MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2013**

MIDTERM EXAMINATION

Answer **two** of the following three questions.

1. Consider an economy with two infinitely lived consumers. There is one good in each period. Consumer i, $i = 1,2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i$$

Here β, $0 < \beta < 1$, is the common discount factor. Each of the consumers is endowed with a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, \ldots) = (3,1,3,1,\ldots)$$
$$(w_0^2, w_1^2, w_2^2, w_3^2, \ldots) = (1,1,1,1,\ldots) .$$

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium. (You are **not** asked to prove this proposition or propositions.)

(d) Calculate the Arrow-Debreu equilibrium for this economy. (This equilibrium is unique, but you do not have to prove this fact.) Use this answer and the answer to part c to calculate the sequential markets equilibrium.

(e) Define a Pareto efficient allocation for this economy. Prove that the allocations in parts a and b are Pareto efficient.

2. Consider an overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \ldots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + 3\log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = 3\log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $(w_1, w_2) = (2, 2)$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Relax now the assumption that the good is not storable. Suppose instead that 1 unit of the good in period t, $t = 1, 2, \ldots$, can be transformed into $\theta > 0$ units of the good in period $t+1$. Define a sequential markets equilibrium for this economy.

3. Consider an economy with a representative consumer with the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. This consumer has an endowment of $\bar{\ell}_t = 1$ units of labor in each period and $\bar{k}_0$ units of capital in period 0. Feasible allocation/production plans satisfy

$$c_t + k_{t+1} \leq \theta k_t^{\alpha} \ell_t^{1-\alpha} .$$

(a) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(b) Define a Pareto efficient allocation/production plan. Prove that a sequential markets allocation/production plan is Pareto efficient.

(c) Write down Bellman's equation that defines the value function for the dynamic programming problem that a Pareto efficient allocation/production plan solves. Explain how you would derive the policy function $k' = g(k)$ from this value function. Guess that the value function has the form $V(k) = a_0 + a_1 \log k$ for some yet-to-be-determined constants a_0 and a_1. Solve for the policy function $k' = g(k)$.

(d) Use the answer to part c to calculate the sequential markets equilibrium of this economy. (That is, provide explicit formulas for all of the objects that make up the definition of a sequential markets equilibrium.)

(e) Suppose now that the utility function is

$$\sum_{t=0}^{\infty} \beta^t \left(\gamma \log c_t + (1-\gamma) \log(1-\ell_t) \right)$$

where $1 \geq \gamma > 0$. Define a sequential markets equilibrium.