

PROBLEM SET #5

1. Consider an economy like that in question 3 on problem set #4 in which the equilibrium allocation is the solution to the optimal growth problem

$$\begin{aligned} \max E \sum_{t=0}^{\infty} \beta^t & \left[ \theta \log C_t + (1-\theta) \log(N_t \bar{h} - L_t) \right] \\ \text{s.t. } C_t + K_{t+1} - (1-\delta)K_t & \leq e^{\bar{z}_t} (\gamma^{1-\alpha})^t A_0 K_t^\alpha L_t^{1-\alpha} \\ C_t, K_t & \geq 0 \\ K_0 & = \bar{K}_0 \\ N_t & = \eta^t N_0. \end{aligned}$$

Here  $z_t$  is a random variable that takes on two values  $\bar{z}_1 = -\zeta$ ,  $\bar{z}_2 = \zeta$ , and whose evolution is governed by the stationary, first order Markov chain with transition matrix

$$\Pi = \begin{bmatrix} 1-\pi & \pi \\ \pi & 1-\pi \end{bmatrix}.$$

Assume, for the sake of specificity, that, at  $t=0$ ,  $z_0 = \bar{z}_1 = -\zeta$ .

- Define an Arrow-Debreu equilibrium for this economy.
- Define a sequential markets equilibrium for this economy.

Redefine variables  $C_t$  and  $K_t$  by dividing by the number of effective working age persons  $\tilde{N}_t = \gamma^t N_t = (\gamma\eta)^t N_0$ . Divide  $L_t$  by  $N_t$ :

$$\begin{aligned} c_t & = C_t / \tilde{N}_t = \gamma^{-t} (C_t / N_t) \\ k_t & = K_t / \tilde{N}_t = \gamma^{-t} (K_t / N_t) \\ \ell_t & = L_t / N_t. \end{aligned}$$

Consider the social planner's problem

$$\begin{aligned} \max E \sum_{t=0}^{\infty} \beta^t & \left[ \theta \log c_t + (1-\theta) \log(\bar{h} - \ell_t) \right] \\ \text{s.t. } c_t + \gamma\eta k_{t+1} - (1-\delta)k_t & \leq e^{\bar{z}_t} A_0 k_t^\alpha \ell_t^{1-\alpha} \\ c_t, k_t \geq 0, \bar{h} \geq \ell_t & \geq 0 \\ k_0 & = \bar{K}_0 / N_0, \end{aligned}$$

with the associated Bellman's equation

$$\begin{aligned}
V(k, z) &= \max \theta \log c + (1-\theta) \log(\bar{h} - \ell) + \beta EV(k', z') \\
\text{s. t. } &c + \gamma \eta k' - (1-\delta)k \leq e^z A_0 k^\alpha \ell^{1-\alpha} \\
&c, k' \geq 0, \bar{h} \geq \ell \geq 0 \\
&k, z \text{ given.}
\end{aligned}$$

c) Suppose that you have solved this dynamic programming problem and have found the policy functions  $k' = g(k, z)$ ,  $c = c(k, z)$ , and  $\ell = \ell(k, z)$ . Explain how you can use these policy functions to calculate the Arrow-Debreu equilibrium. Explain how you can use these policy functions to calculate the sequential markets equilibrium.

2. Consider an economy like that in question 1 in which the equilibrium allocation is the solution to the optimal growth problem with the Bellman's equation

$$\begin{aligned}
V(k, z) &= \max \theta \log c + (1-\theta) \log(\bar{h} - \ell) + \beta EV(k', z') \\
\text{s. t. } &c + \gamma \eta k' \leq e^z A_0 k^\alpha \ell^{1-\alpha} \\
&c, k' \geq 0, \bar{h} \geq \ell \geq 0 \\
&k, z \text{ given.}
\end{aligned}$$

That is,  $\delta = 1$ . Guess that the value function  $V(k, z)$  has the form

$$V_i(k) = V(k, z_i) = a_{0i} + a_{1i} \log k, \quad i = 1, 2$$

for the yet-to-be-determined coefficients  $a_{01}, a_{11}, a_{02}, a_{12}$ .

a) Solve for the value functions  $V_i(k)$ ,  $i = 1, 2$ .

b) Solve for the policy functions  $k' = g_i(k)$ ,  $c = c_i(k)$ , and  $\ell = \ell_i(k)$ .

3. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage  $w$  drawn independently from the time invariant probability distribution  $F(v) = \text{prob}(w \leq v)$ ,  $v \in [0, B]$ ,  $B > 0$ . After receiving the wage offer  $w$  the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit  $b$ , and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}.$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where  $1 > \beta > 0$ . Once a job offer has been accepted, there are no fires or quits.

a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

- b) Using Bellman's equation from part a, characterize the value function  $V(w)$  in a graph and argue that the worker's problem reduces to determining a reservation wage  $\bar{w}$  such that she accepts any wage offer  $w \geq \bar{w}$  and rejects any wage offer  $w < \bar{w}$ .
- c) Consider two economies with different unemployment benefits  $b_1$  and  $b_2$  but otherwise identical. Let  $\bar{w}_1$  and  $\bar{w}_2$  be the reservation wages in these two economies. Suppose that  $b_2 > b_1$ . Prove that  $\bar{w}_2 > \bar{w}_1$ . Provide some intuition for this result.
- d) Consider two economies with different wage distributions  $F_1$  and  $F_2$  but otherwise identical. Let  $\bar{w}_1$  and  $\bar{w}_2$  be the reservation wages in these two economies. Define a mean preserving spread. Suppose that  $F_2$  is a mean preserving spread of  $F_1$ . Prove that  $\bar{w}_2 > \bar{w}_1$ . Provide some intuition for this result.