MACROECONOMIC THEORY **T.J. KEHOE**
ECON 8105 **FALL 2002**

FINAL EXAMINATION

Answer ***two*** of the following three questions.

1. Consider an economy with a representative infinitely lived consumer who has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t \ .$$

The consumer is endowed with one unit of labor in every period and $\bar{k}_0$ units of capital in period 0. The set of feasible consumption and production plans satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \leq \lambda^t A k_t^{\alpha} \ell_t^{1-\alpha} \ .$$

Here $1 > \beta > 0$, $1 \geq \delta \geq 0$, $\lambda > 1$, and $1 > \alpha > 0$.

(a) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(b) Suppose that both consumption and the capital stock grow at, possibly different, constant rates in equilibrium. Prove that they have to grow at the same rate. Derive the relationship between this rate of growth and λ.

(c) Use your answer to part b to define a balanced growth path for this economy.

(d) Suppose now that there is an economy with roughly constant population. In 1980 its national income and product accounts were

Product	
Consumption	80
Investment	20
GDP	100

Income	
Labor Income	70
Capital Income	20
Depreciation	10
GDP	100

Between 1980 and 2000 all of these numbers grew at roughly three percent per year in real terms. Either calibrate the model economy to match this set of balanced growth observations or carefully specify a procedure to do so.

2. Consider an overlapping generations economy in which there is one good in each period. There are two types of consumers born in each period t, $t = 1, 2, \ldots$, who live for two periods. All consumers have the same utility function

$$\log c_t^{it} + \log c_{t+1}^{it}, \ i = 1, 2 .$$

The representative consumer of type 1 has endowments $(w_t^{1t}, w_{t+1}^{1t}) = (5,1)$. The representative consumer of type 2 has endowments $(w_t^{2t}, w_{t+1}^{2t}) = (3,3)$. In addition to these consumers, there is an initial generation 0, made up of two types of consumers who live only in period 1. All consumers in this initial generation have the utility function

$$\log c_1^{i0}, \ i = 1, 2 .$$

The endowments of the representative consumers of the two types are $w_1^{10} = 1$ and $w_1^{20} = 3$.

(a) Suppose that the two representative consumers in generation 0 have endowments fiat money m^1 and m^2. Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Again suppose that the two representative consumers in generation 0 have endowments fiat money m^1 and m^2. Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Suppose that $m^1 = m^2 = 0$. Calculate the Arrow-Debreu equilibrium.

(e) Again suppose that $m^1 = m^2 = 0$. Use your answers to parts c and d to calculate the sequential markets equilibrium.

3. Consider an economy with a representative consumer with the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. This consumer has an endowment of $\bar{\ell}_t = 1$ units of labor in each period and $\bar{k}_0$ units of capital in period 0. Feasible allocation/production plans satisfy

$$c_t + k_{t+1} \leq \theta k_t^{\alpha} \ell_t^{1-\alpha}.$$

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

(b) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Define a Pareto efficient allocation/production plan. Prove either that an Arrow-Debreu allocation/production plan is Pareto efficient or that a sequential markets allocation/production plan is Pareto efficient.

(e) Write down Bellman's equation that defines the value function for the dynamic programming problem that a Pareto efficient allocation/production plan solves. Guess that the value function has the form $V(k) = a_0 + a_1 \log k$ for some yet-to-be-determined constants a_0 and a_1. Solve for both the value function $V(k)$ and the policy function $k' = g(k)$.

(f) Calculate the Arrow-Debreu equilibrium of this economy. Calculate the sequential markets equilibrium.