$  function – overcomes the term given by the  $U$  in the precise sense that the inequality (13) is violated. Though behaviorally very significant, a kink at 0 of  $\gamma$  is thus a special case where the local gain from diversification dominates. This is why it is a sufficient, but not necessary, condition for asymmetry.

## 5 Conclusions

We have derived simple and general conclusions on the relationship between the degree of inequality in an economy and the relative weight of envy and pride in the utility function of the typical agent in an economy where there is no essential heterogeneity among agents. The observed heterogeneity of behavior is produced at equilibrium, and cannot be traced to idiosyncratic differences.

As we remarked in the Introduction, the diversity in consumption behavior caused by pride is the most remarkable part of the result because agents are identical. By point (i) of Theorem 2, in all equilibria necessarily some agents will choose to consume more today, that is, to have a dominant position today, while other agents will choose the opposite, that is, they will save more today in order to consume more tomorrow and have then a dominant position. This diversity in equilibrium behavior is entirely due to the social dimension of preferences. In fact, in this economy there is no trade and agents' actions do not need to equilibrate in terms of resources, as remarked in the Introduction.

A simple modification of the economy that allows trade is to change the “saving technology” by assuming that agents no longer can store the consumption good for future consumption. They can, however, borrow and lend amounts of the consumption good, which is now also a real asset. Agents can save by lending any amount of the consumption good that they do not consume in the first period. As a result, in the (real) asset economy agents interact by trading in the real asset market.

Though for brevity we do not study in detail this economy, it is worth observing that here conformism/anticonformism correspond to no trade/trade. In fact, conformism means that all social equilibria are symmetric and, by the market clearing condition, it is easy to see that in such equilibria there is no trade in the real asset market. In contrast, this market operates in the asymmetric equilibria of the anticonformism case. As a result, in this market economy envy leads to autarky and pride to trade.

## 6 Appendix: Proofs and related material

### 6.1 Social decision theory

In this preliminary section we summarize the essential features of the social decision theory introduced and axiomatized in Maccheroni et al. (2012). In that paper we discuss in detail how our approach compares to social preferences literature (see for example Fehr et al., 1999, 2003, 2006.) We also show how this new approach suggests different and new experimental and empirical tests. Some of these test have already been carried out after the paper appeared.

Following Savage (1954) tradition, acts are measurable functions  $f : S \rightarrow C$  from a state space  $S$  to a consequence space  $C$ .<sup>8</sup> In social decision theory, an agent  $o$  has preferences over acts' profiles  $(f_o, (f_i)_{i \in I})$  that represent the situation in which agent  $o$  takes act  $f_o$ , while each member  $i$  of the agent's reference group  $I$  takes act  $f_i$ .

In the main application provided in the text, where agents choose consumption and effort in two periods, an act of an agent  $i$  is a vector  $f_i = (c_{i,0}, e_{i,0}, ((c_{i,s}, e_{i,s})_{s \in S}))$ . Note that the choice in the first period is, of course, independent of the state, which has yet to realize.

Our agent  $o$  evaluates this situation according to:

$$V(f_o, (f_i)_{i \in I}) = \int_S u(f_o(s)) dP(s) + \int_S \varrho \left( v(f_o(s)), \sum_{i \in I} \delta_{v(f_i(s))} \right) dP(s). \quad (14)$$

The first term of this representation is familiar. The index  $u(f_o(s))$  represents the agent's intrinsic utility of the realized outcome  $f_o(s)$ , and  $P$  represents his subjective probability over the state space  $S$ . The first term thus represents his direct subjective expected utility from act  $f_o$ .

The effect on  $o$ 's welfare of the outcome of the other individuals is reported in the second term. The index  $v(f_o(s))$  represents the social value that  $o$  attaches to outcome  $f_o(s)$ . Given a profile of acts, agent  $o$ 's peers will get outcomes  $(f_i(s))_{i \in I}$  once state  $s$  obtains. If  $o$  does not care about the identity of who gets the value  $v(f_i(s))$ , then he will only be interested in the distribution of these values. This distribution is represented by the term  $\sum_{i \in I} \delta_{v(f_i(s))}$  in (14) above.

The positional index  $\varrho$  is an externality that models agent  $o$ 's relative outcome concerns. It is increasing in the first component and stochastically decreasing in the second. These monotonicity properties of  $\varrho$  reflect  $o$ 's different attitudes towards his own outcomes and those of his peers. In particular,  $\varrho$  is increasing in the first component because  $o$  positively values his own outcome  $f_o(s)$ , while  $\varrho$  is stochastically decreasing in the second one since  $o$  negatively values his peers' outcomes, and so benefits from a stochastically dominated distribution of their outcomes.

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<sup>8</sup>Maccheroni et al. (2012) use an Anscombe and Aumann (1963) version of Savage's setting, where  $C$  is a convex subset of a vector space.

As mentioned in the Introduction, the social emotions that underlie these negative attitudes toward peers' outcome are envy and pride. This is discussed at length by Maccheroni et al. (2012), who provide the behavioral axioms that deliver the choice criterion (14), with these monotonicity properties of the positional index  $\varrho$ .

The index  $v : C \rightarrow \mathbb{R}$  is conceptually different from the utility function  $u : C \rightarrow \mathbb{R}$ , although they may coincide in special cases. The two indices are equal if agent  $o$  evaluates his peers' outcomes only through his own utility function, that is, according to the user value that their outcomes have for him. In contrast, if peers' outcomes are valued beyond their user value, say because of status concerns, then the indices  $u$  and  $v$  may differ as the latter keeps track of this further social concerns about peers' outcomes. For example, we can envy our neighbor's Ferrari both because we would like to drive it (user value) and because of its symbolic/status value. The index  $v$  reflects the overall, cumulative, "outcome externality" that the agent perceives, that is, his overall relative outcome concerns. Thus,  $u$  and  $v$  agree if user value considerations prevail, but they may well differ if symbolic or status considerations matter.

## 6.2 Specifications

Maccheroni et al. (2012) study several possible specifications of the general choice criterion (14). Among them, the following average specification is especially important:

$$V(f_o, (f_i)_{i \in I}) = \int_S u(f_o(s)) dP(s) + \int_S \varrho \left( v(f_o(s)), \frac{1}{|I|} \sum_{i \in I} v(f_i(s)) \right) dP(s). \quad (15)$$

Here, agent  $o$  only cares about the average social value  $|I|^{-1} \sum_{i \in I} v(f_i(s))$ . For example, if  $v(x) = x$ , then (15) becomes

$$V(f_o, (f_i)_{i \in I}) = \int_S u(f_o(s)) dP(s) + \int_S \varrho \left( f_o(s), \frac{1}{|I|} \sum_{i \in I} f_i(s) \right) dP(s),$$

where only the average outcome  $|I|^{-1} \sum_{i \in I} f_i(s)$  appears, as it is the case in many specifications used in applications.

