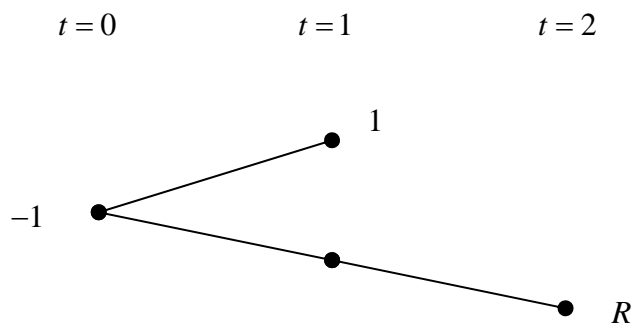## PROBLEM SET #3

1. Consider a simple version of the Diamond-Dybvig model of banking. There are three periods: 0,1,2. There is a single, storable good. There are many consumers who make a total deposit, which we normalize to 1 unit, in $t = 0$. The technology for investing in a project is given by:

The good can be invested in a project that pays $R > 1$ in $t = 2$ for each unit in $t = 0$. The project can be shut down in $t = 1$ and the investment can be salvaged one-for-one. A project that is shut down cannot be restarted. Consumers can store the good. The consumer's utility is $v(c_1, c_2, \theta)$, where θ takes on the value 1 or 2 in $t = 1$

$$v(c_1, c_2, 1) = u(c_1)$$
$$v(c_1, c_2, 2) = \beta u(c_1 + c_2).$$

Here $u(c) = \dfrac{c^{1-\sigma} - 1}{1 - \sigma}$.

a) Suppose that the probability that a consumer has the liquidity shock $\theta = 1$ in $t = 1$ is λ, $0 < \lambda < 1$. Derive the maximum expected utility for a consumer who invests on his own.

b) Suppose now that all the consumer deposit in a bank in $t = 0$. Liquidity shocks are independent across consumers. Now λ is the fraction of depositors who receive the liquidity shock $\theta = 1$ in $t = 1$ as well as the probability that and individual consumer receives the shock. Set up and explain the bank's optimal deposit contract problem of determining c_1^1, c_1^2, c_2^2, where the bank pays c_t^θ in period t to the depositor who has liquidity shock θ.

c) Show that, in the optimal deposit contract, if $R > 1/\beta > 1$ and $\sigma \geq 1$,

$$1 < c_1^1 < c_2^2 < R.$$

Interpret this contract in terms of gross rates of returns r_t on deports withdrawn in period t.

d) Suppose now that the type of a depositor in $t = 1$ is not verifiable, that is, it is private information. Argue that, if other patient depositors start to run on the bank every depositor will want to run on the bank.

e) Suppose that the bank realizes that there is a run as soon as it starts and that, if $f \geq 1/r_1$ depositors try to withdraw at $t = 1$, then it pays out all its deposits to withdrawers. Show that all depositors will want to withdraw if there is a bank run.

f) Suppose that what touches off a bank run is the realization of a sunspot variable ζ that takes on two values:

$$\text{prob}(\zeta = 1) = \pi, \ \text{prob}(\zeta = 0) = 1 - \pi.$$

Depositors run on the bank if $\zeta = 1$ but not if $\zeta = 0$. Calculate the expected utility of depositing in the bank in period $t = 0$. Show that consumers want to deposit if $\pi > 0$ is small enough but not if it is large.

g) Argue that a lender of last resort can stop a bank run in the equilibrium in part f.

h) Discuss the problems of using this model for analyzing actual bank runs because of what Neil Wallace calls the sequential service constraint.

2. Find a specific example of a financial crisis in Kindleberger and Aliber's *Manias, Panics and Crashes* or in Ferguson's *The Ascent of Money*.

a) Try to put this crisis into the taxonomy presented in Kindleberger and Aliber's Chapter 2 and further discussed in their Chapters 3, 4, and 5.

b) Discuss some of the problems of using a formal economic model to analyze this crisis. In particular, discuss what aspects of the crisis the model in question 1 helps you understand and what aspects it does not.

c) Discuss how government intervention could have helped, or did help, alleviate the impact of the crisis on the rest of the economy.

d) Explain how government intervention creates moral hazard. Discuss the problem of morel hazard in relation to part c.