MACROECONOMIC THEORY **T. J. KEHOE**
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PROBLEM SET #5

1. Consider an economy like that in question 3 on problem set #4 in which the equilibrium allocation is the solution to the optimal growth problem

$$\begin{aligned} &\max E\sum_{t=0}^{\infty}\beta^t\left[\theta\log C_t+(1-\theta)\log(N_t\bar{h}-L_t)\right] \\ &\text{s.t.}\quad C_t+K_{t+1}-(1-\delta)K_t\le e^{z_t}(\gamma^{1-\alpha})^t A_0K_t^{\alpha}L_t^{1-\alpha} \\ &C_t,\,K_t\ge 0 \\ &K_0=\bar{K}_0 \\ &N_t=\eta^t N_0\,. \end{aligned}$$

Here z_t is a random variable that takes on two values $\bar{z}_1=-\zeta$, $\bar{z}_2=\zeta$, and whose evolution is governed by the stationary, first order Markov chain with transition matrix

$$\Pi=\begin{bmatrix}1-\pi & \pi\\ \pi & 1-\pi\end{bmatrix}.$$

Assume, for the sake of specificity, that, at $t=0$, $z_0=\bar{z}_1=-\zeta$.

a) Define an Arrow-Debreu equilibrium for this economy.

b) Define a sequential markets equilibrium for this economy.

Redefine variables C_t and K_t by dividing by the number of effective working age persons $\tilde{N}_t=\gamma^t N_t=(\gamma\eta)^t N_0$. Divide L_t by N_t:

$$\begin{aligned} c_t&=C_t/\tilde{N}_t=\gamma^{-t}(C_t/N_t)\\ k_t&=K_t/\tilde{N}_t=\gamma^{-t}(K_t/N_t)\\ \ell_t&=L_t/N_t\,. \end{aligned}$$

Consider the social planner's problem

$$\begin{aligned} &\max\ E\sum_{t=0}^{\infty}\beta^t\left[\theta\log\ c_t+(1-\theta)\log(\bar{h}-\ell_t)\right] \\ &\text{s. t.}\ \ c_t+\gamma\eta k_{t+1}-(1-\delta)k_t\le e^{z_t}A_0k_t^{\alpha}\ell_t^{1-\alpha} \\ &c_t,\ k_t\ge 0,\ \bar{h}\ge\ell_t\ge 0 \\ &k_0=\bar{K}_0/N_0\,, \end{aligned}$$

with the associated Bellman's equation

$$V(k,z) = \max \ \theta \log \ c + (1-\theta)\log(\bar{h} - \ell) + \beta E V(k', z')$$
$$\text{s. t. } \ c + \gamma\eta k' - (1-\delta)k \le e^{z} A_0 k^{\alpha} \ell^{1-\alpha}$$
$$c, \ k' \ge 0, \ \bar{h} \ge \ell \ge 0$$
$$k, \ z \ \text{given.}$$

c) Suppose that you have solved this dynamic programming problem and have found the policy functions $k' = g(k,z)$, $c = c(k,z)$, and $\ell = \ell(k,z)$. Explain how you can use these policy functions to calculate the Arrow-Debreu equilibrium.

d) Explain how you can use these policy functions $k' = g(k,z)$, $c = c(k,z)$, and $\ell = \ell(k,z)$ from part c to calculate the sequential markets equilibrium.

2. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \le v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}.$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

b) Using Bellman's equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \ge \bar{w}$ and rejects any wage offer $w < \bar{w}$.

c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that F_2 is a mean preserving spread of F_1. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.