It is also possible to give choice based conditions such that in (15) we actually have  $\varrho(z, t) = \gamma(z - t)$  for some increasing  $\gamma : \mathbb{R} \rightarrow \mathbb{R}$  with  $\gamma(0) = 0$ . Here relative concerns are modelled through the difference between the value of  $o$ 's outcome and that of the peers' average outcome. This is the simple and convenient specification that, in the two-periods version (2), we use in the present paper. That is, we study an economy where agents rank acts' profiles according to:

$$V(f_o, (f_i)_{i \in I}) = \int_S u(f_o(s)) dP(s) + \int_S \gamma \left( v(f_o(s)) - \frac{1}{|I|} \sum_{i \in I} v(f_i(s)) \right) dP(s). \quad (16)$$

### 6.3 Kinks and comparative envy

As observed in the Introduction, an outcome profile where your peers get a socially better outcome than yours can be viewed as social loss; conversely, a profile where you get more than them can be viewed as a social gain. In particular, individuals might well have different attitudes toward such social gains and losses, similarly to what happens for standard private gains and losses. The distinction between gains and losses is inspired by Prospect Theory, with its focus on *private* gains and losses. While there is no presumption that preferences in social and private domains are the same (there is indeed some evidence that they are not), we keep the basic intuition that the change across domains may change the evaluation of one additional unit, that is, may introduce a discontinuity in marginal utility. Kinks in the objective function add to the mathematical complexity of the analysis, but they are necessary to study appropriately this view.

These different social attitudes play a key role in the paper and it is therefore important to understand how to model them via the positional index  $\varrho$ . Given a “fair” event  $E$  with  $P(E) = 1/2$ , say that an agent  $o$  that features the choice criterion (14) is *more envious than proud* (or *averse to social losses*), relative to a given  $x_o \in C$ , if

$$V(x_o, x_o) \geq V(x_o, x_i E y_i) \quad (17)$$

for all  $x_i, y_i \in C$  such that  $(1/2)v(x_i) + (1/2)v(y_i) = v(x_o)$ .<sup>9</sup> The intuition is that agent  $o$  tends to be more frustrated by envy than satisfied by pride. That is, assuming without loss of generality that  $v(x_i) \geq v(y_i)$ , he is more scared by the social loss  $(x_o, x_i)$  than lured by the social gain  $(x_o, y_i)$ .

Maccheroni et al. (2012) show that agent  $o$  is more envious than proud, relative to an  $x_o \in C$ , if and only if

$$\varrho(v(x_o), v(x_o) + h) \leq -\varrho(v(x_o), v(x_o) - h) \quad \forall h > 0. \quad (18)$$

In particular,<sup>10</sup>

$$D^+ \varrho(v(x_o), v(x_o)) \leq D_- \varrho(v(x_o), v(x_o)). \quad (19)$$

In other words, a concave kink at  $v(x_o)$  reveals a more envious than proud attitude at  $x_o$ . In the special case  $\varrho(z, t) = \gamma(z - t)$  condition (19) becomes

$$D^+ \gamma(0) \leq D_- \gamma(0). \quad (20)$$

Similarly, the convex kink

$$D_+ \gamma(0) \geq D^- \gamma(0) \quad (21)$$

reveals a more proud than envious attitude. When  $\varrho(z, t) = \gamma(z - t)$ , a convex kink at 0 thus reveals a global (i.e., at all points  $x_o$ ) more proud than envious attitude. This is a remarkable feature of the specification (16) that makes it especially tractable.

<sup>9</sup>Here,  $x_i E y_i$  denotes the act that gives  $x_i$  if event  $E$  obtains and  $y_i$  otherwise. It can be proved that the choice of  $E$  is immaterial in the definition (events  $E$  with  $P(E) = 1/2$  are often called ethically neutral in decision theory).

<sup>10</sup>The Dini derivatives that appear in (19), (20), and (21) are discussed in the next Section 6.4.

**Remark** When right  $\gamma'_-(0)$  and left  $\gamma'_+(0)$  derivatives at zero exist, then  $D^-\gamma(0) = D_-\gamma(0) = \gamma'_-(0)$  and  $D^+\gamma(0) = D_+\gamma(0) = \gamma'_+(0)$  (see Appendix 6.4). The kink conditions (20) and (21) then become  $\gamma'_+(0) \leq \gamma'_-(0)$  and  $\gamma'_+(0) \geq \gamma'_-(0)$ , respectively. On a first reading, to fix ideas the reader can focus on this important special case.

## 6.4 Dini derivatives

Let  $a, b \in \mathbb{R}$  with  $a < b$ . If  $f, g : (a, b) \rightarrow \mathbb{R}$ , set

$$\limsup_{x \rightarrow a^+} f(x) \equiv \lim_{\delta \rightarrow 0^+} \sup_{h \in (0, \delta)} f(a+h), \quad \liminf_{x \rightarrow a^+} f(x) \equiv \lim_{\delta \rightarrow 0^+} \inf_{h \in (0, \delta)} f(a+h),$$

and

$$\limsup_{x \rightarrow b^-} f(x) \equiv \lim_{\delta \rightarrow 0^+} \sup_{h \in (0, \delta)} f(b-h), \quad \liminf_{x \rightarrow b^-} f(x) \equiv \lim_{\delta \rightarrow 0^+} \inf_{h \in (0, \delta)} f(b-h).$$

These four limits always exist in  $[-\infty, +\infty]$ ,<sup>11</sup> with the conventions:

$$\begin{aligned} (-\infty) \dot{+} (+\infty) &= (+\infty) \dot{+} (-\infty) = +\infty = (-\infty) \dot{-} (-\infty) = (+\infty) \dot{-} (+\infty), \\ (-\infty) \dot{+} (+\infty) &= (+\infty) \dot{+} (-\infty) = -\infty = (-\infty) \dot{-} (-\infty) = (+\infty) \dot{-} (+\infty). \end{aligned}$$

Using these limits we can define the Dini derivatives. Let  $f : [a, b] \rightarrow \mathbb{R}$ . For all  $c \in [a, b)$ , set

$$D^+f(c) \equiv \limsup_{h \rightarrow 0^+} \frac{f(c+h) - f(c)}{h} \quad \text{and} \quad D_+f(c) \equiv \liminf_{h \rightarrow 0^+} \frac{f(c+h) - f(c)}{h}$$

Analogously, for all  $c \in (a, b]$  set

$$D^-f(c) \equiv \limsup_{h \rightarrow 0^-} \frac{f(c+h) - f(c)}{h} \quad \text{and} \quad D_-f(c) \equiv \liminf_{h \rightarrow 0^-} \frac{f(c+h) - f(c)}{h}$$

In particular,  $\gamma$  has a left derivative  $\gamma'_+(c)$  at  $c$  if and only if  $D^+f(c) = D_+f(c)$ ; in this case,  $\gamma'_+(c) = D^+f(c) = D_+f(c)$ . A similar property holds for the right derivatives. Moreover,  $\gamma$  is differentiable at  $c$  if and only if all four Dini derivatives at  $c$  are equal; in this case,  $\gamma'(c) = D^+f(c) = D_+f(c) = D^-f(c) = D_-f(c)$ .

It is easy to see that, if  $c \in [a, b)$  is a local maximum, then  $D_+f(c) \leq D^+f(c) \leq 0$ . Analogously, if  $c \in (a, b]$  is a local maximum, then  $0 \leq D_-f(c) \leq D^-f(c)$ . In particular, if an interior point  $c \in (a, b)$  is a local maximum, then

$$D_+f(c) \leq D^+f(c) \leq 0 \leq D_-f(c) \leq D^-f(c).$$

This is the form that the first order condition takes in terms of Dini derivatives.

