

PROBLEM SET #4

1. Consider an economy with the following input-output matrix:

	Agr.	Mfg.	Con.	Inv.	Exp.	Total
Agriculture	2	3	8	2	5	20
Manufacturing	6	6	10	4	4	30
Imports	4	5				9
Tariff Revenue	2	1				3
Labor Compensation	3	10				13
Returns to Capital	3	5				8
Total	20	30	18	6	9	

- a) What are the national income and product accounts for this economy?
- b) Suppose that consumers and producers regard domestic goods and imports of goods as imperfect substitutes and that the Armington aggregators are Cobb-Douglas:

$$y_j = \gamma_j y_{j,d}^{\delta_j} y_{j,f}^{1-\delta_j}, \quad j = agr, man.$$

Calibrate these Armington aggregators. Calibrate the tariff rates τ_{agr} , τ_{man} .

- c) Suppose that all tariff revenues are transferred in lump-sum fashion to a representative consumer. Suppose that this consumer's utility function is Cobb-Douglas:

$$\theta_{agr} \log c_{agr} + \theta_{man} \log c_{man} + \theta_{inv} \log c_{inv}.$$

Calibrate the consumer's utility function and endowments $\bar{\ell}$, \bar{k} .

- d) Suppose that net domestic production of each good is governed by a nested production function that produces value added by combining labor and capital using a Cobb-Douglas

function and combines intermediate inputs of the other good and value-added in fixed proportions.

$$y_{j,d} = \min [x_{agr,j} / a_{agr,j}, x_{man,j} / a_{man,j}, \beta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j}], \quad j = agr, man.$$

Calibrate the two production functions.

- e) Suppose that there is a production function that produces the investment good using agriculture and manufactured goods in fixed proportions:

$$y_{inv} = \min [x_{agr,inv} / a_{agr,inv}, x_{man,inv} / a_{man,inv}].$$

Calibrate this production function.

- f) Suppose that the representative consumer in the rest of the world has a Cobb-Douglas utility function,

$$\theta_{agr,f} \log x_{agr,f} + \theta_{man,f} \log x_{man,f} + \theta_{f,f} \log x_{f,f}$$

Suppose too that the rest of the world imposes a 50 percent tariff on the imports of agricultural goods and a 10 percent tariff on imports of manufacturing goods from our economy. Expressed in the same units as the data in the input-output table, the income in the rest of the world is 1000. Calibrate the rest of the world's utility function.

2. Consider the model economy that you have calibrated in question 2.

- a) Define an equilibrium for this economy.

- b) Suppose that the elasticity of substitution between domestic and foreign goods in both the Armington aggregators in question 1, part b

$$y_j = \gamma_j [\delta_j y_{j,d}^\rho + (1 - \delta_j) y_{j,f}^\rho]^{1/\rho}, \quad j = agr, man,$$

and the rest of the world's utility function in question 1, part f

$$(\theta_{agr,f}x_{agr,f}^\rho + \theta_{man,f}x_{man,f}^\rho + \theta_{f,f}x_{f,f}^\rho - 1)/\rho$$

have a constant elasticity of substitution, $\sigma = 1/(1-\rho) = 5$, or $\rho = 0.8$. Redo question 1, parts b and f.

3. Suppose now that you have carefully constructed and calibrated an applied general equilibrium model of this economy and have performed numerical experiments of various options for trade reforms under different specifications of the elasticity of substitution in part b. You are going to brief the Minister of the Economy on the various options for unilaterally reforming trade policy or negotiating a free trade agreement with the United States. On your way to the ministry, you discover to your horror that (1) your computer's printer failed to print out all of the values for the variables in the reports of your numerical experiments and (2) you have forgotten your report that summarizes the results of the experiments and makes recommendations. Fortunately, you have a couple of hours before your briefing.

