

FINAL EXAMINATION

Answer *two* of the following three questions.

1. Consider an economy with a representative infinitely lived consumer who has the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\gamma \log c_t + (1-\gamma) \log(n_t \bar{h} - \ell_t) \right]$$

The set of feasible consumption and production plans satisfy

$$\begin{aligned} c_t + k_{t+1} - (1-\delta)k_t &\leq \lambda^t A k_t^\alpha \ell_t^{1-\alpha} \\ 0 &\leq n_t \bar{h} - \ell_t \leq n_t \bar{h} \\ c_t, k_t &\geq 0 \\ k_0 &= \bar{k}_0. \end{aligned}$$

Here $1 > \beta > 0$, $1 \geq \delta \geq 0$, $\lambda > 0$, $1 > \alpha > 0$, n_t is an exogenously given sequence of population sizes, and \bar{h} is the endowment of hours available for work or leisure in one period.

(a) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium. [10 points]

(b) Assume that in the equilibrium in part a, population n_t and hours worked ℓ_t are constant. Suppose that both consumption and the capital stock grow at, possibly different, constant rates in equilibrium. Prove that they have to grow at the same rate. Derive the relationship between this rate of growth and λ . [10 points]

(c) Use your answer to part b to define a balanced growth path for this economy. [10 points]

(d) Suppose now that there is an economy with roughly constant population. In 1990 its national income and product accounts were

Product		Income	
Consumption	80	Labor Income	70
Investment	<u>20</u>	Capital Income	20
GDP	100	Depreciation	<u>10</u>
		GDP	100

Between 1990 and 2004 all of these numbers grew at roughly three percent per year in real terms. Hours worked per working age person were roughly constant at 30 hours per week. Either calibrate the model economy to match this set of balanced growth observations or carefully specify a procedure to do so. [20 points]

2. Consider an overlapping generations economy in which there is one good in each period and each generation, except the initial one, lives for two periods. The representative consumer in generation t , $t = 1, 2, \dots$, has the utility function

$$\log c_t^t + \log c_{t+1}^t$$

and the endowment $(w_t^t, w_{t+1}^t) = (1, 2)$. The representative consumer in generation 0 lives only in period 1, has the utility function $\log c_1^0$, and has the endowment $w_1^0 = 2$. Goods are not storable, and there is no fiat money.

(a) Define an Arrow-Debreu equilibrium for this economy. Calculate the unique Arrow-Debreu equilibrium. [8 points]

(b) Define a sequential markets equilibrium for this economy. Calculate the unique sequential markets equilibrium. [8 points]

(c) Define a Pareto efficient allocation for this economy. Is the equilibrium allocation in part a Pareto efficient? Explain carefully why or why not. [8 points]

(d) Suppose now that consumers live for three periods, that the representative consumer in generation t , $t = 1, 2, \dots$, has the utility function $\log c_t^t + \log c_{t+1}^t + \log c_{t+2}^t$ and the endowment $(w_t^t, w_{t+1}^t, w_{t+2}^t) = (1, 2, 0)$. There is also a generation -1 that lives only in period 1, whose representative consumer has the utility function $\log c_1^{-1}$ and who has the endowment $w_1^{-1} = 0$. In addition, there is a generation 0, whose representative consumer lives in periods 1 and 2 and who has the utility function $\log c_1^0 + \log c_2^0$ and endowment $(w_1^0, w_2^0) = (2, 0)$. There is no fiat money. Define a sequential markets equilibrium for this economy. [10 points]

(e) In the equilibrium of part d, is it the case that $\hat{c}_{t+2}^t = 0$? Explain why or why not. [8 points]

(f) Relax now the assumption that goods are not storable. Suppose instead that 1 unit of the good in period t , $t = 1, 2, \dots$, can be transformed into $\theta > 0$ units of the good in period $t + 1$. Define a sequential markets equilibrium for the economy in which consumer lives for three periods. [8 points]

3. Consider the social planner's problem of choosing sequences of c_t , ℓ_t , and k_t to solve

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t [\log c_t + \gamma \log(1 - \ell_t)] \\ \text{s.t.} \quad & c_t + k_{t+1} \leq \theta k_t^\alpha \ell_t^{1-\alpha} \\ & c_t, k_t, \ell_t, (1 - \ell_t) \geq 0 \\ & k_0 \leq \bar{k}_0. \end{aligned}$$

(a) Write down the Euler conditions and the transversality condition for this problem. [8 points]

(b) Formulate this social planner's problem as a dynamic programming problem by writing down the relevant Bellman's equation. Guessing that the value function takes the form

$$V(k) = a_0 + a_1 \log k,$$

solve for the policy functions $c = c(k)$, $\ell = \ell(k)$, $k' = k'(k)$. (Hint: the optimal value of ℓ does not vary with k .) [14 points]

(c) Verify that the solution to the social planner's generated by the policy functions in part b satisfy the Euler conditions and transversality condition in part a. [8 points]

(d) Specify an economic environment for which the solution to this social planning problem is a Pareto efficient allocation. Define a sequential markets equilibrium for this economy. Explain how you can use the policy functions from part b to calculate his equilibrium. [10 points]

(e) Define an Arrow-Debreu equilibrium for the economy in part d. Explain how you can use the policy functions from part b to calculate this equilibrium. [10 points]