Next we provide a chain rule that will be useful in the sequel:

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<sup>11</sup>To be precise, we should write  $d > \delta > 0$ , where  $d \in (0, b-a)$ , rather than  $\delta > 0$ . But, no confusion should arise.

**Proposition 2** Let  $v : [\alpha, \beta] \rightarrow [a, b]$  be strictly increasing, onto, and concave. Then, given any  $f : [a, b] \rightarrow \mathbb{R}$ , we have:

(ii)  $D_+(f \circ v)(\gamma) = v'_+(\gamma) D_+f(v(\gamma))$  provided either  $\gamma \in (\alpha, \beta)$  or  $\gamma = \alpha$  and either  $v'_+(\alpha) \neq +\infty$  or  $D_+f(v(\alpha)) > 0$ .

(iii)  $D^-(f \circ v)(\gamma) = v'_-(\gamma) D^-f(v(\gamma))$  provided either  $\gamma \in (\alpha, \beta)$  or  $\gamma = \beta$  and  $v'_-(\beta) \neq 0$ .

Similar properties hold for  $D^+$  and  $D_-$ .

## 6.5 Proofs

For each agent  $i \in I$ , set

$$W_i(x, y, z) \equiv \sum_{s \in S_0} \pi_s u_i(x_s, y_s) + \sum_{s \in S_0} \pi_s [\gamma_i(v(x_s) - z_s)] \quad (22)$$

for all  $(x, y, z) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \times \mathbb{R}^{S+1}$ , where  $\pi_0 = 1$  and  $\pi_s = \beta p_s$  for all  $s \in S$ . Notice that  $V_i(c, e) =$

$$\sum_{s \in S_0} \pi_s u_i(c_{i,s}, e_{i,s}) + \sum_{s \in S_0} \pi_s \gamma_i \left( v(c_{i,s}) - \int_I (v \circ c_s) d\lambda \right) = W_i \left( c_i, e_i, \left[ \int_I (v \circ c_s) d\lambda \right]_{s \in S_0} \right)$$

for all  $(c, e) \in L_+^{S+1} \times L_+^{S+1}$ .

**Lemma 2** If  $(c^*, e^*)$  is a social equilibrium, then  $(c^*, e^*) \in L^{S+1} \times L^{S+1}$  and  $(c_i^*, e_i^*)$  is a solution of problem  $\max_{(x,y) \in B_i} W_i(x, y, z)$  where  $z_s \equiv \int_I (v \circ c_s^*) d\lambda$  for all  $s \in S_0$ , for  $\lambda$ -almost all  $i \in I$ . The converse is true up to a  $\lambda$ -negligible variation of  $(c^*, e^*)$ .<sup>12</sup>

The simple proof is omitted.

**Lemma 3** If H.1 holds, then  $B_i$  is compact and nonempty for all  $i \in I$ .

**Proof.** Since  $F_{i,s}$  is increasing, then

$$B_i = \left\{ (x, y) \in \mathbb{R}_+^{S+1} \times \prod_{s=0}^S [0, h_{i,s}] : F_{i,0}(y_0) \geq x_0, x_s = F_{i,s}(y_s) + R(F_{i,0}(y_0) - x_0) \quad \forall s \in S \right\}$$

that is

$$\begin{aligned} B_i = & \left( [0, F_{i,0}(h_{i,0})] \times \prod_{s=1}^S [0, F_{i,s}(h_{i,s}) + R F_{i,0}(h_{i,0})] \times \prod_{s=0}^S [0, h_{i,s}] \right) \quad (23) \\ & \cap \left\{ (x, y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,0}(y_0) - x_0 \geq 0 \right\} \\ & \cap \left\{ (x, y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,1}(y_1) + R(F_{i,0}(y_0) - x_0) - x_1 = 0 \right\} \\ & \dots \\ & \cap \left\{ (x, y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,S}(y_S) + R(F_{i,0}(y_0) - x_0) - x_S = 0 \right\} \end{aligned}$$

<sup>12</sup>A  $\lambda$ -negligible variation of a function on a measure space  $(I, \Lambda, \lambda)$  is a function that coincides  $\lambda$ -almost everywhere with the original one.

which is compact since the functions  $(x, y) \mapsto F_{i,0}(y_0) - x_0$  and  $(x, y) \mapsto F_{i,s}(y_s) + R(F_{i,0}(y_0) - x_0) - x_s$  are continuous for all  $s \in S$ . Moreover, for all  $y \in \prod_{s=0}^S [0, h_{i,s}]$ ,  $\left( (F_{i,s}(y_s))_{s=0}^S, (y_s)_{s=0}^S \right) \in B_i$ , which implies  $B_i \neq \emptyset$ .  $\blacksquare$

**Proof of Theorem 1.** For each agent  $i$ , consider the strategy set  $B_i \subseteq \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$  and the payoff function  $W_i(x, y, z) = \sum_{s \in S_0} \pi_s u_i(x_s, y_s) + \sum_{s \in S_0} \pi_s [\gamma_i(v(x_s) - z_s)]$  for all  $(x, y, z) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \times \mathbb{R}^{S+1}$ , where  $\pi_0 = 1$  and  $\pi_s = \beta p_s$  for all  $s \in S$ .

Since  $\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$  and  $\mathbb{R}^{S+1}$  are Polish spaces, assumptions 2.1 and 2.2 of Balder (1995) hold. Since  $B_i$  is nonempty and compact for all  $i \in I$ , assumption 2.3 of Balder (1995) holds. For every  $i \in I$ ,  $W_i : \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \times \mathbb{R}^{S+1} \rightarrow \mathbb{R}$  is continuous, and so assumptions 2.4 and 2.6 of Balder (1995) hold.

Assumptions H.1.ii and H.2.iii guarantee that the real valued functions on  $I \times \left( \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$  defined by  $(i, (x, y)) \mapsto F_{i,0}(y_0) - x_0$  and by  $(i, (x, y)) \mapsto F_{i,s}(y_s) + R(F_{i,0}(y_0) - x_0) - x_s$  are Caratheodory functions for all  $s \in S$ , hence they are  $\Lambda \times \mathcal{B} \left( \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$ -measurable and the graphs of the correspondences

$$i \mapsto \left\{ (x, y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,0}(y_0) - x_0 \geq 0 \right\}$$

and

$$i \mapsto \left\{ (x, y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} : F_{i,s}(y_s) + R(F_{i,0}(y_0) - x_0) - x_s = 0 \right\}$$

are  $\Lambda \times \mathcal{B} \left( \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$ -measurable.

Moreover, the functions  $i \mapsto F_s(i, h_s(i))$  are  $\Lambda$ -measurable for all  $s \in S_0$ , since  $i \mapsto (i, h_s(i))$  is  $\Lambda$ -measurable on  $I$  and  $(i, t) \mapsto F_{i,s}(t)$  is a Caratheodory function on  $I \times \mathbb{R}_+$ . Therefore the graph of the correspondence  $i \mapsto [0, F_{i,0}(h_{i,0})] \times \prod_{s=1}^S [0, F_{i,s}(h_{i,s}) + R F_{i,0}(h_{i,0})] \times \prod_{s=0}^S [0, h_{i,s}]$  is  $\Lambda \times \mathcal{B} \left( \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$ -measurable.

