MACROECONOMIC THEORY **T.J. KEHOE**
ECON 8105 **FALL 2002**

MIDTERM EXAMINATION

Answer ***two*** of the following three questions.

1. Consider an economy with a representative infinitely lived consumer who has the utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $0 < \beta < 1$ and $u : R \to R$ is continuously differentiable, strictly concave, and increasing. This consumer is endowed with $\bar{\ell}_t = 1$ units of labor in each period and $\bar{k}_0$ units of capital in period 0. The set of feasible consumption and production plans satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \leq F(k_t, \ell_t) .$$

Here, $0 \leq \delta \leq 1$ and $F : R_+^2 \to R$ is continuously differentiable, homogeneous of degree one, concave, and increasing.

(a) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(b) Define a Pareto efficient allocation for this economy.

(c) Prove that any equilibrium allocation is Pareto efficient. [Hint: you are welcome to use the continuous differentiability of u and F, but you do not need to do so.]

(d) Suppose now that there are two types of consumers. The representative consumer of type i, $i = 1, 2$, has the discount factor β_i, the period utility function u_i, the period labor endowment $\bar{\ell}_t^i = \bar{\ell}^i$, and the initial endowment of capital $\bar{k}_0^i$. Define a sequential markets equilibrium.

(e) Define a Pareto efficient allocation for the economy in part d.

2. Consider an overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \ldots,$ has the utility function over consumption of the single good in periods t and $t+1$

$$u\left(c_t^t, c_{t+1}^t\right) = \log c_t^t + c_{t+1}^t$$

and endowments $\left(w_t^t, w_{t+1}^t\right) = \left(w_1, w_2\right)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0\left(c_1^0\right) = c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b.

(d) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(e) Suppose now that representative consumer born in period t, $t = 1, 2, \ldots,$ has the utility function over consumption of the single good in periods t, $t+1$, and $t+2$

$$u\left(c_t^t, c_{t+1}^t, c_{t+2}^t\right) = \log c_t^t + \gamma \log c_{t+1}^t + c_{t+2}^t$$

and endowments $\left(w_t^t, w_{t+1}^t, w_{t+2}^t\right) = \left(w_1, w_2, w_3\right)$. There are two additional generations alive in period 1. The representative consumer in generation –1 has the utility function $u^{-1}\left(c_1^{-1}\right) = c_1^{-1}$ and endowment $w_1^{-1} = w_3$ of the good in period 1 and endowment m^{-1} of fiat money. The representative consumer in generation 0 has the utility function $u^0\left(c_1^0, c_2^0\right) = \gamma \log c_1^0 + c_2^0$ and endowments $\left(w_1^0, w_2^0\right) = \left(w_2, w_3\right)$ of the goods in periods 1 and 2 and endowment m^0 of fiat money. Define an Arrow-Debreu equilibrium for this economy. Define a sequential markets equilibrium.

3. Consider an economy with a representative consumer with the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. This consumer has an endowment of $\bar{\ell}_t = 1$ units of labor in each period and $\bar{k}_0$ units of capital in period 0. Feasible allocation/production plans satisfy

$$c_t + k_{t+1} \leq \theta k_t^{\alpha} \ell_t^{1-\alpha}.$$

(a) Describe a sequential markets market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(b) Define a Pareto efficient allocation/production plan.

(c) Prove that any Pareto efficient allocation/production plan is an equilibrium allocation/ production. plan.

(d) Write down Bellman's equation that defines the value function for the dynamic programming problem that a Pareto efficient allocation/production plan solves. Guess that the value function has the form $V(k) = a_0 + a_1 \log k$ for some yet-to-be-determined constants a_0 and a_1. Solve for the policy function $k' = g(k)$.

(e) Use the answers to parts c and d to calculate the sequential markets equilibrium of this economy.