

MIDTERM EXAMINATION

Answer *two* of the following three questions.

1. Consider an economy with two infinitely lived consumers. There is one good in each period. Consumer i , $i = 1, 2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t u_i(c_t^i)$$

Here β , $0 < \beta < 1$, is the common discount factor, and u_i , $i = 1, 2$, is a continuously differentiable utility function that satisfies $u_i'(c) > 0$ and $u_i''(c) < 0$ for all $c > 0$ and $\lim_{c \rightarrow 0} u_i'(c) = \infty$. Each of the consumers is endowed with a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, \dots) = (3, 1, 3, 1, \dots)$$
$$(w_0^2, w_1^2, w_2^2, w_3^2, \dots) = (1, 3, 1, 3, \dots).$$

There is no production or storage.

- Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.
- Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.
- Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.
- Prove that the equilibrium consumption sequences are constant, $\hat{c}_t^i = \hat{c}^i$. Explain how you would calculate the values of \hat{c}^1 and \hat{c}^2 .
- Relax now the assumption that the good is not storable. Suppose instead that 1 unit of the good in period t , $t = 0, 1, \dots$, can be transformed into $\theta > 0$ units of the good in period $t + 1$. Define a sequential markets equilibrium for this economy.

2. Consider an overlapping generations economy in which the representative consumer born in period t , $t = 1, 2, \dots$, has the utility function over consumption of the single good in periods t and $t + 1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $w_1 > w_2$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that there are two types of consumers in each generation. The representative consumer of type i , $i = 1, 2$, born in period t , $t = 1, 2, \dots$, has the utility function over consumption of the single good in periods t and $t + 1$

$$u_i(c_t^{it}, c_{t+1}^{it}) = \gamma_i \log c_t^{it} + (1 - \gamma_i) \log c_{t+1}^{it}$$

and endowments $(w_t^{it}, w_{t+1}^{it}) = (w_1^i, w_2^i)$. Suppose that the representative consumer of type i , $i = 1, 2$, in the initial old generation has the utility function

$$u_{i0}(c_1^{i0}) = (1 - \gamma_i) \log c_1^{i0}$$

and endowment $w_1^{i0} = w_2^i$ of the good in period 1 and endowment m^i of fiat money. Define an Arrow-Debreu equilibrium for this economy. Define a sequential markets equilibrium.

3. Consider an economy with a representative consumer with the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. This consumer has an endowment of $\bar{\ell}_t = 1$ units of labor in each period and \bar{k}_0 units of capital in period 0. Feasible allocation/production plans satisfy

$$c_t + k_{t+1} \leq \theta k_t^\alpha \ell_t^{1-\alpha}.$$

- (a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.
- (b) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.
- (c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.
- (d) Define a Pareto efficient allocation/production plan. Prove either that an Arrow-Debreu allocation/production plan is Pareto efficient or that a sequential markets allocation/production plan is Pareto efficient.
- (e) Write down Bellman's equation that defines the value function for the dynamic programming problem that a Pareto efficient allocation/production plan solves. Explain how you would derive the policy function $k' = g(k)$ from this value function. Guess that the value function has the form $V(k) = a_0 + a_1 \log k$ for some yet-to-be-determined constants a_0 and a_1 . Solve for the policy function $k' = g(k)$.
- (f) Use the answer to part e to calculate the sequential markets equilibrium of this economy. [That is, provide explicit formulas for all of the objects that make up the definition of a sequential markets equilibrium.]