By (23), for all  $i \in I$ , the graph of the correspondence  $B : i \mapsto B_i$  is  $\Lambda \times \mathcal{B} \left( \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$ -measurable and assumption 2.5 of Balder (1995) holds.

For every  $z \in \mathbb{R}^{S+1}$ ,  $W_{(\cdot)}(\cdot, z) : (i, (x, y)) \mapsto W_i((x, y), z)$  is a Caratheodory function on  $I \times \left( \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \right)$ . Hence, it is  $\Lambda \times \mathcal{B}(\mathbb{R}^n)$ -measurable, and so assumption 2.7 of Balder (1995) holds.

Now define  $g_s : I \times \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1} \rightarrow \mathbb{R}$  by  $g_s(i, (x, y)) = v(x_s)$  for all  $s \in S_0$ . Let  $s \in S_0$ , clearly  $g_s(i, (\cdot, \cdot))$  is continuous on  $\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$  for all  $i \in I$  and  $g_s(\cdot, (x, y))$  is constant – hence  $\Lambda$ -measurable – on  $I$  for all  $(x, y) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$ . Therefore  $g_s$  is a Caratheodory function for all  $s \in S_0$ , in particular, it is  $\Lambda \times \mathcal{B}(\mathbb{R}^n)$ -measurable.

For all  $i \in I$ ,  $\inf_{(x,y) \in B_i} g_s(i, (x, y)) = \inf_{(x,y) \in B_i} v(x_s) \geq v(0)$ ,  $\sup_{(x,y) \in B_i} g_s(i, (x, y)) = \sup_{(x,y) \in B_i} v(x_s) \leq \sup_{x_s \in [0, F_{i,s}(h_{i,s}) + R F_{i,0}(h_{i,0})]} v(x_s) = v(F_{i,s}(h_{i,s}) + R F_{i,0}(h_{i,0}))$ , and  $i \mapsto v(F_{i,s}(h_{i,s}) + R F_{i,0}(h_{i,0}))$  is  $\Lambda$ -measurable and bounded (by H.2.i). Therefore, assumption 3.4.2 of Balder (1995) holds.

Finally, nonatomicity of  $\lambda$  guarantees that assumption 3.4.1 of Balder (1995) holds too.

Therefore, by Theorem 3.4.1 of Balder (1995) there exists a  $\lambda$ -measurable almost everywhere selection  $(c^*, e^*)$  of the correspondence  $B : i \mapsto B_i$  such that for  $\lambda$ -almost all  $i$ ,

$(c_i^*, e_i^*) \in \arg \max_{(x,y) \in B_i} (\sum_{s \in S_0} \pi_s u_i(x_s, y_s) + \sum_{s \in S_0} \pi_s [\gamma_i(v(x_s) - m_s(c^*, e^*))])$  where  $m_s(c^*, e^*) = \int_I g_s(\iota, (c_{\iota,s}^*, e_{\iota,s}^*)) d\lambda(\iota) = \int_I v(c_{\iota,s}^*) d\lambda(\iota)$ . Since  $B_i$  is never empty, wlog, we can assume that  $(c_i^*, e_i^*) \in B_i$  for all  $i \in I$ . Then, by Corollary, we only have to check that  $(c^*, e^*)$  is bounded. This is easily obtained, setting  $\Xi = \max_{s \in S_0} (\sup_{i \in I} (F_{i,s}(h_{i,s}) + h_{i,s}))$  which is finite by H.2.i, and observing that, for all  $i \in I$ ,  $B_i \subseteq [0, F_{i,0}(h_{i,0})] \times \prod_{s=1}^S [0, F_{i,s}(h_{i,s}) + R F_{i,0}(h_{i,0})] \times \prod_{s=0}^S [0, h_{i,s}] \subseteq [0, \Xi] \times \prod_{s=1}^S [0, (1+R)\Xi] \times \prod_{s=0}^S [0, \Xi]$ . ■

**Proof of Lemma 1.** For all  $i \in I$ , problem (6) is equivalent to  $\max_{0 \leq y \leq h_i} u_i(F_i(y), y)$ . Setting  $U_i(y) = u_i(F_i(y), y)$  for all  $y \in \mathbb{R}_+$  it is easily checked that

$$U_i'(y) = \nabla u_i(F_i(y), y) \begin{bmatrix} F_i'(y) \\ 1 \end{bmatrix} = F_i'(y) \frac{\partial u_i}{\partial x}(F_i(y), y) + \frac{\partial u_i}{\partial y}(F_i(y), y)$$

and

$$U_i''(y) = \begin{bmatrix} F_i'(y) \\ 1 \end{bmatrix}^\top \nabla^2 u_i(F_i(y), y) \begin{bmatrix} F_i'(y) \\ 1 \end{bmatrix} + F_i''(y) \frac{\partial u_i}{\partial x}(F_i(y), y).$$

H.3 guarantees that  $U_i'' < 0$  on  $(0, h_i)$ , in particular,  $U_i$  is concave on  $[0, h_i]$  and  $U_i'$  is strictly decreasing on  $(0, h_i)$ .

If  $U_i'$  never vanishes, since derivatives have the Darboux property, then  $U_i$  is either strictly increasing or decreasing on  $[0, h_i]$  and the maximum is achieved at  $y_i^* = h_i$  or  $y_i^* = 0$  (and nowhere else).

If  $U_i'$  vanishes at some  $y_i^*$  in  $(0, h_i)$ , then  $y_i^*$  is the unique maximum ( $U_i'$  is strictly decreasing on  $(0, h_i)$ ).

We can conclude that if an equilibrium profile  $(c^*, e^*)$  exists, then it is,  $\lambda$ -a.e. unique since it must satisfy  $c_i^* = F_i(y_i^*)$  and  $e_i^* = y_i^*$  for  $\lambda$ -almost all  $i \in I$ . ■

Next proposition shows that H.1, H.2, and H.4-(iii) imply that  $W : E \rightarrow L$  is well defined.

**Proposition 3** *If H1-H2 hold, then  $W_{(\cdot)}(e) : i \mapsto W_i(e)$  is  $\Lambda$ -measurable for all  $e \in E$ . Moreover,  $W(e) \in L$  for all  $e \in E$  provided*

$$\sup_{i \in I} \left( \sup_{(x,y,z) \in [0,n]^2 \times [-n,n]} |u_i(x, y) + \gamma_i(z)| \right) < \infty \quad \forall n \in \mathbb{N}. \quad (24)$$

**Proof.** Fix  $e \in E$ . The function  $i \mapsto F_i(e(i))$  is  $\Lambda$ -measurable, since  $i \mapsto (i, e(i))$  is  $\Lambda$ -measurable from  $I$  to  $I \times \mathbb{R}_+$  and  $(i, t) \mapsto F_i(t)$  is a Caratheodory function from  $I \times \mathbb{R}_+$  to  $\mathbb{R}_+$ . H.2-(i) implies that the function  $i \mapsto F_i(e(i))$  is also bounded.

Set  $m^* = \int v(F_i(e_i)) d\lambda(\iota)$ .

