MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2009**

FINAL EXAMINATION

Answer ***two*** of the following three questions.

1. Consider an overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \ldots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. (Notice, in particular, that the utility function is not $\log c_t^t + \log c_{t+1}^t$.) Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $w_1 < 1$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that there are two types of consumers in each generation. The representative consumer of type i, $i = 1, 2$, born in period t, $t = 1, 2, \ldots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u_i(c_t^{it}, c_{t+1}^{it}) = \log c_t^{it} + \gamma_i c_{t+1}^{it}$$

and endowments $(w_t^{it}, w_{t+1}^{it}) = (w_1^i, w_2^i)$. Suppose that the representative consumer of type i, $i = 1, 2$, in the initial old generation has the utility function

$$u_{i0}(c_1^{i0}) = c_1^{i0}$$

and endowment $w_1^{i0} = w_2^i$ of the good in period 1 and endowment m^i of fiat money. Define an Arrow-Debreu equilibrium for this economy. Define a sequential markets equilibrium.

2. Consider the optimal growth problem

$$\begin{aligned} &\max \sum_{t=0}^{\infty} \beta^t \log c_t \\ \text{s.t.} \quad & c_t + k_{t+1} \leq \theta k_t^{\alpha} \\ & c_t, k_t \geq 0 \\ & k_0 = \bar{k}_0 . \end{aligned}$$

Here $1 > \alpha > 0$, $1 > \beta > 0$, $\theta > 0$

(a) Write down the Euler conditions and the transversality condition for this problem. Calculate the steady state values of *c* and *k*.

(b) Write down the functional equation that defines the value function for this problem. Guess that the value function has the form $a_0 + a_1 \log k$. Calculate the policy function.

(c) Verify that the policy function in part b generates a path for capital that satisfies the Euler conditions and transversality condition in part a.

(d) Define sequential markets equilibrium for this economy. Define an Arrow-Debreu equilibrium.

(e) Consider now the optimal growth problem

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t \left(\gamma \log c_t + (1-\gamma) \log(1-\ell_t) \right) \\ \text{s.t.} \quad & c_t + k_{t+1} \leq \theta k_t^{\alpha} \ell_t^{1-\alpha} \\ & c_t, k_t \geq 0 \\ & k_0 = \bar{k}_0 . \end{aligned}$$

Write down the functional equation that defines the value function for this problem. Guess that the value function has the form $a_0 + a_1 \log k$. Calculate the policy function. [Hint: Guess that $\ell_t = \bar{\ell}$.]

3. Consider an economy with a representative, infinitely lived consumer who has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. The consumer owns one unit of labor in each period and $\bar{k}_0$ units of capital in period 0. Suppose that feasible consumption/investment plans satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \leq \theta k_t^{\alpha} \ell_t^{1-\alpha} .$$

(a) Suppose that the consumer borrows b_{t+1} bonds in period t to be paid off in period $t+1$. The consumer's initial endowment of bonds is $\bar{b}_0 = 0$, the wage rate in period t is w_t, the rental rate on capital is r_t^k, and the interest rate on bonds is r_t^b. Write down the consumer's utility maximization problem in a sequential markets economy. Explain why you need to include a constraint to rule out Ponzi schemes. Write down the Euler conditions and the transversality conditions for this problem. Define a sequential markets equilibrium with borrowing and lending for this economy. Prove that in equilibrium $r_t^k - \delta = r_t^b$ if $k_t > 0$.

(b) Suppose that the consumer sells his endowment of capital to the firm in period 0. Thereafter, firms buy and sell capital from each other. Describe the production set for the Arrow-Debreu economy, the set of feasible $k_0, k_1, \ldots, \ell_0, \ell_1, \ldots, c_0, c_1, \ldots$. Define the Arrow-Debreu equilibrium for this economy.

(c) Carefully state theorems that relate the equilibrium allocations in parts a and b.

(d) Suppose that now there are two types of consumers. The representative consumer of type 1 has the endowment of labor $(\bar{\ell}_0^1, \bar{\ell}_1^1, \bar{\ell}_2^1, \bar{\ell}_3^1, \ldots) = (2,1,2,1,\ldots)$ and the endowment of capital $\bar{k}_0^1$. The representative consumer of type 2 has the endowment of labor $(\bar{\ell}_0^2, \bar{\ell}_1^2, \bar{\ell}_2^2, \bar{\ell}_3^2, \ldots) = (1,2,1,2,\ldots)$ and the endowment of capital $\bar{k}_0^2$. Define a sequential markets equilibrium for this economy.

(e) Does the equilibrium allocation/production plan in part d solve a dynamic programming problem? If it does, write down the Bellman's equation for this problem. If it does not, explain carefully why it does not.