- a) Fill in the equilibrium values of the variables that are missing in the following tables.
- b) Write brief report, summarizing what the experiments indicate. Make recommendations for policy. (You probably want to indicate that there is a lot of controversy about the estimates for the Armington elasticities, but that you believe them to be high, more like 5 than 1. Explain why.)

Numerical Experiments I: Unilateral Reform

Tariffs

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
τ_{agr}	0.5000	0.2000	0.2000	0.0000	0.0000
τ_{man}	0.2000	0.1000	0.1000	0.0000	0.0000
$\tau_{agr,f}$	0.5000	0.5000	0.5000	0.5000	0.5000
$\tau_{man,f}$	0.1000	0.1000	0.1000	0.1000	0.1000

Prices

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
\hat{p}_{agr}	1.0000	0.9818	0.9687	0.9705	0.9389
\hat{p}_{man}	1.0000		1.0250	1.0236	1.0489
\hat{p}_{inv}	1.0000		1.0063	1.0059	
$\hat{p}_{agr,d}$	1.0000	1.0039		1.0055	1.0494
$\hat{p}_{man,d}$	1.0000	1.0017	1.0291	1.0014	1.0605
\hat{e}	1.0000	1.1650	1.1013	1.3405	1.2095
\hat{r}	1.0000	1.0006	1.0340	0.9997	1.0788
\hat{w}	1.0000	1.0004	1.0472	0.9983	1.0949

Here prices are normalized:

$$\frac{\theta_{agr}}{\theta_{agr} + \theta_{man}} \hat{p}_{agr} + \frac{\theta_{man}}{\theta_{agr} + \theta_{man}} \hat{p}_{man} = 1 \text{ (CPI).}$$

Domestic production

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
$\hat{y}_{agr,d}$	14.0000	14.0117	13.1445	14.0901	
$\hat{x}_{agr,agr}$	2.0000		1.8778	2.0129	1.8566
$\hat{x}_{man,agr}$	6.0000	6.0050	5.6333		5.5699
$\hat{\ell}_{agr}$	3.0000	3.0028	2.7989	3.0213	2.7644
\hat{k}_{agr}	3.0000	3.0023		3.0173	2.8057
$\hat{y}_{man,d}$	24.0000	23.9920		23.9382	24.6873
$\hat{x}_{agr,man}$	3.0000	2.9990	3.0733		3.0859
$\hat{x}_{man,man}$	6.0000		6.1465	5.9846	6.1718
$\hat{\ell}_{man}$	10.0000		10.2011	9.9787	
\hat{k}_{man}	5.0000	4.9977	5.1655	4.9827	

Investment

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
\hat{y}_{inv}	6.0000	5.6198	6.0269	5.2131	
$\hat{x}_{agr,inv}$	2.0000	1.8733			1.8824
$\hat{x}_{man,inv}$	4.0000	3.7466		3.4754	3.7647

Total supply and international trade

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
\hat{y}_{agr}	20.0000	20.4676	24.7388		32.3907
\hat{y}_{man}	30.0000	29.6081		29.2757	32.6113
$\hat{y}_{agr,d}$	14.0000	14.0117	13.1445	14.0901	
$\hat{y}_{man,d}$	24.0000	23.9920		23.9382	24.6873
$\hat{y}_{agr,f}$	4.0000	4.3119	7.9530	4.5293	13.8676
$\hat{y}_{man,f}$	5.0000	4.6881	5.6390	4.4707	6.6340
\hat{T}	3.0000		2.3727		0.0000
$\hat{x}_{agr,f}$	5.0000	5.9335	9.4317	6.9061	17.4483
$\hat{x}_{man,f}$	4.0000	4.5932	5.6892		8.0220

Consumption, savings, and welfare

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
\hat{c}_{agr}	8.0000			7.2041	8.1175
\hat{c}_{man}	10.0000	9.2654	9.8610	8.5385	9.0828
\hat{c}_{inv}	6.0000	5.6198	6.0269		5.6471
real income	1.0000	0.9393	1.0095	0.8729	