For every  $i \in I$ , the real valued function on  $\mathbb{R}_+^2$  defined by  $(x, y) \mapsto u_i(x, y) + \gamma_i(v(x) - m^*)$  is continuous, and for every  $(x, y) \in \mathbb{R}_+^2$ , the real valued function on  $I$  defined by  $i \mapsto$

$u_i(x, y) + \gamma_i(v(x) - m^*)$  is  $\Lambda$ -measurable. Hence, the real valued function on  $I \times \mathbb{R}_+^2$  defined by

$$(i, (x, y)) \mapsto u_i(x, y) + \gamma_i(v(x) - m^*) \quad (25)$$

is  $\Lambda \times \mathcal{B}(\mathbb{R}_+^2)$ -measurable (being a Caratheodory function).

Conclude that  $i \mapsto (i, F_i(e_i), e_i)$  is  $\Lambda$ -measurable from  $I$  to  $I \times \mathbb{R}_+^2$  (since  $i \mapsto F_i(e(i))$  is  $\Lambda$ -measurable), and its composition with (25) delivers measurability of  $W_{(\cdot)}(e)$ .

Finally  $\sup_{i \in I} |W_i(e)| = \sup_{i \in I} |u_i(F_i(e_i), e_i) + \gamma_i(v(F_i(e_i)) - m^*)|$ . By H.2.i, it follows that  $\Xi = \sup_{i \in I} (F_i(h_i) + h_i) < \infty$  hence  $(0, 0) \leq (F_i(e_i), e_i) \leq (F_i(h_i), h_i) \leq (\Xi, \Xi)$  for all  $i \in I$ , moreover  $v(F_i(e_i)) \in [v(0), v(\Xi)]$  and  $v(F_i(e_i)) - m^* \in [v(0) - m^*, v(\Xi) - m^*]$  for all  $i \in I$ , thus  $\sup_{i \in I} |W_i(e)| \leq \sup_{i \in I} \left( \sup_{(x,y,z) \in [0,\Xi]^2 \times [v(0)-m^*, v(\Xi)-m^*]} |u_i(x, y) + \gamma_i(z)| \right)$  which is finite if (24) holds.  $\blacksquare$

**Lemma 4** *If H.1-H.3 and H.4.i hold, then all social equilibria  $(c^*, e^*)$  are such that  $c_i^* \geq \hat{c}_i$  and  $e_i^* \geq \hat{e}_i$   $\lambda$ -a.e. If  $(c^*, e^*)$  is internal and  $D^+ \gamma_i > 0$ , then,  $e_i^* > \hat{e}_i$  and  $c_i^* > \hat{c}_i$   $\lambda$ -a.e..*

**Proof.** Notice that, by Lemma 2, if a pair  $(c^*, e^*) \in L \times L$  is a social equilibrium, then, setting  $m^* = \int (v \circ c^*) d\lambda$ ,  $(c_i^*, e_i^*)$  is a solution of problem

$$\max_{(x,y) \in B_i} u_i(x, y) + \gamma_i(v(x) - m^*). \quad (26)$$

For all  $i \in I$ , problem (26) is equivalent to  $\max_{0 \leq y \leq h_i} u_i(F_i(y), y) + \gamma_i(v(F_i(y)) - m^*)$ .

If  $(c^*, e^*)$  is a social equilibrium and  $(\hat{c}, \hat{e})$  is the asocial equilibrium, then for  $\lambda$ -almost all  $i \in I$ ,

$$\begin{aligned} e_i^* &\in \arg \max_{0 \leq y \leq h_i} u_i(F_i(y), y) + \gamma_i(v(F_i(y)) - m^*), \\ c_i^* &= F(e_i^*), \quad m^* = \int v(F_i(e_i^*)) d\lambda(\iota), \\ \hat{e}_i &\in \arg \max_{0 \leq y \leq h_i} u_i(F_i(y), y), \quad \hat{c}_i = F(\hat{e}_i). \end{aligned}$$

If  $e_i^* = h_i$  or  $\hat{e}_i = 0$ , then  $e_i^* \geq \hat{e}_i$  and  $c_i^* = F_i(e_i^*) \geq F_i(\hat{e}_i) = \hat{c}_i$ .

If  $e_i^* = 0$  and  $\hat{e}_i > 0$ , then:

- either  $U_i'$  never vanishes in  $(0, h_i)$ ,<sup>13</sup> then  $U_i$  is strictly increasing (it cannot be strictly decreasing, otherwise  $\hat{e}_i = 0$ ), but also  $\gamma_i(v(F_i(\cdot)) - m^*)$  is increasing and the inclusion  $0 \in \arg \max_{0 \leq y \leq h_i} u_i(F_i(y), y) + \gamma_i(v(F_i(y)) - m^*) = \arg \max_{0 \leq y \leq h_i} U_i(y) + \gamma_i(v(F_i(y)) - m^*)$  is absurd,
- or  $U_i'$  vanishes at  $\hat{e}_i \in (0, h_i)$ , then  $U_i'$  – being strictly decreasing – must be positive in a right neighborhood of 0, again  $u_i(F_i(\cdot), \cdot) + \gamma_i(v(F_i(\cdot)) - m^*)$  is strictly increasing in a right neighborhood of 0, which is absurd.

<sup>13</sup> $U_i$  is defined in the proof of Lemma 1.

It follows that, if  $e_i^* = 0$ , then  $\hat{e}_i = 0$ , thus  $e_i^* \geq \hat{e}_i$  and  $c_i^* = F_i(e_i^*) \geq F_i(\hat{e}_i) = \hat{c}_i$ .

Finally, if  $e_i^* \in (0, h_i)$ , then

$$D^+ [u_i(F_i(\cdot), \cdot) + \gamma_i(v(F_i(\cdot)) - m^*)](e_i^*) \leq 0 \quad (27)$$

that is  $U_i'(e_i^*) + F_i'(e_i^*) v'(F_i(e_i^*)) D^+ \gamma_i(v(F_i(e_i^*)) - m^*) \leq 0$  and

$$U_i'(e_i^*) \leq -F_i'(e_i^*) v'(F_i(e_i^*)) D^+ \gamma_i(v(F_i(e_i^*)) - m^*). \quad (28)$$

By monotonicity,  $D^+ \gamma_i \geq 0$ , therefore  $U_i'(e_i^*) \leq 0$ , which implies  $e_i^* \geq \hat{e}_i$  because from the proof of Lemma 1 we know that  $U_i$  is concave on  $[0, h_i]$  with a unique maximum. Again  $c_i^* = F_i(e_i^*) \geq F_i(\hat{e}_i) = \hat{c}_i$ .

