Urban Economics: Economics 4621

University of Minnesota, Spring 2008

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Answer Sheet to Problem Set 1

Question 1.

The density gradients for the various demographic groups and geography groupings are displayed in the following table:

	city data	tract-level data
Total Population	-.141	-.148
White Population	-.130	-.104
Black Population	-.326	-.358

The R^2 for the regressions are displayed in the following table:

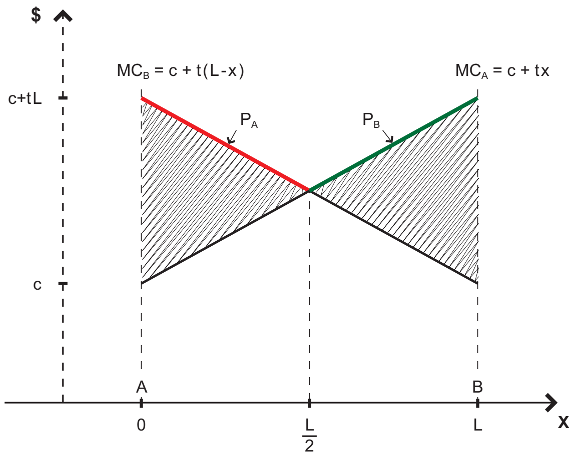
	city data	tract-level data
Total Population	.470	.607
White Population	.433	.387
Black Population	.673	.626

Question 2.

When each point is a market in which firms engage in Bertrand competition, standard result applies such that the firm with the lowest (after-delivery) marginal cost grabs the market and charges its price at the second lowest (after-delivery) marginal cost. Imagine that a firm A is located at 0, and a firm B is located at D. The after-delivery marginal cost for both firms on the area of $[0, D]$ is

$$MC_A = c + tx,$$
$$MC_B = c + t(D - x).$$

A corresponding graph is shown below (with the understanding that L is D). Thus, the area of the triangle marked on the area of $[0, D/2]$ represents the gross profit (profit gross of fixed cost) of A on this area. If firms are equal distance apart, A's gross profit is obviously twice the area of the triangle. So, take fixed cost into account, each firms' profit would be $\frac{tD^2}{2} - F$.



Recall that this is a two-stage game. The first stage is when firms decide whether to enter and pick their locations. The second stage is that firms compete in each local market in Bertrand competition. We usually solve backward in sequential game like this. What I have done so far is to describe what would be going on when the two firms are given the distance D. Since firms have this knowledge of what would be going on in the second stage, given that they would enter, they would pick an optimal location given others' location. It is easy to verify that it is optimal to locate in the middle in between one's neighbor. Thus, firms must be equal distance apart.

Given free entry and exit, the equilibrium distance must be that all firms earn nonnegative profit (pinning down minimum distance, i.e., the zero-profit distance) and that the distance cannot be so large that new entrant can come in to get positive profit (the equilibrium distance cannot be larger than twice the zero-profit distance). Zero-profit distance is $\sqrt{\frac{2F}{t}}$. Thus, the range of equilibrium distance is $[\sqrt{\frac{2F}{t}}, 2\sqrt{\frac{2F}{t}}]$.

Assuming density $m = 1$, from the lecture note on social planner's problem, the socially optimal distance is $2\sqrt{\frac{F}{t}}$. Thus, the socially optimal distance can be decentralized. One equilibrium distance is just socially optimal, while other distances are either larger or smaller. Thus, entry of firms could be either just right, insufficient, or excessive.

Question 3.

The equilibrium conditions are:

(1) Given that a particular individual locates at u, the individual picks the consumption bundle $(X(u), L(u))$ that maximizes utility subject to the budget constraint

$$pX + L(u)R(u) = w - tu.$$

(2) Individuals are indifferent as to where they locate for any u between 0 and $\hat{u}$ (the higher computing costs of further away locations are exactly offset by the benefit of the lower land rents).

(3) $R(\hat{u}) = \overline{R}$; i.e., the price of land at the boundary of the city equals the price of farm land.

(4) The demand for land by individuals equals the supply of land (The 2 in formula below adds up the deman in both sides of the city.)

$$H = 2\int_0^{\hat{u}} \frac{1}{L(u)} du$$