MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2011**

FINAL EXAMINATION

Answer ***two*** of the following three questions.

1. Consider an overlapping generations economy in which there is one good in each period and each generation, except the initial one, lives for two periods. The representative consumer in generation t, $t = 1,2,\ldots$, has the utility function

$$\log c_t^t + \log c_{t+1}^t$$

and the endowment $(w_t^t, w_{t+1}^t) = (2,1)$. The representative consumer in generation 0 lives only in period 1, has the utility function $\log c_1^0$, and has the endowment $w_1^0 = 1$. Goods are not storable, and there is no fiat money.

a) Define an Arrow-Debreu equilibrium for this economy. Calculate the unique Arrow-Debreu equilibrium.

b) Define a sequential markets equilibrium for this economy. Calculate the unique sequential markets equilibrium.

c) Define a Pareto efficient allocation for this economy. Is the equilibrium allocation in part a Pareto efficient? Explain carefully why or why not.

d) Suppose now that consumers live for three periods, that the representative consumer in generation t, $t = 1,2,\ldots$, has the utility function

$$\log c_t^t + \log c_{t+1}^t + \log c_{t+2}^t$$

and the endowment $(w_t^t, w_{t+1}^t, w_{t+2}^t) = (2,2,1)$. There is also a generation -1 that lives only in period 1, whose representative consumer has the utility function $\log c_1^{-1}$ and who has the endowment $w_1^{-1} = 1$. In addition, there is a generation 0, whose representative consumer lives in periods 1 and 2 and who has the utility function $\log c_1^0 + \log c_2^0$ and endowment $(w_1^0, w_2^0) = (2,1)$. There is no fiat money. Define a sequential markets equilibrium for this economy.

e) Relax now the assumption that goods are not storable. Suppose instead that 1 unit of the good in period t, $t = 1,2,\ldots$, can be transformed into $\theta > 0$ units of the good in period $t+1$. Define a sequential markets equilibrium for this economy with storage in which consumers live for three periods. .

2. Consider the optimal growth problem

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t$$
$$\text{s.t.} \quad c_t + k_{t+1} - (1-\delta)k_t \le \theta k_t^{\alpha}$$
$$c_t, k_t \ge 0$$
$$k_0 = \bar{k}_0 .$$

Here $1 > \beta > 0$, $1 > \delta > 0$, $1 > \alpha > 0$, $\theta > 0$.

a) Write down the Euler conditions and the transversality condition for this problem. Calculate the nontrivial steady state values of *c* and *k*. (The trivial steady state is $\hat{c} = \hat{k} = 0$.)

b) Let $C(K)$ be the set of bounded continuous functions on $K \subset R_+$. Define a contraction mapping $T : C(K) \to C(K)$. State Blackwell's sufficient conditions for the mapping T to be a contraction mapping.

c) Let

$$T(V)(k) = \max \log c + \beta V(k')$$
$$\text{s.t.} \quad c + k' - (1-\delta)k \le \theta k^{\alpha}$$
$$c, k' \ge 0 .$$

Explain how you can choose the set $K \subset R_+$ so that $T(V)$ is bounded above without restricting the set of solutions to the optimal growth problem when $V = T(V)$. Ignore the fact that $T(V)$ is not bounded below. Prove that T satisfies Blackwell's sufficient conditions.

d) Suppose for the moment that $\delta = 1$. Guess that the value function has the form $a_0 + a_1 \log k$. Calculate the value function $V(k) = T(V)(k)$ and the policy function $k' = g(k)$.

e) Suppose now that productivity is stochastic:

$$c_t + k_{t+1} - (1-\delta)k_t = \theta_t k_t^{\alpha},$$

where θ_t can take on two values, $\bar{\theta}_1$ and $\bar{\theta}_2$, where $\bar{\theta}_1 > \bar{\theta}_2 > 0$. Transition probabilities are given by the stationary Markov matrix

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}.$$

Write down the functional equation that defines the value function for this problem.

f) Assume that $\theta_0 = \bar{\theta}_1$. Define sequential markets equilibrium for the economy in part e. Define an Arrow-Debreu equilibrium.

3. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \leq v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}.$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

b) Using Bellman's equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \geq \bar{w}$ and rejects any wage offer $w < \bar{w}$.

c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that F_2 is a mean preserving spread of F_1. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.