MACROECONOMIC THEORY   **T. J. KEHOE**
ECON 8105   **FALL 2012**
MIDTERM EXAMINATION

Answer *two* of the following three questions.

1. Consider an economy with two infinitely lived consumers. There is one good in each period.
Consumer i, $i = 1, 2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i$$

Here β, $0 < \beta < 1$, is the common discount factor. Each of the consumers is endowed with a
sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, ...) = (3, 1, 3, 1, ...)$$
$$(w_0^2, w_1^2, w_2^2, w_3^2, ...) = (2, 2, 2, 2, ...) .$$

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are
open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open,
who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the
equilibrium concept in part a with that in part b. Be sure to specify the relationships between the
objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Calculate the Arrow-Debreu equilibrium for this economy. (This equilibrium is unique, but
you do not have to prove this fact.) Use this answer and the answer to part c to calculate the
sequential markets equilibrium.

(e) Define a Pareto efficient allocation for this economy. Prove that the allocations in parts a
and b are Pareto efficient.

2. Consider an overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \ldots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $w_2 < w_1$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Relax now the assumption that the good is not storable. Suppose instead that 1 unit of the good in period t, $t = 0, 1, \ldots$, can be transformed into $\theta > 0$ units of the good in period $t+1$. Define a sequential markets equilibrium for this economy.

3. Consider an economy in which the social planner solves the problem

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t$$
$$\text{s.t. } c_t + k_{t+1} \leq \theta k_t^{\alpha}$$
$$c_t, k_t \geq 0$$
$$k_0 \leq \bar{k}_0.$$

where $1 > \beta > 0$, $1 > \delta > 0$, $\theta > 0$.

(a) Write down the Euler conditions and the transversality condition for this problem.

(b) Guess that the optimal policy function $k_{t+1} = g(k_t)$ has the form

$$k_{t+1} = A\theta k_t^{\alpha}.$$

Use the Euler conditions from part a to determine the value of the constant A as a function of the parameters.

(c) Specify an economic environment (preferences, technology, endowments, and market structure) for which the allocation in part c is an equilibrium allocation. Define an equilibrium.

(d) Use the solution to part b to calculate the values of all equilibrium variables in part c.