MACROECONOMIC THEORY **T. J. KEHOE**
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PROBLEM SET #1

1. Consider an economy two infinitely lived consumers, each of whom has the same utility function,

$$u(c_0^i, c_1^i, \ldots) = \sum_{t=0}^{\infty} \beta^t \log \ c_t^i$$

where $0 < \beta < 1$. Suppose that consumer 1 has the endowments

$$(w_0^1, w_1^1, w_2^1, w_3^1, \ldots) = (4,\ 2,\ 4,\ 2, \ldots),$$

and consumer 2 has the endowments

$$(w_0^2, w_1^2, w_2^2, w_3^2, \ldots) = (2,\ 4,\ 2,\ 4, \ldots).$$

a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

b) Define a Pareto efficient allocation for this economy. Calculate a Pareto efficient allocation by maximizing a weighted sum of utilities, $\alpha_1 u_1 + \alpha_2 u_2$.

c) Define an Arrow-Debreu equilibrium with transfers. Find the transfer payments necessary to implement the Pareto efficient allocation in part b as equilibrium with transfers. Demonstrate that the transfer payments are homogeneous of degree one in (α_1, α_2) and sum to 0.

d) Find the transfer payments necessary to implement the allocation $(c_t^1, c_t^2) = (3,\ 3)$ as an equilibrium with transfers.

e) Calculate the (unique) Arrow-Debreu equilibrium of this economy.

f) Define a sequential markets equilibrium. Calculate the unique sequential markets equilibrium of the economy.

2. Consider a simple overlapping generations economy in which the consumer born in period t, $t = 1,\ 2, \ldots$, has the utility function

$$u(c_t^t, c_{t+1}^t) = c_t^t + \left[\left(c_{t+1}^t\right)^b - 1\right]/b,$$

where $b < 1$. Suppose that his endowment is $(w_t^t, w_{t+1}^t) = (w_1, w_2)$.

a) What is the utility function in the case where $b = 0$? [Hint: use l'Hôpital's rule.]

b) Write down the utility maximization problem in an environment with Arrow-Debreu markets. Derive the excess demand functions $y(p_t, p_{t+1})$ and $y(p_t, p_{t+1})$. Demonstrate that they are homogeneous of degree zero and that they satisfy Walras's law.

c) Suppose that the first generation has an excess demand function of the form

$$z_0(p_1, m) = \frac{m}{p_1}.$$

Explain the role of m. Define an Arrow-Debreu equilibrium of this model. Write down the equilibrium conditions using the excess demand functions.

d) Find an expression for the offer curve for this model. (Hint: you have to solve for y as a function of z.)

e) Suppose that $w_1 = 1$ and $w_2 = 0.25$. Draw the offer curve for the three cases $b = 0.5$, $b = 0$, and $b = -1$.

3. Consider an overlapping generations economy in which the representative consumer in generation t, $t = 1, 2, \ldots$, has preferences over the consumption of the single good in each of the two periods of her life given by the utility function

$$u(c_t^t, c_{t+1}^t) = \log\ c_t^t + \log\ c_{t+1}^t.$$

This consumer is endowed with quantities of labor $(\ell_t^t, \ell_{t+1}^t) = (\ell_1, \ell_2)$. In addition there is a generation 0 who representative consumer lives only in period 1 and has the utility function

$$u^0(c_1^0) = \log\ c_1^0,$$

and the endowment of ℓ_2 units of labor and $\bar{k}_1$ units of capital in period 1. In addition, this consumer has an endowment of fiat money m, which can be positive, negative or zero.

The production function is

$$f(k_t, \ell_t) = \theta k_t^{\alpha} \ell_t^{1-\alpha},$$

and capital depreciates at the rate δ per period, $0 \le \delta \le 1$.

a) Define a sequential market equilibrium for this economy.

b) Define an Arrow-Debreu equilibrium for this economy. State and prove two theorems that establish the equivalence between a sequential market equilibrium and an Arrow-Debreu equilibrium.

c) Reduce the equilibrium conditions to a second-order difference equation in k_t, that is, an equation in k_{t+1}, k_t, k_{t-1} that includes no other endogenous variables.

d) Suppose that $m = 0$. Reduce the equilibrium conditions to a first-order difference equation in k_t. (Hint: in this case you know that the savings of generation t in period t in the sequential market equilibrium must equal k_{t+1}.)

4. Suppose that $\theta = 100$, $\alpha = 0.4$, $\delta = 0.8$, $\ell_1 = 1$, and $\ell_2 = 0$ in question 3.

a) Define a steady state for this economy. Calculate the two steady states.

b) Suppose that $\bar{k}_1 = 10$ and $m = 0$. Use your answer to question 3d to calculate the equilibrium in the first 10 periods both by hand and on the computer.

c) Suppose now that $\ell_2 = 0.5$. Repeat parts a and b, doing your calculations on the computer.

d) Suppose now that $\ell_2 = 2$. Repeat parts a and b, doing your calculations on the computer.