MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2014**

FINAL EXAMINATION

Answer ***two*** of the following four questions.

1. Consider an overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \ldots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \gamma \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \gamma \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

(b) Describe a sequential market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $\gamma = 2$ and $(w_1, w_2) = (4, 5)$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that, rather than endowments of consumption goods, the consumers have endowments of labor $(\bar{\ell}_t^t, \bar{\ell}_{t+1}^t) = (\bar{\ell}_1, \bar{\ell}_2)$ and $\bar{\ell}_1^0 = \bar{\ell}_2$, The representative consumer in the initial old generation has an endowment of capital $\bar{k}_1^0$ and an endowment m of fiat money. Final output, which can be consumed or invested is produced using the production function $\theta k_t^\alpha \ell_t^{1-\alpha}$, $\theta > 0$, $0 < \alpha < 1$, and a fraction δ, $0 \le \delta \le 1$, of capital depreciates every period. Define a sequential markets equilibrium for this economy.

2. Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital as well as labor. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t c_t^{\rho} .$$

Here $0 < \beta < 1$ and $0 < \rho < 1$. The consumer is endowed with 1 unit of labor in each period and with $\bar{k}_0$ units of capital in period 0. Feasible allocations satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \leq \theta k_t^{\alpha} \ell_t^{1-\alpha}$$
$$c_t, k_t \geq 0 .$$

Here $\theta > 0$, $0 < \alpha < 1$, and $0 \leq \delta \leq 1$.

(a) Formulate the problem of maximizing the representative consumer's utility subject to feasibility conditions as a dynamic programming problem. Write down the appropriate Bellman's equation.

(b) Let $K = [0, \tilde{k}]$. Explain how you can use the feasibility condition to choose $\tilde{k}$ to be the maximum sustainable capital stock. Let $C(K)$ be the space of continuous bounded functions on K. Endow $C(K)$ with the topology induced by the sup norm

$$d(V,W) = \sup_{k \in K} \left| V(k) - W(k) \right| \text{ for any } V, W \in C(K) .$$

Define a contraction mapping $T : C(K) \to C(K)$.

(c) State Blackwell's sufficient conditions for T to be a contraction. (You do not need to prove that these conditions are sufficient for T to be a contraction.)

(d) Using the Bellman's equation from part a, define the mapping for the value function iteration algorithm,

$$V_{n+1} = T(V_n) ,$$

where $T : C(K) \to C(K)$; that is $V = T(V)$ is the Bellman's equation. (You do not need to prove that $T(V) \in C(K)$ for all $V \in C(K)$.) Prove that T satisfies Blackwell's sufficient conditions to be a contraction.

(e) Specify an economic environment for which the solution to the social planner's problem in part a is a Pareto efficient allocation/production plan. Define a sequential markets equilibrium for this environment. Explain how you could use the value function iteration algorithm $V_{n+1} = T(V_n)$ to calculate the unique sequential markets equilibrium. (You do not have to prove that this equilibrium is unique.)

3. Consider the social planner's problem of choosing sequences of c_t, ℓ_t, and k_t to solve

$$\max \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \gamma \log(1-\ell_t) \right]$$
$$\text{s.t.} \quad c_t + k_{t+1} \le \theta k_t^{\alpha} \ell_t^{1-\alpha}$$
$$c_t, k_t \ge 0, \; 1 \ge \ell_t \ge 0$$
$$k_0 \le \bar{k}_0 .$$

(a) Write down the Euler conditions and the transversality condition for this problem.

(b) Formulate this social planner's problem as a dynamic programming problem by writing down the relevant Bellman's equation. Guessing that the value function takes the form

$$V(k) = a_0 + a_1 \log k ,$$

solve for the policy functions $c = c(k)$, $\ell = \ell(k)$, $k' = k'(k)$. (Hint: the optimal value of ℓ does not vary with k.)

(c) Verify that the solution to the social planner's generated by the policy functions in part b satisfy the Euler conditions and transversality condition in part a.

(d) Specify an economic environment for which the solution to this social planning problem is a Pareto efficient allocation. Define a sequential markets equilibrium for this economy. Explain how you can use the policy functions from part b to calculate his equilibrium.

(e) Define an Arrow-Debreu equilibrium for the economy in part d. Explain how you can use the policy functions from part b to calculate this equilibrium.

4. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \le v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}.$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

(a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

(b) Using Bellman's equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \ge \bar{w}$ and rejects any wage offer $w < \bar{w}$.

(c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

(d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Define what it means for F_2 to be a mean preserving spread of F_1.

(e) Suppose that F_2 is a mean preserving spread of F_1. Let $\bar{w}_1$ be the reservation wage in the economy with wage distribution F_1 and $\bar{w}_2$ be the reservation wage in economy with wage distribution F_2. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.