

MIDTERM EXAMINATION

Answer *two* of the following three questions.

1. Consider an economy with two infinitely lived consumers. There is one good in each period. Consumer i , $i = 1, 2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i.$$

Here β , $0 < \beta < 1$, is the common discount factor. Each of the consumers is endowed with a sequence of goods:

$$\begin{aligned} (w_0^1, w_1^1, w_2^1, w_3^1, \dots) &= (2, 1, 2, 1, \dots) \\ (w_0^2, w_1^2, w_2^2, w_3^2, \dots) &= (1, 4, 1, 4, \dots). \end{aligned}$$

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Calculate the Arrow-Debreu equilibrium for this economy. (This equilibrium is unique, but you do not have to prove this fact.) Use this answer and the answer to part c to calculate the sequential markets equilibrium.

(e) Suppose now that there is a production technology that transforms labor and capital into output that can be consumed or saved as capital:

$$y_t = \theta k_t^\alpha \ell_t^{1-\alpha},$$

where $\theta > 0$ and $1 > \alpha > 0$. Capital depreciates at the rate δ , $1 > \delta > 0$, every period. The consumers' endowments of labor are

$$\begin{aligned} (\bar{\ell}_0^1, \bar{\ell}_1^1, \bar{\ell}_2^1, \bar{\ell}_3^1, \dots) &= (2, 1, 2, 1, \dots) \\ (\bar{\ell}_0^2, \bar{\ell}_1^2, \bar{\ell}_2^2, \bar{\ell}_3^2, \dots) &= (1, 4, 1, 4, \dots). \end{aligned}$$

Their endowments of capital in period 0 are $\bar{k}_0^i > 0$, $i = 1, 2$. Define a sequential markets equilibrium for this economy.

2. Consider an overlapping generations economy in which the representative consumer born in period $t, t = 1, 2, \dots$, has the utility function over consumption of the single good in periods t and $t + 1$

$$u(c_t^t, c_{t+1}^t) = c_t^t + \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. (Notice that the utility function is not $\log c_t^t + \log c_{t+1}^t$.) Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $w_2 > 1$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that there are two types of consumers of equal measure in each generation. The representative consumer of type 1 born in period $t, t = 1, 2, \dots$, has the utility function over consumption of the single good in periods t and $t + 1$

$$u_1(c_t^{1t}, c_{t+1}^{1t}) = c_t^{1t} + \log c_{t+1}^{1t},$$

while the representative consumer of type 2 has the utility function

$$u_2(c_t^{2t}, c_{t+1}^{2t}) = \log c_t^{2t} + c_{t+1}^{2t}.$$

The endowments of these consumers are $(w_t^{it}, w_{t+1}^{it}) = (w_1^i, w_2^i), i = 1, 2$. The representative consumers of type 1 and 2 who live only in period 1 have utility functions $\log c_1^{10}$ and c_1^{20} , endowments $w_1^{10} = w_2^1$ and $w_1^{20} = w_2^2$ of the good in period 1, and endowments m^1 and m^2 of fiat money. Define an Arrow-Debreu equilibrium for this economy. Define a sequential markets equilibrium.

3. Consider an economy in which the social planner solves the problem

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \beta^t \log c_t \\ \text{s.t. } & c_t + k_{t+1} - (1 - \delta)k_t \leq \theta k_t \\ & c_t, k_t \geq 0 \\ & k_0 \leq \bar{k}_0. \end{aligned}$$

where $1 > \beta > 0$, $1 > \delta > 0$, $\theta > 0$. (Notice that the production function is not θk_t^α .)

(a) Write down the Euler conditions and the transversality condition for this problem.

(b) Write down Bellman's equation that defines the value function for the social planner's problem expressed as a dynamic programming problem. Explain how you would derive the policy function $k' = g(k)$ from this value function. Guess that the value function has the form

$$V(k) = a_0 + a_1 \log k$$

for some yet-to-be-determined constants a_0 and a_1 . Solve for the policy function $k' = g(k)$.

(c) Verify that the sequence of capital stocks $\hat{k}_{t+1} = g(\hat{k}_t)$, where $\hat{k}_0 = \bar{k}_0$, and the associated sequence of consumption levels

$$\hat{c}_t = (\theta + 1 - \delta)\hat{k}_t - g(\hat{k}_t)$$

satisfy the Euler conditions and the transversality condition in part a.

(d) Specify an economic environment (preferences, technology, endowments, and market structure) for which the allocation in part c is an equilibrium allocation. Define an equilibrium.