Suppose that  $(c^*, e^*)$  is an internal social equilibrium and  $D^+ \gamma_i > 0$ . Then (28) delivers  $U_i'(e_i^*) < 0$ , which implies  $e_i^* > \hat{e}_i$  because from the proof of Lemma 1 we know that  $U_i$  is concave on  $[0, h_i]$  with a unique maximum. It follows that  $c_i^* = F_i(e_i^*) > F_i(\hat{e}_i) = \hat{c}_i$ . ■

**Proposition 4** *If assumptions H.1-H.4 hold, then all internal social equilibrium profiles  $(e^*, c^*)$  are strongly inefficient.*

**Proof.** Suppose, *per contra*, that  $(c^*, e^*) \in L \times L$  is a social equilibrium with  $e^* \in \text{int}(E)$  and  $(c^*, e^*)$  is not strongly inefficient. Let  $f : L \rightarrow \mathbb{R}$  be defined by  $f(\xi) = \text{essinf}_\lambda [\xi - W(e^*)]$  for all  $\xi \in L$ . Then,  $f(W(e^*)) = 0$  and  $f(W(e)) \leq 0$  for all  $e \in E$ . Moreover,  $f$  is a concave niveloid.<sup>14</sup>

Next we show that there is no concave niveloid  $f : L \rightarrow \mathbb{R}$  such that  $e^*$  solves the problem  $\max_{e \in E} (f \circ W)(e)$ , which is absurd.

First observe that Gateaux differentiability of  $W$  guarantees that for all  $e \in \text{int}E$  there exists a linear and continuous operator  $\nabla W(e) : L \rightarrow L$  such that

$$\lim_{t \rightarrow 0} \frac{W(e + tk) - W(e)}{t} = \nabla W(e)(k) \in L \quad (29)$$

for all  $k \in L$ . Arbitrarily choose  $e \in \text{int}E$  and  $k \in L$ , (29) means that

$$\lim_{t \rightarrow 0} \left\| \frac{W(e + tk) - W(e)}{t} - \nabla W(e)(k) \right\|_{\text{sup}} = 0.$$

A fortiori, for all  $i \in I$ ,  $\frac{W_i(e+tk) - W_i(e)}{t} \rightarrow \nabla W(e)(k)_i$  as  $t \rightarrow 0$ , but for all  $i \in I$

$$\frac{W_i(e + tk) - W_i(e)}{t} = \frac{u_i(F_i(e_i + tk_i), e_i + tk_i) + \gamma_i(v(F_i(e_i + tk_i)) - \int v(F_l(e_l + tk_l)) d\lambda(\iota))}{t} - \frac{u_i(F_i(e_i), e_i) + \gamma_i(v(F_i(e_i)) - \int v(F_l(e_l)) d\lambda(\iota))}{t}.$$

<sup>14</sup>A functional  $f : M \rightarrow \mathbb{R}$  is a niveloid (see Maccheroni, Marinacci, and Rustichini, 2006) if, for all  $\psi$  and  $\varphi$  in  $M$  and  $c \in \mathbb{R}$ , we have: (i)  $\varphi \geq \psi$  implies  $f(\varphi) \geq f(\psi)$ ; (ii)  $f(\varphi + c) = f(\varphi) + c$ .

It is relatively easy to show that, this implies

$$\begin{aligned} \nabla W(e)(k)_i = & k_i U'_i(e_i) + \gamma'_i(v(F_i(e_i)) - \int v(F_\iota(e_\iota)) d\lambda(\iota)) \times \\ & \times (v'(F_i(e_i)) F'_i(e_i) k_i - \int v'(F_\iota(e_\iota)) F'_\iota(e_\iota) k_\iota d\lambda(\iota)) \end{aligned} \quad (30)$$

for  $\lambda$ -almost all  $i \in I$ .

If  $f : L \rightarrow \mathbb{R}$  is concave niveloid, then it is Lipschitz and its superdifferential at each point consists of probability charges that are absolutely continuous with respect to  $\lambda$ .

By a chain rule for the Clarke differential (see Theorem 2.3.10 in Clarke, 1983), we have that  $f \circ W$  is Lipschitz near  $e$  and  $\partial(f \circ W)(e) \subseteq \partial f(W(e)) \circ \nabla W(e)$ . That is, for all  $\mu \in \partial(f \circ W)(e)$  there is  $\nu \in \partial f(W(e))$  such that  $\mu = \nu \circ \nabla W(e)$ . Therefore, for all  $k \in L$

$$\begin{aligned} \mu(k) = \nu(\nabla W(e)(k)) &= \int_I \nabla W(e)(k) d\nu \\ &= \int_I \left[ k_i U'_i(e_i) + \gamma'_i \left( v(F_i(e_i)) - \int v(F_\iota(e_\iota)) d\lambda(\iota) \right) \left( v'(F_i(e_i)) F'_i(e_i) k_i - \int v'(F_\iota(e_\iota)) F'_\iota(e_\iota) k_\iota d\lambda(\iota) \right) \right] d\nu(i). \end{aligned}$$

If, as assumed *per contra*,  $e^*$  is a local maximum of  $f \circ W$  on  $E$ , then  $\partial(f \circ W)(e^*) \ni 0$ , and there exists a probability charge  $\nu \in \partial f(W(e^*))$  such that  $\nu \circ \nabla W(e^*) = 0$ , that is, for all  $k \in L$

$$\int_I \left[ k_i U'_i(e_i^*) + \gamma'_i \left( v(F_i(e_i^*)) - \int v(F_\iota(e_\iota^*)) d\lambda(\iota) \right) \left( v'(F_i(e_i^*)) F'_i(e_i^*) k_i - \int v'(F_\iota(e_\iota^*)) F'_\iota(e_\iota^*) k_\iota d\lambda(\iota) \right) \right] d\nu(i) = 0 \quad (31)$$

But, for all  $i \in I$ , problem (26) is equivalent to

$$\max_{0 \leq y \leq h_i} u_i(F_i(y), y) + \gamma_i(v(F_i(y)) - m^*). \quad (32)$$

Therefore, if  $(c^*, e^*)$  is an internal social equilibrium,

- $c_i^* = F_i(e_i^*)$  for all  $i \in I$ .
- $e_i^*$  is a solution of problem (32) for  $\lambda$ -almost all  $i \in I$ .
- $m^* = \int v(F_\iota(e_\iota^*)) d\lambda(\iota)$ .

In particular, first order conditions implied by the second point, see (27) and recall that now  $\gamma_i$  is differentiable, amount to  $U'_i(e_i^*) + \gamma'_i(v(F_i(e_i^*)) - m^*) v'(F_i(e_i^*)) F'_i(e_i^*) = 0$  for  $\lambda$ -almost all  $i \in I$ . Which plugged into (31) delivers,

$$\int v'(F_\iota(e_\iota^*)) F'_\iota(e_\iota^*) k_\iota d\lambda(\iota) \int_I -\gamma'_i(v(F_i(e_i^*)) - m^*) d\nu(i)$$

= 0 for all  $k \in L$ .

