ECON 4261: Introduction to Econometrics
Spring 2006

Problem Set 1
Due: February 2, 2006

NOTE: All assignments must be typed. Graphs and calculations may be handwritten. **No late homeworks will be accepted.**

Exercise 1 (10 points) Let A be an $(m \times n)$ matrix, with $m > n$. Show that the matrix $A'A$ is symmetric, and that if A has full rank, $A'A$ is positive definite.

Exercise 2 (20 points) Characterize each of the following statements as true or false. If a statement is true, show the proof. If it is false, give a counterexample. Assume that A and B are square matrices.

(i) If A and B are non-singular, then AB is non-singular.

(ii) $(AB)' = A'B'$.

(iii) If $AB + BA = 0$, then $A^2B^3 = B^3A^2$.

(iv) If $A^2 = 0$, then $A = 0$.

(v) If A and B are symmetric, then AB is symmetric.

Exercise 3 (20 points) Let X and Y be two random variables. Consider the following joint density function

$$f(x,y) = \begin{cases} A(x^2+y^2) & \text{for } x \in [0,1] \text{ and } y \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

(i) Find A.

(ii) Find the marginal density $f(x)$.

(iii) Find the conditional density $f(y \mid x)$.

(iv) Find $Cov(X, Y)$.

(v) Are X and Y independent?

(vi) Find $P(\frac{1}{2} \leq x \leq 1, \frac{1}{2} \leq y \leq 1)$.

Exercise 4 (20 points) Let

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 4 & 2 \\ 0 & 7 & 3 \end{bmatrix}$$

(i) Find $A'A$. Show your work.

(ii) Find A^{-1}. Show your work.

(iii) Calculate the *trace* of A.

Exercise 5 (10 points) Let X be a random variable with the following density function

x	$P(X = x)$
2	0.08
5	0.12
7	0.22
9	0.03
14	0.28
21	0.17
25	0.10

(i) Calculate $E(X)$. Show your work.

(ii) Calculate $Var(X)$.Show your work.

(iii) Calculate the Standard Deviation of X. Show your work.

Exercise 6 (20 points) Let X, and Y be two random variables such that $X \sim U[0, 1]$, and $Y = 3X + 2$. Calculate the values for: $(E[X])^2$, $E[X^2]$, $E[Y]$, $Var[X]$, $Var[Y]$, $Cov[X, Y]$, and $Var[(2X + Y)^2]$. Show your work.

Exercise 7 (10 points) Give an example of two random variables X and Y such that $Cov(X, Y) = 0$, but X and Y are not independent.

Exercise 8 (10 points) Show that if X and Y are two independent random variables, then $Var(X + Y) = Var(X) + Var(Y)$.