MACROECONOMIC THEORY **T.J. KEHOE**
ECON 8105 **FALL 2009**

PROBLEM SET #3

1. Find annual time series data on real output, real investment, hours worked, and working age population for the some country not the United States. (If you cannot find data on hours worked, use data on employment and assume that all workers worked a fixed amount, like 40 hours per week.)

a) Use the data for real investment to construct a series for the capital stock following the rule

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$K_{T_0} = \bar{K}_{T_0} .$$

where T_0 is the first year for which you have data on output and investment. Choose $\bar{K}_{T_0}$ so that

$$K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10} .$$

b) Repeat part a, but choose $\bar{K}_{T_0}$ so that

$$K_{T_0} / Y_{T_0} = \left(\sum_{t=T_0}^{T_0+9} K_t / Y_t \right) / 10 .$$

c) Compare the two series constructed in parts a and b.

2. Suppose that the aggregate production function for the country that you are studying takes the form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} .$$

If you have sufficient data for other variables for this country, calibrate an annual deprecation rate δ and a capital share α. Otherwise, use the values $\delta = 0.05$ and $\alpha = 0.30$ in this question and in question 3.

a) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person:

$$\frac{Y_t}{N_t} = A_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t} \right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}$$

b) Discuss what happens during different time periods.

3. Consider an economy in which the equilibrium solves the optimal growth problem

$$\begin{aligned}
&\max \textstyle\sum_{t=0}^{\infty} \beta^t \left[\theta \log C_t + (1-\theta)\log(N_t \bar{h} - L_t)\right] \\
&\text{s.t.} \quad C_t + K_{t+1} - (1-\delta)K_t \le (\gamma^{1-\alpha})^t A_0 K_t^{\alpha} L_t^{1-\alpha} \\
&C_t, K_t \ge 0 \\
&K_0 = \bar{K}_0 \\
&N_t = \eta^t N_0 .
\end{aligned}$$

a) Define a balanced growth path for this economy. Write down conditions that characterize this balanced growth path. Verify that the balanced growth path exhibits characteristics consistent with Kaldor's stylized facts on economic growth.

b) Calibrate the parameters of this economy — $\beta, \theta, \gamma, A_0$, and η — so that that the behavior of this economy matches that in the data for the country in questions 1 and 2 over some decade. Do the data for this country look like those of a balanced growth path? Discuss.

4. Consider the optimal growth problem

$$\begin{aligned}
&\max \sum_{t=0}^{\infty} (0.6)^t \log c_t \\
&\text{s.t.} \quad c_t + k_{t+1} \le 20 k_t^{0.3} \\
&c_t, k_t \ge 0 \\
&k_0 = \bar{k}_0 .
\end{aligned}$$

a) Write down the Euler conditions and the transversality condition for this problem. Calculate the steady state values of *c* and *k*.

b) Write down the functional equation that defines the value function for this problem. Guess that the value function has the form $a_0 + a_1 \log k$. Calculate the value function and the policy function. Verify that the policy function generates a path for capital that satisfies the Euler conditions and transversality condition in part a.

c) Let capital take values for the discrete grid (2, 4, 6, 8, 10). Make the original guess $V_0(k) = 0$ for all *k*, and perform the first three steps of the value function iteration

$$V_{i+1}(k) = \max \log\left(20k^{0.3} - k'\right) + 0.6 V_i(k').$$

d) Perform the value function iterations until

$$\max_k \left| V_{i+1}(k) - V_i(k) \right| < 10^{-5}.$$

Report the value function and the policy function that you obtain. Compare these results with what you obtained in part b. (Hint: you probably want to use a computer.)

e) Repeat part d for the grid of capital stocks $(0.05, 0.10, \ldots, 9.95, 10)$. Compare your answer with those of parts b and d. (Hint: you need to use a computer).

f) Repeat part e for the problem

$$\begin{aligned} &\max \sum_{t=0}^{\infty} (0.6)^t \log c_t \\ \text{s.t.} \quad & c_t + k_{t+1} - 0.5k_t \le 20k_t^{0.3} \\ & c_t, k_t \ge 0 \\ & k_0 = \bar{k}_0 . \end{aligned}$$

(There is now no comparison with an analytical answer to be made, however.)