Since  $v'$  and  $F'_\iota$  are positive and  $\lambda$  is  $\sigma$ -additive, then  $\int v'(F_\iota(e_\iota^*)) F'_\iota(e_\iota^*) k_\iota d\lambda(\iota) > 0$  for some  $k \in L$  (e.g.  $k_\iota = 1$  for all  $\iota \in I$ ) and it must be the case that  $\int_I -\gamma'_i(v(F_i(e_i^*)) - m^*) d\nu(i) = 0$ , which is absurd since  $\gamma'_i$  is bounded away from 0.  $\blacksquare$

**Proof of Proposition 1.** It follows from Lemma 4 and Proposition 4. ■

**Proof of Theorem 2.** Define  $V : [0, \bar{x}_0] \rightarrow \mathbb{R}$  by

$$V(x) = U(x) + \gamma(v(x) - m_0^*) + \beta \sum_{s \in S} p_s [\gamma(v(\bar{x}_s + R(\bar{x}_0 - x)) - m_s^*)]$$

and set  $y_s = \bar{x}_s + R(\bar{x}_0 - x)$  for all  $s \in S$ . For all  $x \in [0, \bar{x}_0]$ ,

$$\begin{aligned} D_+V(x) &\geq \liminf_{h \rightarrow 0^+} \frac{U(x+h) - U(x)}{h} \\ &\quad + \liminf_{h \rightarrow 0^+} \frac{\gamma(v(x+h) - m_0^*) - \gamma(v(x) - m_0^*)}{h} \\ &\quad + \beta \sum_{s \in S} p_s \liminf_{h \rightarrow 0^+} \frac{\gamma(v(y_s - Rh) - m_s^*) - \gamma(v(y_s) - m_s^*)}{h}. \end{aligned}$$

Analogously, for all  $x \in (0, \bar{x}_0]$ ,

$$\begin{aligned} D^-V(x) &\leq \limsup_{h \rightarrow 0^-} \frac{U(x+h) - U(x)}{h} \\ &\quad + \limsup_{h \rightarrow 0^-} \frac{\gamma(v(x+h) - m_0^*) - \gamma(v(x) - m_0^*)}{h} \\ &\quad + \beta \sum_{s \in S} p_s \limsup_{h \rightarrow 0^-} \frac{\gamma(v(y_s - Rh) - m_s^*) - \gamma(v(y_s) - m_s^*)}{h} \end{aligned}$$

Consider any symmetric consumption profile, where all agents consume the same amount  $x^* \in [0, \bar{x}_0]$  in the first period (i.e.  $c_i^* = x^*$  for all  $i \in I$ ), and  $y_s^* = \bar{x}_s + R(\bar{x}_0 - x^*)$  in each state in the second period. Then, for each  $s \in S$ ,

$$m_0^* = m_0^*(c^*) = \int_I v(x^*) d\lambda(\iota) = v(x^*)$$

and

$$m_s^* = m_s^*(c^*) = \int_I v(\bar{x}_s + R(\bar{x}_0 - x^*)) d\lambda(\iota) = v(y_s^*).$$

For  $x^* \in [0, \bar{x}_0]$ ,

$$\begin{aligned} D_+V(x^*) &\geq U'_+(x^*) + \liminf_{h \rightarrow 0^+} \frac{\gamma(v(x^* + h) - v(x^*)) - \gamma(v(x^*) - v(x^*))}{h} \\ &\quad + \beta \sum_{s \in S} p_s \liminf_{h \rightarrow 0^+} \frac{\gamma(v(y_s^* - Rh) - v(y_s^*)) - \gamma(v(y_s^*) - v(y_s^*))}{h} \\ &= U'_+(x^*) + \liminf_{h \rightarrow 0^+} \frac{\gamma(v(x^* + h) - v(x^*)) - \gamma(0)}{h} \\ &\quad + \beta \sum_{s \in S} p_s \liminf_{h \rightarrow 0^+} \frac{\gamma(v(y_s^* - Rh) - v(y_s^*)) - \gamma(0)}{h} \end{aligned}$$

The function  $v_0 : \mathbb{R}_+ \rightarrow \mathbb{R}$  defined by  $v_0(h) = v(x^* + h) - v(x^*)$  for all  $h \in [0, +\infty)$  is continuous, concave, differentiable on  $(0, +\infty)$  with  $v'_0 > 0$ . Moreover,  $v_0(0) = 0$  and

$(v_0)'_+(0) = v'_+(x^*)$ . Therefore, since Assumption H.6 allows to apply a chain rule for Dini derivatives, we have

$$\begin{aligned} \liminf_{h \rightarrow 0^+} \frac{\gamma(v(x^* + h) - v(x^*)) - \gamma(0)}{h} &= D_+(\gamma \circ v_0)(0) \\ &= (v_0)'_+(0) D_+\gamma(v_0(0)) = v'_+(x^*) D_+\gamma(0). \end{aligned}$$

For all  $s \in S$ , the function  $v_s : [-y_s^*/R, +\infty) \rightarrow \mathbb{R}$  by  $v_s(h) = v(y_s^* + Rh) - v(y_s^*)$  is continuous, concave, differentiable on  $(-y_s^*/R, +\infty)$  with  $v'_s > 0$  and  $v_s(0) = 0$ . Moreover,

$$\begin{aligned} (v_s)'_-(0) &= \lim_{h \rightarrow 0^-} \frac{v_s(h) - v_s(0)}{h} = \lim_{h \rightarrow 0^-} \frac{v(y_s^* + Rh) - v(y_s^*)}{h} \\ &= R \lim_{h \rightarrow 0^-} \frac{v(y_s^* + Rh) - v(y_s^*)}{Rh} = R \lim_{t \rightarrow 0^-} \frac{v(y_s^* + t) - v(y_s^*)}{t} = Rv'_-(y_s^*). \end{aligned}$$

Thus, since Assumption H.6 allows to apply a chain rule for Dini derivatives, we have

$$\begin{aligned} &\liminf_{h \rightarrow 0^+} \frac{\gamma(v(y_s^* - Rh)) - v(y_s^*) - \gamma(0)}{h} \\ &= \lim_{\delta \rightarrow 0^+} \inf_{h \in (0, \delta)} \frac{\gamma(v(y_s^* - Rh)) - v(y_s^*) - \gamma(0)}{h} \\ &= - \lim_{\delta \rightarrow 0^+} \sup_{h \in (0, \delta)} \frac{\gamma(v(y_s^* - Rh)) - v(y_s^*) - \gamma(0)}{-h} \\ &= - \lim_{\delta \rightarrow 0^+} \sup_{h \in (-\delta, 0)} \frac{\gamma(v(y_s^* + Rh)) - v(y_s^*) - \gamma(0)}{h} \\ &= - \limsup_{h \rightarrow 0^-} \frac{(\gamma \circ v_s)(h) - (\gamma \circ v_s)(0)}{h} \\ &= -D^-(\gamma \circ v_s)(0) = -(v_s)'_-(0) D^-\gamma(0) = -Rv'_-(y_s^*) D^-\gamma(0) \end{aligned}$$

Hence,

$$D_+V(x^*) \geq U'_+(x^*) + v'_+(x^*) D_+\gamma(0) - \beta R D^-\gamma(0) \sum_{s \in S} p_s v'_-(y_s^*). \quad (33)$$

Analogously, for  $x^* \in (0, \bar{x}_0]$

$$\begin{aligned} D^-V(x^*) &\leq U'_-(x^*) + \limsup_{h \rightarrow 0^-} \frac{\gamma(v(x^* + h) - v(x^*)) - \gamma(0)}{h} \\ &\quad - \beta \sum_{s \in S} p_s \liminf_{h \rightarrow 0^+} \frac{\gamma(v(y_s^* + Rh) - v(y_s^*)) - \gamma(0)}{h}. \end{aligned}$$