Real income index:

$$\hat{Y} = \frac{\hat{c}_{agr}^{\theta_{agr}} \hat{c}_{man}^{\theta_{man}} \hat{c}_{inv}^{\theta_{inv}}}{8^{\theta_{agr}} 10^{\theta_{man}} 6^{\theta_{inv}}}.$$

Numerical Experiments II: Free Trade Agreement

Tariffs

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
τ_{agr}	0.5000	0.2000	0.2000	0.0000	0.0000
τ_{man}	0.2000	0.1000	0.1000	0.0000	0.0000
$\tau_{agr,f}$	0.5000	0.1000	0.1000	0.0000	0.0000
$\tau_{man,f}$	0.1000	0.0500	0.0500	0.0000	0.0000

Prices

	benchmark		partial liberalization	free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
\hat{P}_{agr}	1.0000	0.9659	0.9455	0.9488	0.8982
\hat{P}_{man}	1.0000		1.0436	1.0410	1.0814
\hat{P}_{inv}	1.0000	1.0068	1.0109		
$\hat{P}_{agr,d}$	1.0000	1.0433	1.0764	1.0602	1.1442
$\hat{P}_{man,d}$	1.0000	1.0547	1.0930	1.0750	1.1726
\hat{e}	1.0000	1.0087		1.0985	1.0772
\hat{r}	1.0000	1.0903	1.1857	1.1245	1.3670
\hat{w}	1.0000	1.0799	1.1210	1.1085	1.2153

Domestic production

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
$\hat{y}_{agr,d}$	14.0000	14.6498	17.8033	14.9679	21.9883
$\hat{x}_{agr,agr}$	2.0000	2.0928	2.5433		3.1412
$\hat{x}_{man,agr}$	6.0000	6.2785	7.6300		9.4236
$\hat{\ell}_{agr}$	3.0000		3.9236	3.2304	4.9972
\hat{k}_{agr}	3.0000		3.7094	3.1845	4.4427
$\hat{y}_{man,d}$	24.0000	23.5541	21.3798	23.3355	
$\hat{x}_{agr,man}$	3.0000	2.9443	2.6725	2.9169	
$\hat{x}_{man,man}$	6.0000	5.8885	5.3449	5.8339	4.6170
$\hat{\ell}_{man}$	10.0000	9.8457		9.7696	8.0028
\hat{k}_{man}	5.0000	4.8758		4.8155	3.5573

Investment

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
\hat{y}_{inv}	6.0000	6.0631	7.3206	5.7923	6.5503
$\hat{x}_{agr,inv}$	2.0000	2.0210		1.9308	2.1834
$\hat{x}_{man,inv}$	4.0000	4.0421		3.8615	4.3669

Total supply and international trade

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
\hat{y}_{agr}	20.0000	22.6047	48.6541	23.8925	105.3496
\hat{y}_{man}	30.0000	30.2277	33.6669	30.1221	34.5968
$\hat{y}_{agr,d}$	14.0000	14.6498	17.8033	14.9679	21.9883
$\hat{y}_{man,d}$	24.0000	23.5541	21.3798	23.3355	
$\hat{y}_{agr,f}$	4.0000			6.1910	64.4880
$\hat{y}_{man,f}$	5.0000	5.5972	10.6900	5.7090	14.6288
\hat{T}	3.0000	1.6563	5.5426	0.0000	0.0000
$\hat{x}_{agr,f}$	5.0000	7.1202	30.5618		87.7951
$\hat{x}_{man,f}$	4.0000	4.1147	3.9931		5.8886

Consumption, savings, and welfare

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 5$
\hat{c}_{agr}	8.0000	8.4264		8.2233	9.9213
\hat{c}_{man}	10.0000	9.9039	11.8185		10.3007
\hat{c}_{inv}	6.0000	6.0631	7.3206	5.7923	
real income	1.0000			0.9736	1.1119