PROBLEM SET #3

1. Consider an economy in which the equilibrium solves the optimal growth problem

$$\max \sum_{t=0}^{\infty} \beta^t \log C_t / N_t$$

$$\text{s.t.} \quad C_t + K_{t+1} - (1-\delta)K_t \le (\gamma^{1-\alpha})^t \theta K_t^\alpha N_t^{1-\alpha}$$

$$C_t, K_t \ge 0$$

$$K_0 = \overline{K}_0$$

$$N_t = \eta^t N_0.$$

(a) Define a balanced growth path for this economy. Write down conditions that characterize this balanced growth path. Verify that the balanced growth path exhibits characteristics consistent with Kaldor's stylized facts on economic growth.

(b) Calibrate the parameters of this economy — β, δ, γ, θ, α, and η — so that the balanced growth path matches as closely as possible the national income and product accounts data of some country not the United States.

2. Consider the optimal growth problem

$$\max \sum_{t=0}^{\infty} (0.6)^t \log c_t$$

$$\text{s.t.} \quad c_t + k_{t+1} \le 20 k_t^{0.3}$$

$$c_t, k_t \ge 0$$

$$k_0 = \overline{k}_0.$$

(a) Write down the Euler conditions and the transversality condition for this problem. Calculate the steady state values of c and k.

(b) Write down the functional equation that defines the value function for this problem. Guess that the value function has the form $a_0 + a_1 \log k$. Calculate the value function and the policy function. Verify that the policy function generates a path for capital that satisfies the Euler conditions and transversality condition in part a.

(c) Let capital take values for the discrete grid (2, 4, 6, 8, 10). Make the original guess $V_0(k) = 0$ for all k, and perform the first three steps of the value function iteration

$$V_{i+1}(k) = \max \log(20 k^{0.3} - k') + 0.6 V_i(k').$$

(d) Perform the value function iterations until

$$\max_k \left| V_{i+1}(k) - V_i(k) \right| < 10^{-5}.$$

Report the value function and the policy function that you obtain. Compare these results with what you obtained in part b. (Hint: you probably want to use a computer.)

(e) Repeat part d for the grid of capital stocks $(0.05, 0.10, \ldots, 9.95, 10)$. Compare your answer with those of parts b and d. (Hint: you need to use a computer).

(f) Repeat part e for the problem

$$\max \sum_{t=0}^{\infty} (0.6)^t \log c_t$$

$$\text{s.t.} \quad c_t + k_{t+1} - 0.5k_t \le 20k_t^{0.3}$$

$$c_t, k_t \ge 0$$

$$k_0 = \bar{k}_0.$$

(There is now no comparison with an analytical answer to be made, however.)