The function  $v_0 : [-x^*, +\infty) \rightarrow \mathbb{R}$  defined by  $v_0(h) = v(x^* + h) - v(x^*)$  is continuous, concave, differentiable on  $(-x^*, +\infty)$  with  $v'_0 > 0$ . Moreover,  $v_0(0) = 0$  and

$$(v_0)'_-(0) = \lim_{h \rightarrow 0^-} \frac{v_0(h) - v_0(0)}{h} = \lim_{h \rightarrow 0^-} \frac{v(x^* + h) - v(x^*)}{h} = v'_-(x^*)$$

Thus, since Assumption H.6 allows to apply a chain rule for Dini derivatives, we have

$$\begin{aligned} \limsup_{h \rightarrow 0^-} \frac{\gamma(v(x^* + h) - v(x^*)) - \gamma(0)}{h} &= D^-(\gamma \circ v_0)(0) \\ &= (v_0)'_-(0) D^-\gamma(v_0(0)) = v'_-(x^*) D^-\gamma(0). \end{aligned}$$

For all  $s \in S$ ,  $v_s : \mathbb{R}_+ \rightarrow \mathbb{R}$  defined by  $v_s(h) = v(y_s^* + Rh) - v(y_s^*)$  for all  $h \in [0, +\infty)$  is continuous, concave, differentiable on  $(0, +\infty)$  with  $v_s' > 0$ . Moreover,  $v_s(0) = 0$  and  $(v_s)'_+(0) = Rv'_+(y_s^*)$ . Thus, since Assumption H.6 allows to apply a chain rule for Dini derivatives, we have

$$\liminf_{h \rightarrow 0^+} \frac{\gamma(v(y_s^* + Rh) - v(y_s^*)) - \gamma(0)}{h} = D_+(\gamma \circ v_s)(0) = RD_+\gamma(0)v'_+(y_s^*)$$

and so

$$D^-V(x^*) \leq U'_-(x^*) \dot{+} v'_-(x^*) D^-\gamma(0) \dot{-} \beta RD_+\gamma(0) \sum_{s \in S} p_s v'_+(y_s^*). \quad (34)$$

The rest of the proof is divided in three cases.

**Case (i)** Suppose  $x^* \in (0, \bar{x}_0)$  is a local maximum. Then  $D_+V(x^*) \leq 0 \leq D^-V(x^*)$ , and so, by (33) and (34),

$$\begin{aligned} U'(x^*) + v'(x^*) D_+\gamma(0) - \beta RD^-\gamma(0) \sum_{s \in S} p_s v'(y_s^*) &\leq \\ U'(x^*) \dot{+} v'(x^*) D^-\gamma(0) \dot{-} \beta RD_+\gamma(0) \sum_{s \in S} p_s v'(y_s^*) & \end{aligned}$$

because  $u$  and  $v$  are differentiable on  $(0, +\infty)$ , and so is  $U$  at  $x^*$  with  $U'_+(x^*) = U'_-(x^*) = U'(x^*)$ . In turn, this inequality implies

$$v'(x^*) D_+\gamma(0) - \beta RD^-\gamma(0) \sum_{s \in S} p_s v'(y_s^*) \leq v'(x^*) D^-\gamma(0) \dot{-} \beta RD_+\gamma(0) \sum_{s \in S} p_s v'(y_s^*)$$

that is,

$$(D_+\gamma(0) - D^-\gamma(0)) v'(x^*) \leq (D^-\gamma(0) - D_+\gamma(0)) \beta R \sum_{s \in S} p_s v'(y_s^*) \quad (35)$$

Since  $x^* \in (0, \bar{x}_0)$ , we have  $y_s^* > \bar{x}_s > 0$  for all  $s$ . Hence,  $v'(x^*) > 0$  and  $\sum_{s \in S} p_s v'(y_s^*) > 0$ . Since  $D_+\gamma(0) > D^-\gamma(0)$ , from (35) it follows  $D^-\gamma(0) - D_+\gamma(0) > 0$ , a contradiction. Thus,  $x^*$  is not a local maximum.

**Case (ii)** Let  $x^* = 0$ . Then,  $y_s^* = \bar{x}_s + R\bar{x}_0 > 0$  for all  $s$  and so  $v'(y_s^*) = v'_+(y_s^*) = v'_-(y_s^*) > 0$  for all  $s$ . This implies

$$\begin{aligned} D_+V(x^*) &\geq U'_+(x^*) + v'_+(x^*) D_+\gamma(0) - \beta RD^-\gamma(0) \sum_{s \in S} p_s v'(y_s^*) \\ &\geq U'_+(x^*) + v'_+(x^*) D_+\gamma(0) - \beta RD^-\gamma(0) v'_+(x^*) > 0 \end{aligned}$$

because  $U'_+(x^*) \geq 0$ ,  $\sum_{s \in S} p_s v'(y_s^*) > 0$ , and  $D_+\gamma(0) > D^-\gamma(0) \geq 0$ . Thus,  $D_+V(0) > 0$  and so  $x^* = 0$  is not a local maximum.

**Case (iii)** Let  $x^* = \bar{x}_0$ . Then  $x^* > 0$  and  $y_s^* = \bar{x}_s > 0$  for all  $s$  and so  $v'(x^*) > 0$  and  $v'(y_s^*) = v'_+(y_s^*) = v'_-(y_s^*) > 0$  for all  $s$ . This implies,

$$\begin{aligned} D^-V(x^*) &\leq U'_-(x^*) \dot{+} v'(x^*) D^- \gamma(0) \dot{-} \beta R D_+ \gamma(0) \sum_{s \in S} p_s v'(y_s^*) \\ &\leq U'_-(x^*) \dot{+} v'(x^*) D^- \gamma(0) \dot{-} \beta R D_+ \gamma(0) v'(x^*) < 0 \end{aligned}$$

because  $U'_-(x^*) \leq 0$ ,  $D_+ \gamma(0) > D^- \gamma(0)$ , and  $v'(x^*) \leq \sum_{s \in S} p_s v'(y_s^*)$  by assumption H.6-(ii). Thus,  $x^* = \bar{x}_0$  is not a local maximum.

(ii) Let  $c^* : I \rightarrow \mathbb{R}$  be a social equilibrium. Set  $m_0^* = \int_I v(c_i^*) d\lambda(\iota)$  and  $m_s^* = \int_I v(\bar{x}_s + R(\bar{x}_0 - c_i^*)) d\lambda(\iota)$  for all  $s \in S$ . Define  $V : [0, \bar{x}_0] \rightarrow \mathbb{R}$  by

$$V(x) = U(x) + \gamma(v(x) - m_0^*) + \beta \sum_{s \in S} p_s [\gamma(v(\bar{x}_s + R(\bar{x}_0 - x)) - m_s^*)].$$

Then,  $c_i^*$  is a solution of problem

$$\max_{x \in [0, \bar{x}_0]} V(x) \tag{36}$$

for  $\lambda$ -almost all  $i \in I$ . The strict concavity of  $u$  implies that of  $U$ . Along with the concavity of  $\gamma$  and  $\nu$ , this implies that  $V$  is strictly concave on  $[0, \bar{x}_0]$ . Hence, the solution to problem (36) is unique, say  $x^*$ . As a result,  $c_i^* = x^*$  for  $\lambda$ -almost all  $i \in I$ . ■

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