MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2012**

FINAL EXAMINATION

Answer ***two*** of the following four questions.

1. Consider an overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \dots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = c_t^t + \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. (Notice, in particular, that the utility function is not $\log c_t^t + \log c_{t+1}^t$.) Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $w_2 < 1$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that there are two types of consumers in each generation. The representative consumer of type i, $i = 1, 2$, born in period t, $t = 1, 2, \dots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u_i(c_t^{it}, c_{t+1}^{it}) = c_t^{it} + \gamma_i \log c_{t+1}^{it}$$

and endowments $(w_t^{it}, w_{t+1}^{it}) = (w_1^i, w_2^i)$. Suppose that the representative consumer of type i, $i = 1, 2$, in the initial old generation has the utility function

$$u_{i0}(c_1^{i0}) = \gamma_i \log c_1^{i0}$$

and endowment $w_1^{i0} = w_2^i$ of the good in period 1 and endowment m^i of fiat money. Define an Arrow-Debreu equilibrium for this economy. Define a sequential markets equilibrium.

2. Consider the optimal growth problem

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t$$
$$\text{s.t.} \quad c_t + k_{t+1} - (1-\delta)k_t \le \theta k_t^{\alpha}$$
$$c_t, k_t \ge 0$$
$$k_0 = \bar{k}_0 .$$

Here $1 > \beta > 0$, $1 > \delta > 0$, $1 > \alpha > 0$, $\theta > 0$.

(a) Write down the Euler conditions and the transversality condition for this problem. Calculate the nontrivial steady state values of c and k. (The trivial steady state is $\hat{c} = \hat{k} = 0$.)

(b) Let $B(K)$ be the set of bounded functions on $K \subset R_+$. Define a contraction mapping $T : B(K) \to B(K)$. State Blackwell's sufficient conditions for the mapping T to be a contraction mapping.

(c) Let

$$T(V)(k) = \max \log c + \beta V(k')$$
$$\text{s.t.} \quad c + k' - (1-\delta)k \le \theta k^{\alpha}$$
$$c, k' \ge 0 .$$

Explain how you can choose the set $K \subset R_+$ so that $T(V)$ is bounded above without restricting the set of solutions to the optimal growth problem when $V = T(V)$. Ignore the fact that $T(V)$ is not bounded below. Prove that T satisfies Blackwell's sufficient conditions.

(d) Suppose for the moment that $\delta = 1$. Guess that the value function has the form $a_0 + a_1 \log k$. Calculate the value function $V(k) = T(V)(k)$ and the policy function $k' = g(k)$.

(e) Suppose now that productivity is stochastic:

$$c_t + k_{t+1} - (1-\delta)k_t = \theta_t k_t^{\alpha} ,$$

where θ_t can take on two values, $\bar{\theta}_1$ and $\bar{\theta}_2$, where $\bar{\theta}_1 > \bar{\theta}_2 > 0$. Transition probabilities are given by the stationary Markov matrix

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}.$$

Write down the functional equation that defines the value function for this problem.

(f) Assume that $\theta_0 = \bar{\theta}_1$. Define sequential markets equilibrium for the economy in part e.

3. Consider an economy with two types of consumers. There are equal measures of each type. The representative consumer of type i, $i = 1, 2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\theta \log c_t^i + (1-\theta)\log(\bar{h}_t^i - \ell_t^i) \right]$$

where $0 < \beta < 1$ and $0 < \theta < 1$. This consumer has an endowment of $\bar{k}_0^i$ units of capital in period 0. Labor endowments are

$$(\bar{h}_0^1, \bar{h}_1^1, \bar{h}_2^1, \bar{h}_3^1, \ldots) = (2,\ 1,\ 2,\ 1, \ldots)$$

and

$$(\bar{h}_0^2, \bar{h}_1^2, \bar{h}_2^2, \bar{h}_3^2, \ldots) = (1,\ 2,\ 1,\ 2, \ldots).$$

Feasible allocation/production plans satisfy

$$c_t + k_{t+1} \leq A k_t^{\alpha} \ell_t^{1-\alpha},$$

where variables without superscripts denote aggregates.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(b) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Define a Pareto efficient allocation and production plan. Prove that the equilibrium allocation and production plan in part a is Pareto efficient.

(e) Does the equilibrium allocation and production plan in part a solve a social planner's problem? If so, explain why it does and write down Bellman's equation. If not, explain carefully why not.

4. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \le v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases} .$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

(a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

(b) Using Bellman's equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \ge \bar{w}$ and rejects any wage offer $w < \bar{w}$.

(c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

(d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that F_2 is a mean preserving spread of F_1. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.