

MIDTERM EXAMINATION

Answer *two* of the following three questions.

1. Consider an economy with two infinitely lived consumers. There is one good in each period. Consumer  $i$ ,  $i = 1, 2$ , has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i$$

Here  $\beta$ ,  $0 < \beta < 1$ , is the common discount factor. Each of the consumers is endowed with a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, \dots) = (3, 1, 3, 1, \dots)$$
$$(w_0^2, w_1^2, w_2^2, w_3^2, \dots) = (1, 4, 1, 4, \dots).$$

There is no production or storage.

- (a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.
- (b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.
- (c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.
- (d) Calculate the Arrow-Debreu equilibrium for this economy. (This equilibrium is unique, but you do not have to prove this fact.) Use this answer and the answer to part c to calculate the sequential markets equilibrium.
- (e) Define a Pareto efficient allocation for this economy. Prove that the allocations in parts a and b are Pareto efficient.

2. Consider an overlapping generations economy in which the representative consumer born in period  $t$ ,  $t = 1, 2, \dots$ , has the utility function over consumption of the single good in periods  $t$  and  $t + 1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \log c_{t+1}^t$$

and endowments  $(w_t^t, w_{t+1}^t) = (w_1, w_2)$ . Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \log c_1^0$$

and endowment  $w_1^0 = w_2$  of the good in period 1 and endowment  $m$  of fiat money.

- (a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.
- (b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.
- (c) Suppose that  $m = 0$ . Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.
- (d) Define a Pareto efficient allocation. Suppose that  $w_2 > w_1$ . Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.
- (e) Relax now the assumption that the good is not storable. Suppose instead that 1 unit of the good in period  $t$ ,  $t = 1, 2, \dots$ , can be transformed into  $\theta > 0$  units of the good in period  $t + 1$ . Define a sequential markets equilibrium for this economy.

3. Consider an economy with a representative consumer with the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where  $0 < \beta < 1$ . This consumer has an endowment of  $\bar{\ell}_t = 1$  units of labor in each period and  $\bar{k}_0$  units of capital in period 0. Feasible allocation/production plans satisfy

$$c_t + k_{t+1} \leq \theta k_t^\alpha \ell_t^{1-\alpha}.$$

(a) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(b) Define a Pareto efficient allocation/production plan. Write down Bellman's equation that defines the value function for the dynamic programming problem that a Pareto efficient allocation/production plan solves. Explain how you would derive the policy function  $k' = g(k)$  from this value function. Guess that the value function has the form

$$V(k) = a_0 + a_1 \log k$$

for some yet-to-be-determined constants  $a_0$  and  $a_1$ . Solve for the policy function  $k' = g(k)$ .

(c) Use the answer to part b to calculate the sequential markets equilibrium of this economy.

(d) Suppose now that the representative consumer faces the choice of selling his labor services or consuming them as leisure. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t (\gamma \log c_t + (1-\gamma) \log x_t)$$

where  $x_t$  is leisure. Define a sequential markets equilibrium for this economy.

(e) Define a Pareto efficient allocation/production plan for the economy in part d. Write down Bellman's equation that defines the value function for the dynamic programming problem that a Pareto efficient allocation/production plan solves.